

Statistical properties of reversals and chrons in numerical dynamos and implications for the geodynamo

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ABSTRACT

We analyse a series of very long runs (equivalent to up to 50 Myr) produced by chemically-driven dynamos. All runs assume homogeneous boundary conditions, an electrically conducting inner-core (except for one run) and only differ by the choice of the Rayleigh number Ra^* . Introducing dynamo-based definitions of reversals, chrons and related concepts, such as “failed reversals” and “segments” (bounded by reversals or failed reversals), we investigate the distributions of chron and segment lengths, those of reversal and failed reversal durations, the way dipole field behaves through reversals and failed reversals, and the possible links between the axial dipole intensity and chron or segment lengths. We show that chron and segment lengths are very well described in terms of a Poisson process (with no occurrence of superchrons), while distributions of reversal and failed reversal durations are better fitted by log-normal distributions. We found that reversal rates generally increase in proportion to $Rm - Rm_c$, Rm being the magnetic Reynolds number and Rm_c a critical value. In contrast, reversal and failed reversal durations appear to be mainly controlled by the core’s magnetic diffusion timescale. More generally, we show that much of the reversing behaviour of these dynamos can be understood by examining their signature in a (g_1^0, g_1^1, h_1^1) phase-space plot. This reveals that the run with an insulating inner-core is very different and has only two distinct modes of opposite polarity, which we argue is the reason it displays less reversals and failed reversals, and has a clear tendency to produce an intensity “overshoot” and some systematic pattern in the dipole pole behaviour through reversals and failed reversals. This contrasts with conducting inner-core runs, which display an additional central unstable mode, the importance of which increases with Rm , and which is responsible for the more complex reversing behaviour of these dynamos. Available paleomagnetic data suggest that the current geodynamo could have such a (small) central mode, which would thus imply a strong sensitivity of the frequency and complexity of reversals and of the likelihood of failed reversals, to changes in the geodynamo’s driving parameters through geological times.

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1. Introduction

Since the advent of the first 3D numerical geodynamo simulations (Glatzmaier and Roberts, 1995a,b; Kageyama et al., 1995), numerous dynamo models have been published, gradually succeeding in reproducing salient properties of the geomagnetic field, such as polarity reversals (see, e.g., Christensen et al., 2007; Wicht et al., 2009, 2010 for reviews). Despite limitations in computing capacities – the explored parameter space is still very remote from where the Earth’s dynamo lies and an excessive viscosity is often used to suppress small scales that cannot be resolved – investiga-

tion of these dynamos have also already led to some understanding of the scaling laws controlling the evolution of key dynamo quantities (e.g. Christensen and Aubert, 2006; Olson and Christensen, 2006; Aubert et al., 2009) and to the identification of parameter regimes hopefully relevant for geophysical investigations (Christensen et al., 2010). Some studies focused on describing the mechanism of reversals and excursions (e.g. Wicht and Olson, 2004; Aubert et al., 2008a; Olson et al., 2011), while others strove to assess the effect of boundary conditions (e.g. Glatzmaier et al., 1999; Olson and Christensen, 2002; Aubert et al., 2008b; Takahashi et al., 2008). Thanks to a permanent increase in computational power, it has also become possible to produce long dynamo solutions over a time equivalent to tens of millions of years (e.g. Wicht et al., 2009; Driscoll and Olson, 2011). These solutions have been used to investigate factors possibly controlling the reversal frequency, with some emphasis on the role played by the inhomogeneous heat flow pattern at the core-mantle boundary,

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in attempts to account for the observed sequence of reversals (e.g. Driscoll and Olson, 2009a,b, 2011; Olson et al., 2010, 2013).

In this paper, we do not seek to reproduce the reversal sequence of the geomagnetic field. Rather, we aim at better characterising reversal and chron properties in very long runs produced by such models, and intend to investigate how these properties depend on parameters such as the conducting or non-conducting nature of the inner-core, or the strength of dynamo forcing. We first focus on the distribution of chron lengths – a topic which has already been investigated in numerous studies, both in paleomagnetism and numerical dynamo simulations (see, e.g., Amit et al., 2010 for a review). In the present context, however, we further introduce the concept of “segments” bounded by either a “successful” or “failed” reversal (defined in a way we later specify) and investigate the distribution of such segments. We next explore the properties of reversals and such “failed” reversals, particularly the distribution of their duration and the way the dipole field behaves through such events, looking for special features such as overshoots, similar to what the data suggests occurs during geomagnetic reversals (e.g. Valet et al., 2005, 2012) and to what was found in experimental dynamos and simple models (e.g. Pétrelis et al., 2009). We also investigate possible correlations between chron (or segment) length and field intensity – a question puzzling paleomagnetists and still essentially unsettled (see, e.g., Tauxe et al., 2007 for a review).

To this purpose, we rely on a number of dynamo simulations with homogeneous boundary conditions run over periods of time equivalent to 40–50 Myr. One simulation is with an insulating inner-core. This control run allows us to investigate the impact of this assumption, often used in recent analyses of long dynamo runs (e.g. Driscoll and Olson, 2009a,b; Olson et al., 2010). The other simulations are with a conducting inner-core and only differ by their forcing amplitude (i.e. the value of the Rayleigh number). They will allow us to investigate the impact of this parameter.

The paper is organised as follows. We first present the numerical models and the various diagnostic parameters and statistical tools we employ, placing emphasis on the definitions we use for the concepts of reversal, failed reversal, stable low intensity event, chron and segment (Section 2). We next present the results of our investigations of chron and segment lengths (Section 3.1), of reversal and failed reversal durations (Section 3.2), of dipole field behaviour through such events (Section 3.3), and of possible links between the field intensity and chron and segment lengths (Section 3.4), before putting these results in perspective with the help of useful phase-space plots (Section 3.5). Finally, these results are discussed (Section 4) and their implications for paleomagnetism and the geodynamo provided with concluding comments (Section 5).

2. Models and tools

2.1. Numerical models

We use the code PARODY (Dormy et al., 1998; Aubert et al., 2008a) and consider simulations close to the chemically-driven dynamo initially studied by Olson (2007) and extensively used to tentatively account for the geomagnetic polarity timescale (Driscoll and Olson, 2009a,b, 2011; Olson et al., 2010, 2011). The choice of these authors was mainly motivated by the fact that, contrary to thermally-driven dynamos having the usual drawback of being weakly dipolar when displaying polarity reversals (Kutzner and Christensen, 2002), chemically-driven dynamos are more strongly dipolar. These dynamos are also not far from the Earth-like path described by Christensen et al. (2010), and relatively cheap to compute. They are from these standpoints convenient candidates for comparison with paleomagnetic data. As they suffer from the clas-

sical shortcomings of numerical dynamos (e.g. Christensen, 2011), comparison with such data must naturally be done with great care.

Following the conventions of Aubert et al. (2008a), the four control parameters of the simulations are the modified Rayleigh number Ra^* (a measure of the strength with which the dynamo is driven), the Ekman number E (a measure of the relative importance of viscous to Coriolis forces), the hydrodynamic Prandtl number Pr (ratio between kinematic and thermal diffusivities) and the magnetic Prandtl number Pm (ratio between kinematic and magnetic diffusivities). Our numerical dynamos are all characterised by the fixed parameters $E = 6.5 \times 10^{-3}$, $Pr = 1$ and $Pm = 20$. Table 1 provides the Ra^* used for each simulation, together with the following relevant output parameters: the magnetic Reynolds number $Rm = UD/\eta$, where U is the time-averaged rms velocity over the outer-core shell, D the shell thickness (we assume an aspect ratio of 0.35 as in Aubert et al., 2008a) and η the magnetic diffusivity; the dipolarity f_{dip}^{13} defined as the time-averaged rms amplitude of the dipole relative to the field up to degree 13 at the core-mantle boundary; and f_{ax}^{eq} , the ratio of the equatorial dipole to the axial dipole at the core-mantle boundary.

The first model has an insulating inner-core and is virtually identical to the dynamos investigated by Olson (2007) and Olson et al. (2009). The others have a conducting inner-core of the same conductivity as the outer-core and operate in the same parameter range as the dynamos investigated by Driscoll and Olson (2011) and Olson et al. (2011); Olson et al., 2013. Note that the additional studies of Driscoll and Olson (2009a,b) and Olson et al. (2010) are also in the same parameter range, but assume an insulating inner-core.

All our simulations operate with a lateral resolution of 44 spherical harmonic degrees, which ensures that the ratio between the minimum and maximum of the spatial power spectra (integrated over the outer-core's shell) is typically higher than 250000 for the velocity field and higher than 2500 for the magnetic field. Unless otherwise noted, all magnetic quantities are expressed in units of $(\rho\mu_0\eta\Omega)^{1/2}$ (as in Christensen et al. (2001), where ρ is the core density, μ_0 the magnetic permeability, and Ω the rotation rate assumed for the dynamo) and all temporal quantities are expressed in units of the magnetic diffusion time $\tau_\eta = D^2/\eta$. The time steps, which are determined according to an adaptive criterion (Christensen et al., 1999), are on the order of $1.25 \times 10^{-5}\tau_\eta$ for the model with an insulating inner-core, on the order of $8.75 \times 10^{-6}\tau_\eta$ for the models with a conducting inner-core.

2.2. Timescales of interest

To describe the timescales of the Earth's magnetic field for a given spherical harmonic degree n , it is convenient to define the correlation times with

$$\tau_n = \sqrt{\frac{\langle \sum_{m=0}^n [(g_n^m)^2 + (h_n^m)^2] \rangle}{\langle \sum_{m=0}^n [(\dot{g}_n^m)^2 + (\dot{h}_n^m)^2] \rangle}}, \quad (1)$$

in which $\{g_n^m, h_n^m\}$ and $\{\dot{g}_n^m, \dot{h}_n^m\}$ are respectively the Gauss coefficients and their time derivatives of spherical harmonic degree n and order m , whereas the angle brackets denote time averaging (Hulot and Le Mouél, 1994; Christensen and Tilgner, 2004). They provide a statistical measure of how long it would take for the observed field at a given spherical harmonic degree n to be completely renewed, and decrease with increasing spherical harmonic degree (meaning that, the smaller the length scales, the shorter the correlation times). Similar quantities τ_n can be defined for the field produced by any dynamo. Considering both numerical geodynamo simulations and geomagnetic field models, it was shown that this decrease is statistically compatible with an inverse linear law $\tau_n = \tau_{SV}/n$ ($n \geq 2$), whose single parameter

Table 1

Properties of the numerical models under investigation. The modified Rayleigh number Ra^* is an input parameter. The magnetic Reynolds number Rm , the dipolarity f_{dip}^{13} and the ratio f_{ax}^{eq} are all output parameters, just like τ_{SV} (secular-variation timescale) and τ_e (error-growth timescale). T is the total duration of each run. All times are expressed in units of the magnetic diffusion time τ_η (see main text for complete definitions).

Model	Inner-core	T/τ_η	Ra^*	Rm	f_{dip}^{13}	f_{ax}^{eq}	τ_{SV}/τ_η	τ_e/τ_η
(1)	Insulating	3790	5.5×10^{-3}	110	0.58	0.25	2.90×10^{-2}	6.50×10^{-2}
(2)	Conducting	3150	5.1×10^{-3}	108	0.57	0.28	3.05×10^{-2}	8.50×10^{-2}
(3)	Conducting	2400	5.5×10^{-3}	117	0.54	0.33	2.85×10^{-2}	7.75×10^{-2}
(4)	Conducting	2690	5.9×10^{-3}	124	0.54	0.33	2.65×10^{-2}	7.00×10^{-2}
(5)	Conducting	2490	6.3×10^{-3}	133	0.50	0.33	2.40×10^{-2}	4.25×10^{-2}

τ_{SV} is then known as the secular-variation timescale (Lhuillier et al., 2011a). This parameter τ_{SV} can thus conveniently be used as an alternative to τ_η to scale time in dynamo outputs, and ensure consistency between model and the Earth's secular-variation timescales, an important issue for geomagnetic data assimilation approaches (Fournier et al., 2010, 2011; Aubert and Fournier, 2011). Based on the “observatory” part (i.e. the period 1840–1990) of *gufm1* (Jackson et al., 2000), τ_{SV} was found to be on the order of 500 yr for the geodynamo. We will use this value when converting time in years (see, e.g. Fig. 3).

Another timescale of interest in numerical geodynamo simulations is the error-growth timescale τ_e , which corresponds to the inverse of the average growth rate at which small errors introduced in a reference solution are exponentially amplified. This growth rate is a robust quantity independent of the conditions (type, amplitude, time) under which the small errors are introduced, and thus gives an insight into the predictability of the model (Hulot et al., 2010a; Lhuillier et al., 2011b). Contrary to the correlation times, this timescale cannot be obtained as a direct output of the simulation, but needs the introduction of small controlled perturbations in a reference solution and the study of the subsequent divergence between reference and perturbed solutions.

In this paper, both the secular-variation timescale τ_{SV} and the error-growth timescale τ_e will be of interest. Their non-dimensional values are provided in Table 1 for the numerical models under investigation. The timescale τ_{SV} is a temporal average over the entire solution, whereas τ_e is a mean timescale deduced from ten independent measurements of the growth rate of small errors of relative amplitude 10^{-10} introduced at ten distinct times of the simulation, within periods of stable polarity.

2.3. Defining reversals, failed reversals, stable low intensity events, chrons and segments

In order to define and detect reversals, “failed” reversals and “stable low intensity” events in our dynamo solutions, we adopt the following algorithm in six steps:

1. estimation of the mean μ and the standard deviation σ of the absolute value of the axial dipole moment (ADM) over the whole solution;
2. determination of time intervals $\mathcal{I}_i = [a_i; b_i]$ of low intensity during which the ADM is lower than $\mu - \sigma$;
3. detection of reversals, “failed” reversals and “stable low intensity” events among the low-intensity zones $\mathcal{I}_i$:
 - a reversal $\mathcal{R}_i = [a_i; b_i]$ (of duration $\Delta t = b_i - a_i$) is identified if the Dipole Pole (DP, defined as the outgoing pole of the dipole field) is in a different hemisphere at $t = a_i$ and $t = b_i$ (see blue zones in Fig. 1(a));
 - a failed reversal $\mathcal{F}_i = [a_i; b_i]$ (of duration $\Delta t = b_i - a_i$) is identified if the Dipole Pole is in the same hemisphere at $t = a_i$ and $t = b_i$, and has reached the equator at least once within $\mathcal{I}_i$ (see dark gray zones in Fig. 1(a));

- remaining low intensity intervals $\mathcal{I}_i = [a_i; b_i]$ (of duration $\Delta t = b_i - a_i$) during which the ADM is below $\mu - \sigma$ but never reaches zero will be referred to as Stable Low Intensity (SLI) events (see light grey zones in Fig. 1(a)).
4. determination of chrons $C_i = [b_{i-1}; a_i]$ as time intervals between two consecutive successful reversals, and of segments $S_i = [b_{i-1}; a_i]$ as time intervals between two consecutive successful or failed reversals (see arrows in Fig. 1(a));
 5. facultative step: filtering of segments shorter than Δt (see below for details);

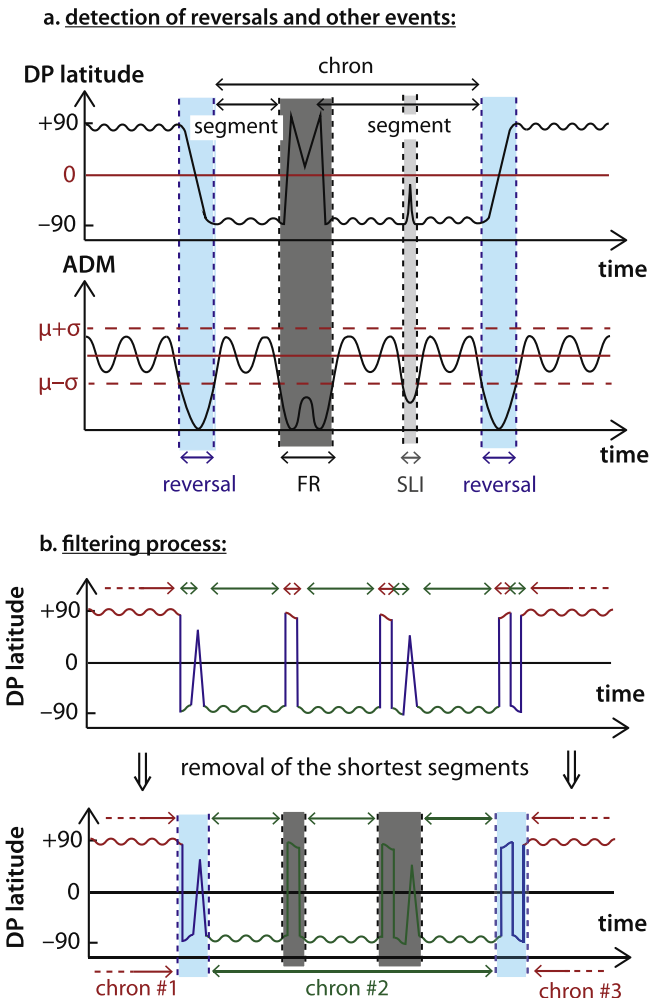


Fig. 1. Sketch of the algorithm employed: (a) detection of reversals, failed reversals (FR) and stable low intensity (SLI) events, according to a criterium based on both intensity and polarity; (b) filtering process used to merge the shortest segments with reversals or failed reversals (see Section 2.3 for more details).

6. computation of μ and σ during segments only (i.e. excluding time spent in either successful or failed reversals), and repetition four times of steps 2–5 to ensure convergence of the algorithm.

The facultative step implying a filtering process is illustrated in Fig. 1(b). If several successive segments are shorter than Δt , the segments are merged to form a single reversal or failed reversal (depending on the polarity of the previous and next segments longer than Δt). In this paper, we alternatively take $\Delta t = 0$ (case a, no filtering), Δt equal to the shortest duration of a reversal (case b) or Δt equal to the likeliest duration of reversal (case c). Note that there is no universal way to define reversals, failed reversals and SLI events, and that other choices are possible (e.g. Wicht, 2005; Driscoll and Olson, 2009a). Our choice is motivated by the fact that we want to have a simple way to define events, only based on threshold values coming from the numerical model itself, and not from a priori values (such as the angle of departure from the geomagnetic pole for the latitude criterium, or some pre-defined arbitrary minimal duration allowed for chrons).

2.4. Statistical distributions of interest

Given a sequence of reversals, it is possible to proceed to a statistical characterisation of the chron length, which can indirectly provide information on the reversal process. The simplest characterisation involves considering a Poisson process (Cox, 1968, 1969), according to which the probability density function (pdf) of the chron length x is

$$p(x) = \frac{1}{\mu^*} \exp\left(-\frac{x}{\mu^*}\right), \quad (2)$$

where μ^* is the expected mean chron length. Then, the dynamo is assumed to have no memory of the previous event, and the probability that a reversal occurs does not depend on the time elapsed since the last reversal (e.g., Merrill et al., 1996). A more general way to account for the distribution of chron length involves considering a Gamma process (Naidu, 1971; Phillips, 1977), according to which the pdf of the chron length x is

$$p(x) = \left(\frac{k}{\mu}\right)^k \frac{x^{k-1}}{\Gamma(k)} \exp\left(-k\frac{x}{\mu}\right), \quad (3)$$

where $\Gamma(k)$ is the Gamma function of real index k , and μ the expected mean chron length. The Poisson process is an end-member case of the Gamma process when $k = 1$. But, contrary to the Poisson event, the Gamma process has a memory of the previous event when $k \neq 1$ (e.g. Merrill et al., 1996). A value of $k > 1$ inhibits the probability to reverse just after the previous reversal, whereas a value of $k < 1$ encourages the dynamo to reverse just after the previous reversal. For the sake of clarity, we note hereinafter k_{chr} and μ_{chr} the estimated parameters of the Gamma distribution for the chron length (i.e. characterising the process responsible for just reversals, independently of failed reversals that may occur meanwhile). By extension, we also define the estimated parameters k_{seg} and μ_{seg} , still computed from Eq. (3) but for the segment length (i.e. characterising a possible common process responsible for both reversals and failed reversals).

Concerning the duration of reversals and failed reversals themselves (i.e. the lengths of the $\mathcal{R}_i$ and $\mathcal{E}_i$ intervals as defined in the previous section), we shall see in Section 3 that they can reasonably be modelled by a log-normal distribution, according to which the pdf of the duration x is

$$p(x) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\ln(x/m)}{\sigma}\right)^2\right], \quad (4)$$

where m is the geometric mean (and also the median) of the distribution and σ the scale parameter.

3. Results

For the five numerical models defined in Table 1, we first checked that the secular-variation timescale τ_{SV} is to first order inversely proportional to the magnetic Reynolds number Rm (Fig. 2(a), in accordance with Christensen and Tilgner, 2004; Lhuillier et al., 2011b). We also checked that the error-growth timescale τ_e is roughly inversely proportional to Rm (Fig. 2(b), in accordance with Hulot et al., 2010a; Lhuillier et al., 2011b). Note in particular that model (1) with an insulating inner core leads to τ_{SV} and τ_e values quite consistent with the trends defined by the four other models, all with a conducting inner core. Note also that all these chemically-driven dynamos operate with low Rm and produce a τ_e/τ_{SV} ratio on the order of 2, which is much larger than the value of 0.05 expected for dynamos with low Ekman number, such as the geodynamo (Hulot et al., 2010a).

Having characterised the τ_{SV} and τ_e timescales, we next applied the algorithm detailed in Section 2.3 to all numerical models. Portions of the temporal evolution of models (1), (2) and (5) are shown in Fig. 3 in the case of filtering (b) to illustrate the behaviour of these dynamos and provide examples of reversals, failed reversals and SLI events identified by our algorithm. These are the events we now wish to characterise in statistical terms. Parameters recovered from such analysis are all reported in Table 2.

3.1. Distribution of chron and segment lengths

In this section, we first deal with the distribution of chron and segment lengths. For each model and each of the three filtering lev-

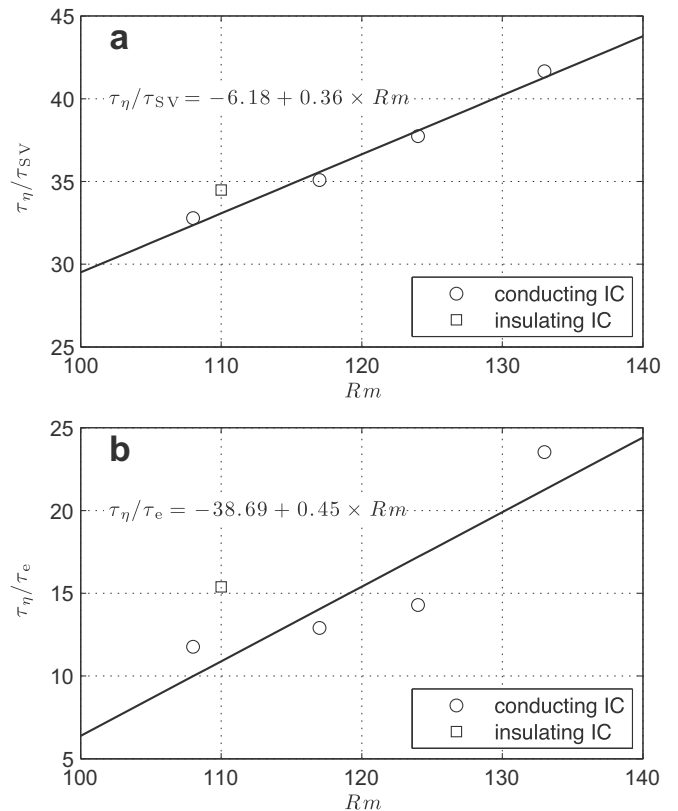


Fig. 2. $\tau_{\eta}/\tau_{\text{SV}}$ (inverse of the secular-variation timescale) and τ_{η}/τ_e (inverse of the error-growth timescale) as a function of Rm for the five numerical models under investigation (insert: best linear fits, also plotted as a line).

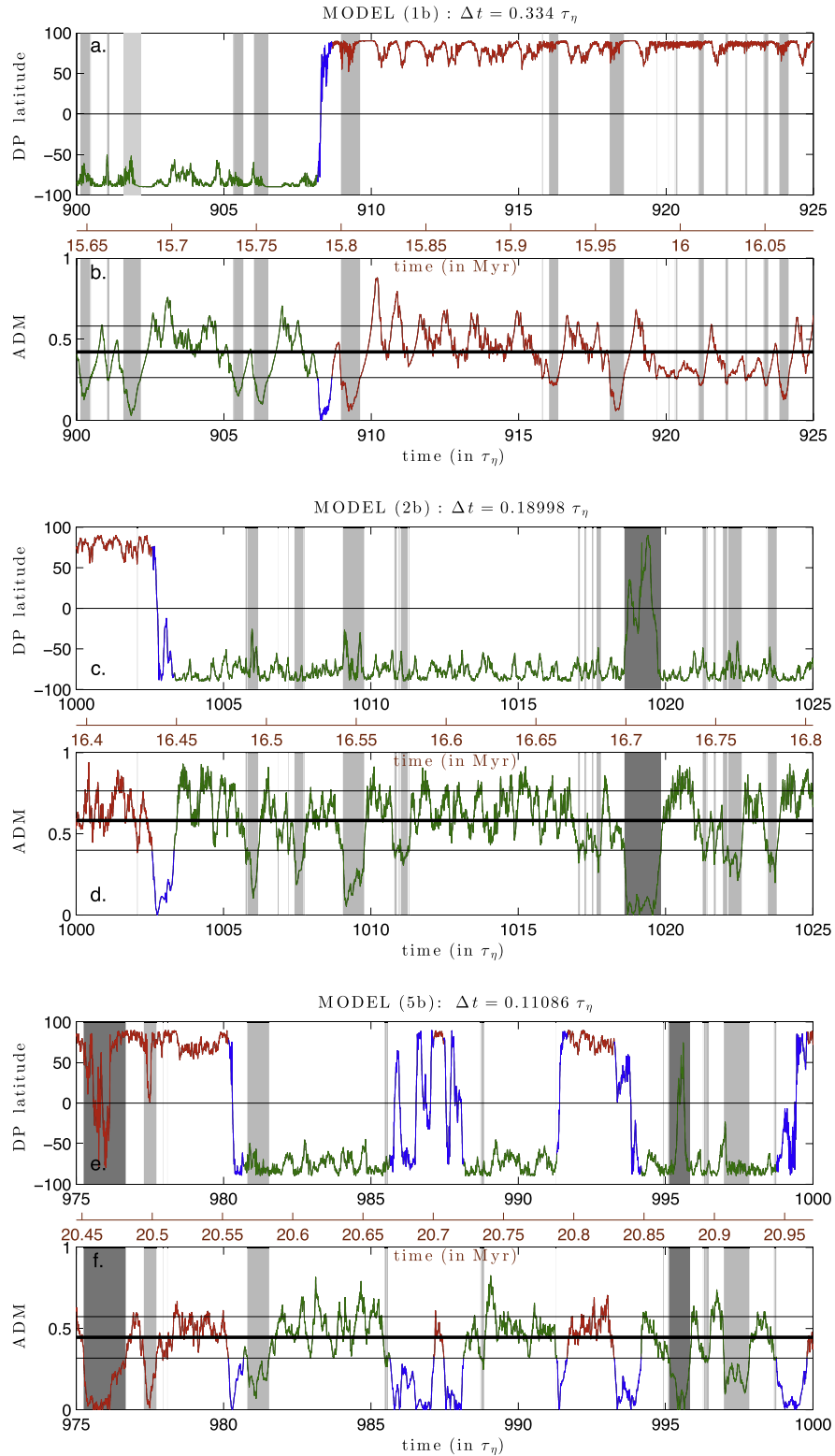


Fig. 3. Examples of the temporal evolution of the Dipole Pole (DP) latitude and of the ADM for models (1), (2) and (5), with events identified according to the algorithm detailed in Section 2.3 for filtering (b) (i.e. by the shortest duration of reversals, the Δt value of which is also provided): chrons of positive polarity are depicted in red, chrons of negative polarity in green, and reversals in blue. Failed reversals during chrons are further highlighted in dark grey, while Stable Low Intensity (SIL) events are highlighted in light grey. The time is expressed both in units of the magnetic diffusion time τ_η (black axis) and in units of equivalent Myr (red axis) using a scaling based on the secular-variation timescale τ_{SV} set to 500 yr (see Section 2.2). (For interpretation of the references to colour in this figure caption, the reader is referred to the web version of this article.)

els mentioned in Section 2.3, we search for Gamma distributions that best fit the distributions of chron and segment lengths. Fig. 4 shows examples of such distributions and fits for models

(1), (2) and (5) in the case of filtering (b). As can be seen, Gamma fits always provide very decent representations of the distributions. Table 2 lists the corresponding maxima of likelihood for

parameters k and μ (as defined by Eq. (3), with their 95% dispersion bars) for all models and filterings.

First focus on the distribution of chron lengths (Fig. 4(a), (c) and (e); and Table 2). One can see that, for any given numerical model, k_{chr} always increases with the filtering level, as expected (McFadden, 1984). For model (2) for instance, k_{chr} evolves from $0.84 \pm_{0.13}^{0.11}$ in the absence of filtering to $1.28 \pm_{0.22}^{0.19}$ for filtering (c), with an intermediate value of $1.10 \pm_{0.18}^{0.15}$ for filtering (b). The k_{chr} index, however, is always close to a unit value, implying that the distributions are never far from being Poissonian. Interestingly, filtering by the shortest duration of reversals (filtering (b)) always leads to a match with a Poisson distribution. This strongly suggests, first, that chrons lasting less than the quickest of all reversals should indeed be considered as part of an ongoing reversal, and second, that chrons do end (and therefore reversals occur) as a result of a process with no memory beyond the length of a reversal.

Considering the distribution of segment lengths is also of interest (Fig. 4(b), (d) and (f); and Table 2). Such segments differ from chrons only by the fact that they start and end by either a reversal or a failed reversal. Just like for chrons, the distributions of segment lengths can be well fitted by Gamma distributions, with k_{seg} again close to a unit value and filtering (b) always leading to distributions closest to Poissonian (see Table 2). This observation strongly suggests that a common process is responsible for producing both reversals and failed reversals, i.e., events characterised by the DP latitude crossing zero, with no memory beyond the length of the events themselves. Once such an event has ended, the dynamo is back in a state where another similar event has a constant instantaneous probability of occurrence, as if resulting from a Poisson process. Note that these observations do not imply that all events are equally likely to end up as reversals or failed reversals. As it turns out, this is generally not the case.

The ratio $N_{\text{frev}}/N_{\text{rev}}$ of the number of failed reversals to the number of reversals illustrates this point (see Table 2). If one considers filtering (b) as most appropriate, $N_{\text{frev}}/N_{\text{rev}}$ is on the order of 0.20 for model (1) with an insulating inner-core, and range from 0.44 to 0.57 for models (2) to (5) with a conducting inner-core. This shows that once the DP latitude has reached zero, the dynamos under investigation are more likely to proceed to a full reversal than to revert to their original polarity. This is particularly true for model (1), which hardly produces any failed reversals, in contrast to models (2) and (3), though models (1) and (2) share very similar

magnetic Reynolds number Rm , and models (1) and (3) are driven by the same modified Rayleigh number Ra^* (recall Table 1). Clearly, the conducting or non-conducting character of the inner-core plays an important role in defining the proportion of failed and successful reversals.

Other parameters worth investigating are the expected mean chron (μ_{chr}) and segment (μ_{seg}) lengths recovered from the Gamma fits (see Table 2). Plotting $\tau_{\eta}/\mu_{\text{chr}}$ (i.e. the average reversal frequency scaled with respect to the diffusion time) as a function of Rm (Fig. 4(g)) again highlights the strong contrast between model (1) and all other models. Chrons in model (1) roughly last three times longer than in model (2), and six times longer than in model (3). Fig. 4(g) otherwise suggests a general increasing trend of $\tau_{\eta}/\mu_{\text{chr}}$ with Rm (and therefore Ra^*) for all dynamos with a conducting inner-core, in line with the findings of Driscoll and Olson (2011).

Plotting $\tau_{\eta}/\mu_{\text{seg}}$ as a function of Rm (Fig. 4(h)) brings further insight. Because so few failed reversals occur in the case of model (1), the contrast between segment lengths produced by this model and models (2) and (3) is even stronger (with factors on the order of 3.6 and 8). But what best emerges from Fig. 4(h) is a much clearer linear trend for $\tau_{\eta}/\mu_{\text{seg}}$ as a function of Rm than for $\tau_{\eta}/\mu_{\text{chr}}$. This result can be understood in view of our previous findings. If both reversals and failed reversals are initially triggered as a result of a common Poisson process, time spent in failed reversals should not be counted in the duration of the chron within which the failed reversal occurred. Chron lengths thus happen to be slightly biased towards larger values, especially when the duration of reversals and failed reversals becomes significant compared to the average chron or segment lengths. As we shall later see, this is the case for models (4) and particularly (5), both of which thus tend to produce $\tau_{\eta}/\mu_{\text{chr}}$ biased towards weaker values. Using the more reliable linear trend inferred from Fig. 4(h), we finally note that this trend suggests that reversal and failed reversal frequencies could drop to zero when reaching $Rm = Rm_c \approx 90$. This illustrates how sensitive the reversing properties of these dynamos are to relatively minor changes in their driving parameters.

3.2. Duration of reversals and failed reversals

We now turn to the distribution of reversal durations. As can be seen in Fig. 5 (for the same cases as in Fig. 4), these distributions

Table 2
Chron, segment, and events statistics for the models under investigation: number N_{rev} (resp. N_{frev}) of reversals (resp. failed reversals), mean chron (resp. segment) duration μ_{chr} (resp. μ_{seg}), Gamma index k_{chr} (resp. k_{seg}) for the distribution of chron (resp. segment) durations, geometric mean m_{rev} (resp. m_{frev}) of reversal (resp. failed reversal) durations, scale parameter σ_{rev} (resp. σ_{frev}) of the corresponding log-normal distributions. Also provided is the number N_{SLI} of SLI events, their average duration m_{SLI} and the fraction of time r_{SLI} spent in SLI events within segments. The estimated parameters of the Gamma (resp. log-normal) distributions are provided with their rounded 95% dispersion bars. The letters a,b,c refer to the three filtering levels mentioned in Section 2.3. Note also that N_{rev} , N_{frev} and N_{SLI} refer to the entire runs, the length of which depends on the model (recall Table 1).

Model	N_{rev}	N_{frev}	$\mu_{\text{chr}}/\tau_{\eta}$	k_{chr}	$\mu_{\text{seg}}/\tau_{\eta}$	k_{seg}	$m_{\text{rev}}/\tau_{\eta}$	σ_{rev}	$m_{\text{frev}}/\tau_{\eta}$	σ_{frev}	N_{SLI}	$m_{\text{SLI}}/\tau_{\eta}$	r_{SLI}
(1a)	123	25	$29.97 \pm_{13.23}^{17.04}$	$1.04 \pm_{0.20}^{0.25}$	$24.84 \pm_{10.10}^{12.72}$	$1.07 \pm_{0.19}^{0.24}$	$0.58 \pm_{0.02}^{0.03}$	0.24 ± 0.03	$0.62 \pm_{0.05}^{0.06}$	0.21 ± 0.05	3727	0.14	0.14
(2a)	298	140	$9.72 \pm_{2.91}^{3.44}$	$0.84 \pm_{0.13}^{0.13}$	$6.30 \pm_{1.58}^{1.82}$	$0.79 \pm_{0.09}^{0.10}$	$0.69 \pm_{0.04}^{0.05}$	0.56 ± 0.04	$0.91 \pm_{0.07}^{0.08}$	0.46 ± 0.05	2548	0.16	0.15
(3a)	464	192	$4.55 \pm_{1.11}^{1.26}$	$0.88 \pm_{0.09}^{0.10}$	$2.96 \pm_{0.61}^{0.69}$	$0.88 \pm_{0.08}^{0.09}$	$0.51 \pm_{0.03}^{0.03}$	0.61 ± 0.04	$0.82 \pm_{0.05}^{0.05}$	0.44 ± 0.04	2296	0.13	0.15
(4a)	547	223	$4.13 \pm_{0.93}^{1.05}$	$0.97 \pm_{0.10}^{0.11}$	$2.60 \pm_{0.50}^{0.55}$	$1.03 \pm_{0.09}^{0.10}$	$0.62 \pm_{0.03}^{0.04}$	0.68 ± 0.04	$1.03 \pm_{0.06}^{0.06}$	0.45 ± 0.04	2594	0.11	0.14
(5a)	457	253	$4.22 \pm_{1.03}^{1.18}$	$0.88 \pm_{0.09}^{0.11}$	$2.07 \pm_{0.46}^{0.41}$	$0.87 \pm_{0.08}^{0.08}$	$0.87 \pm_{0.06}^{0.07}$	0.84 ± 0.05	$1.51 \pm_{0.12}^{0.12}$	0.61 ± 0.05	1976	0.10	0.13
(1b)	123	25	$29.97 \pm_{13.23}^{17.04}$	$1.04 \pm_{0.20}^{0.25}$	$24.84 \pm_{10.10}^{12.72}$	$1.07 \pm_{0.19}^{0.24}$	$0.58 \pm_{0.02}^{0.03}$	0.24 ± 0.03	$0.62 \pm_{0.05}^{0.06}$	0.21 ± 0.05	3727	0.14	0.14
(2b)	270	130	$10.73 \pm_{3.33}^{3.95}$	$1.10 \pm_{0.15}^{0.18}$	$6.89 \pm_{1.78}^{2.04}$	$1.19 \pm_{0.14}^{0.16}$	$0.74 \pm_{0.05}^{0.06}$	0.60 ± 0.05	$1.01 \pm_{0.08}^{0.08}$	0.45 ± 0.05	2543	0.16	0.15
(3b)	432	192	$4.91 \pm_{1.23}^{1.41}$	$0.98 \pm_{0.11}^{0.12}$	$3.10 \pm_{0.65}^{0.73}$	$1.08 \pm_{0.10}^{0.11}$	$0.52 \pm_{0.03}^{0.03}$	0.63 ± 0.04	$0.86 \pm_{0.06}^{0.06}$	0.47 ± 0.04	2289	0.13	0.15
(4b)	515	229	$4.40 \pm_{1.01}^{1.13}$	$1.10 \pm_{0.11}^{0.13}$	$2.69 \pm_{0.52}^{0.57}$	$1.22 \pm_{0.11}^{0.12}$	$0.63 \pm_{0.04}^{0.04}$	0.69 ± 0.04	$1.05 \pm_{0.06}^{0.07}$	0.47 ± 0.04	2583	0.11	0.14
(5b)	421	239	$4.53 \pm_{1.15}^{1.32}$	$1.01 \pm_{0.13}^{0.13}$	$2.22 \pm_{0.45}^{0.51}$	$1.19 \pm_{0.11}^{0.12}$	$0.96 \pm_{0.08}^{0.08}$	0.87 ± 0.05	$1.54 \pm_{0.12}^{0.13}$	0.63 ± 0.05	1965	0.10	0.13
(1c)	121	25	$30.46 \pm_{13.53}^{17.46}$	$1.06 \pm_{0.21}^{0.26}$	$25.17 \pm_{10.28}^{12.95}$	$1.14 \pm_{0.19}^{0.26}$	$0.58 \pm_{0.03}^{0.03}$	0.25 ± 0.03	$0.65 \pm_{0.07}^{0.08}$	0.28 ± 0.06	3726	0.14	0.14
(2c)	248	132	$11.72 \pm_{3.76}^{4.49}$	$1.28 \pm_{0.22}^{0.29}$	$7.23 \pm_{1.90}^{2.20}$	$1.43 \pm_{0.17}^{0.20}$	$0.77 \pm_{0.06}^{0.06}$	0.61 ± 0.05	$1.08 \pm_{0.09}^{0.09}$	0.49 ± 0.05	2538	0.16	0.15
(3c)	360	194	$5.93 \pm_{1.61}^{1.86}$	$1.24 \pm_{0.17}^{0.17}$	$3.47 \pm_{0.77}^{0.86}$	$1.52 \pm_{0.15}^{0.17}$	$0.57 \pm_{0.04}^{0.04}$	0.66 ± 0.05	$0.96 \pm_{0.07}^{0.08}$	0.53 ± 0.05	2261	0.13	0.15
(4c)	447	227	$5.09 \pm_{1.24}^{1.17}$	$1.37 \pm_{0.15}^{0.15}$	$2.94 \pm_{0.59}^{0.66}$	$1.58 \pm_{0.15}^{0.16}$	$0.69 \pm_{0.05}^{0.05}$	0.75 ± 0.05	$1.15 \pm_{0.07}^{0.08}$	0.49 ± 0.04	2542	0.11	0.14
(5c)	331	216	$5.82 \pm_{1.64}^{1.91}$	$1.15 \pm_{0.15}^{0.17}$	$2.61 \pm_{0.58}^{0.65}$	$1.80 \pm_{0.19}^{0.21}$	$1.14 \pm_{0.11}^{0.12}$	0.92 ± 0.07	$1.82 \pm_{0.16}^{0.18}$	0.70 ± 0.06	1899	0.10	0.13

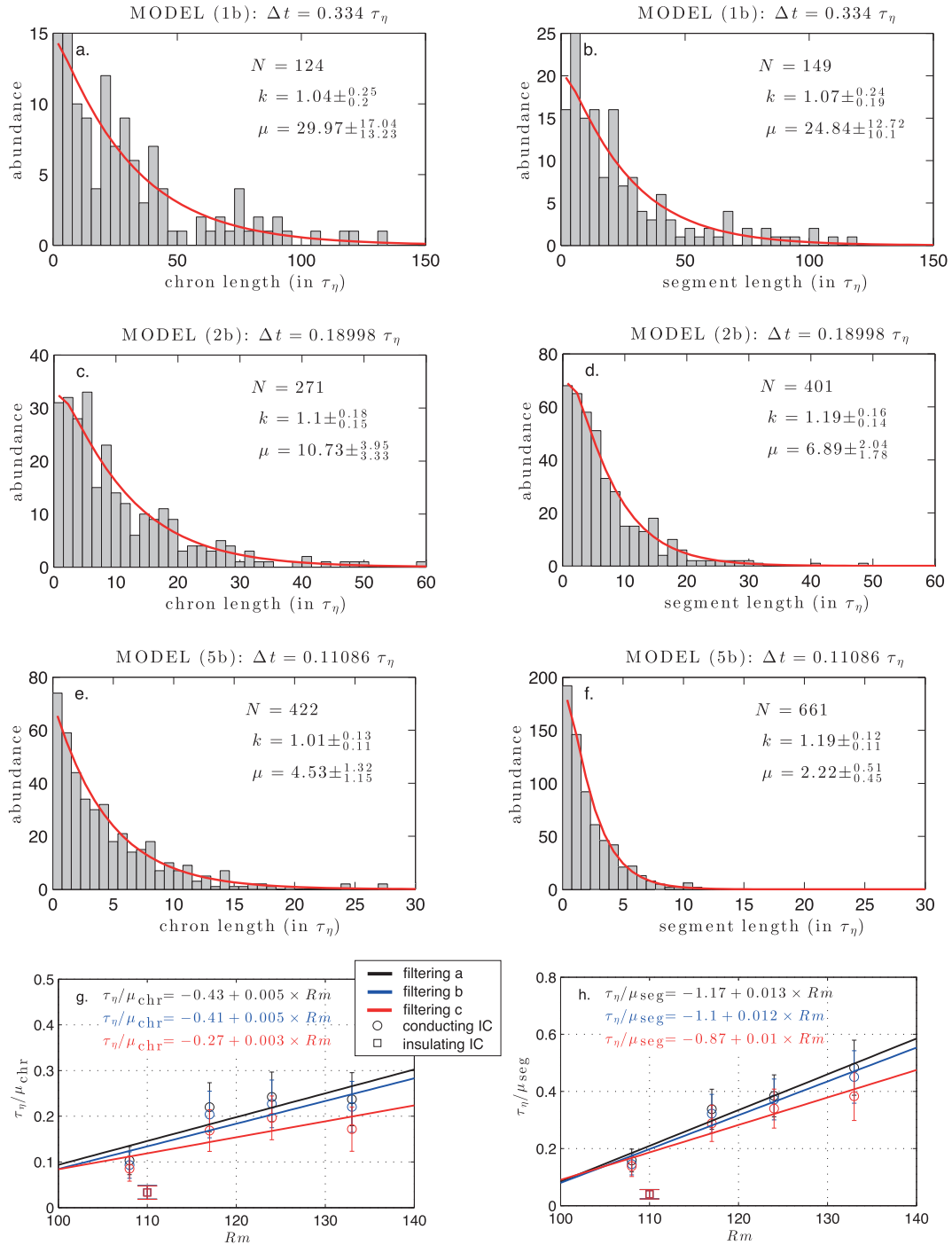


Fig. 4. Analysis of chron and segment lengths for models (1), (2) and (5), using filtering (b): distribution of chron lengths in panels a, c and e; distribution of segment lengths in panels b, d and f. These empirical distributions are fitted assuming a Gamma process (red curves, see Eq. (3)) with index k and mean duration μ expressed in units of τ_η . The corresponding estimates are provided with their 95% dispersion bars (inserts). N is the total number of chrons (resp. segments). Also shown with their uncertainties are $\tau_\eta / \mu_{\text{chr}}$ (panel g) and $\tau_\eta / \mu_{\text{seg}}$ (panel h) as a function of Rm with best linear fits (colour-coded inserts and lines) for the five dynamos (1–5) and the three types of filtering (a–c). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

can reasonably be fitted by log-normal distributions of the form given by Eq. (4). We checked that this is the case for all models, and for any of the (a), (b) and (c) types of filtering. Values of the corresponding best-fitting parameters are reported in Table 2 (recall Section 2.4 for the definition of these parameters). Of particular interest is the geometric mean (and also median) m_{rev} of these distributions, which gives a simple measure of the typical duration of the reversals for each model. The mean durations m_{rev} slightly increase with the filtering level when considering a given model. This

can be explained by the fact that the filtering of the shortest segments is done by assigning such segments to neighbouring reversals and failed reversals, thus increasing the reversal durations. To investigate whether the mean durations m_{rev} of reversals display a similar behaviour as the mean durations μ_{chr} of chrons when Rm increases, we plotted the quantity $\tau_\eta / m_{\text{rev}}$ as a function of Rm (Fig. 5(g)). As can be seen, contrary to $\tau_\eta / \mu_{\text{chr}}$, $\tau_\eta / m_{\text{rev}}$ does not display any monotonic trend. Rather, a maximum is observed for model (3), which thus experiences the reversals with the shortest

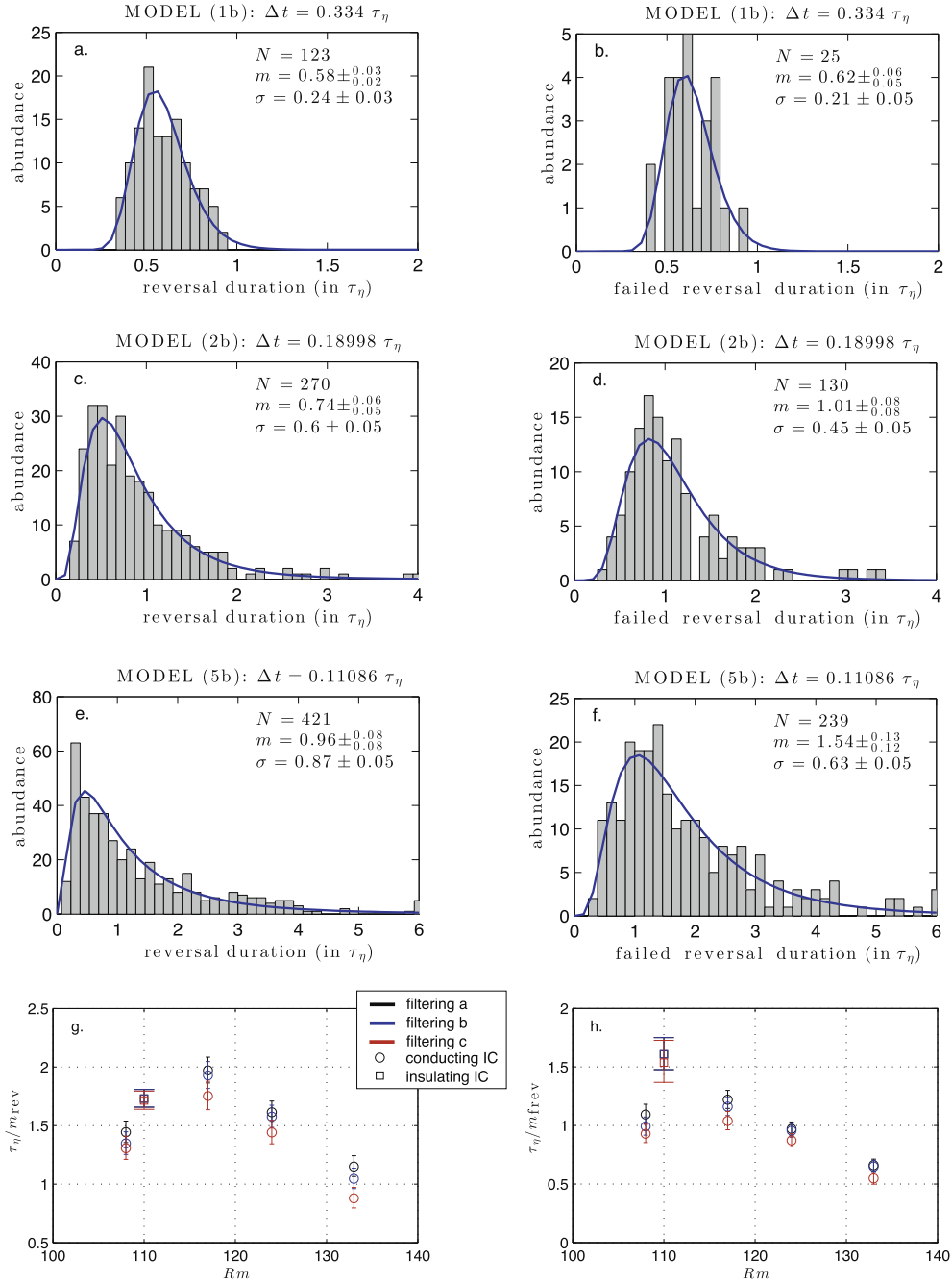


Fig. 5. Analysis of reversal and failed reversal durations for models (1), (2) and (5), using filtering (b): distribution of reversal durations in panels a, c and e; distribution of failed reversal durations in panels b, d and f. These empirical distributions are fitted assuming a log-normal law (blue curves, see Eq. (4)) with scale parameter σ and geometric mean m expressed in units of τ_η . The corresponding estimates are provided with their 95% dispersion bars (inserts). N is the total number of reversals (resp. failed reversals). Also shown with their uncertainties are τ_η/m_{rev} (panel g) and $\tau_\eta/m_{\text{frev}}$ (panel h) as a function of Rm for the five dynamo models (1–5) and the three types of filtering (a, b, and c). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

durations. Note that model (1), with an insulating inner-core, roughly fits the same general trend as all other models, even though these have a conducting inner-core.

A similar analysis can be done for durations of failed reversals (see Fig. 5(b), (d) and (f); and Table 2). Although the agreement between the empirical distribution and the best fitting log-normal distribution is no longer as good for model (1) as when we considered reversal durations (an observation which can be explained by the little number of failed reversals observed for this model), the overall fits to log-normal distributions remain satisfactory. As in the case of m_{rev} , we observe that the mean duration m_{frev} of failed

reversals slightly increases with the filtering level when considering a given model. We also plotted the quantity $\tau_\eta/m_{\text{frev}}$ as a function of Rm (Fig. 5(h)), which again reveals a non monotonic trend, with a maximum for model (3). Model (3) thus also experiences failed reversals with the shortest mean durations. Model (1), with an insulating inner-core, no longer fits the general trend defined by the other models. It is also worth noting that, for any given model and chosen filtering, failed reversals last longer than reversals on average. This is marginally true for model (1), which leads to $m_{\text{frev}}/m_{\text{rev}}$ ratios on the order of 1.1, but very clear for all other models with ratios ranging from 1.3 to 1.7. This can be

qualitatively understood by inspection of Fig. 1(a), where it can be seen that for a reversing event to end as a failed reversal, the field has to spend some additional time (compared to a true reversal) in the reversed polarity before reverting to its original polarity.

Extrapolations from the bell-shaped trends observed in Fig. 5(g) and (h) are clearly uncertain. Nevertheless, extrapolating the trend as R_m increases beyond the $R_m = 117$ value of model (3) leads to interesting insight. Comparing Fig. 5(g) and (h) with Fig. 4(h) reveals that, as R_m increases, both reversal and failed reversal durations increase while segment lengths decrease. For model (5) indeed, segment lengths are already short enough and reversal and failed reversal durations long enough, that all are of comparable size ($\mu_{\text{seg}} = 2.22\tau_\eta$, $m_{\text{rev}} = 0.96\tau_\eta$ and $m_{\text{frev}} = 1.54\tau_\eta$ in the case of filtering (b)). It can thus be anticipated that models with R_m even larger would end up being in a permanent state of reversal, never producing any well defined segments or chrons.

3.3. Dipole field behaviour during reversals and failed reversals

We now turn to a detailed examination of the dynamics of the dipole field during reversals and failed reversals. For each reversal identified in each model run, we computed the central time t_0 of the interval defining the event ($[a_i; b_i]$ as defined in Section 2.3) and selected an interval of total width $6\tau_\eta$ centred on that time. We next synchronised all selected intervals based on their central value and stacked all corresponding ADMs and DP latitudes to produce average ADM and DP latitude curves. In the case of DP latitudes, however, we first did this separately for reversals starting from a positive polarity and for those starting from a negative polarity. But as this revealed exactly the same behaviour, to within a polarity change, we eventually produced a single final joint DP latitude stack by simply changing the polarity of events starting from a negative polarity. Fig. 6 shows plots of such average curves for models (1), (2) and (5), when considering filtering (b). In each case, the first twenty of all intervals used to produce the stacked curves are also plotted to illustrate the variety of behaviours of the individual ADM and DP latitude curves during reversals. Similar plots built to investigate failed reversals are shown in Fig. 7. Models (3) and (4), and filtering (a) and (c) were also investigated (not shown). This revealed only a mild influence of the filtering used and analogous behaviours to that observed for models (2) and (5), except for the length of the events, consistent with our previous findings (recall Table 2 and Fig. 5(g) and (h)).

Most striking in Figs. 6 and 7 is the very different behaviour displayed by the dipole field in model (1) compared to all other models. The ADM of model (1) clearly displays an asymmetric dynamical behaviour through both reversals and failed reversals. It first decreases to zero and then displays a very distinct overshoot before settling in the next chron (or segment). Also, Fig. 6 shows that during reversals the DP latitude goes on fluctuating close to its stable polarity average value for some time while the ADM already started decreasing, before crossing zero in a short time and reaching a value very close to -90° . It then remains close to this value with little variability while the ADM is regaining strength, only to start fluctuating again about the new stable polarity average latitude at the time of the ADM overshoot. A very similar behaviour is observed during failed reversals, except for the fact that the DP latitude goes back to a value very close to 90° , instead of reaching -90° (Fig. 7). Then again the DP remains close to this latitude while the ADM is regaining strength, only to start fluctuating again about the original average stable polarity latitude at the time of the ADM overshoot.

Now, consider model (2), very close to model (1) but with a conducting inner-core. As can be seen in Figs. 6 and 7, the ADM has a much more symmetric behaviour during both reversals and failed reversals; it no longer displays any systematic overshoot. Further-

more, individual reversals and failed reversals are less similar from one event to the next, which is also reflected in the DP latitude behaviour. In particular it may sometimes happen that double and triple events occur while the ADM is low, i.e., events starting from a polarity, leading to a nearly successful reversal but immediately followed by another reversal (leading in effect to a failed reversal), or even followed again by another reversal, finally qualifying as a successful reversal. Note also how often the ADM may reach low values before and after a reversal or failed reversal, sometimes because of a nearby event (in which case the ADM drops to zero, and the DP latitude crosses the 0° value), but also often as a result of what we defined as SLI events (recall Section 2.3), i.e. times when the ADM is low but does not reach zero and the dipole remains of the same polarity. This general behaviour is typical of all models with conducting inner-core we investigated, the difference among models being essentially in the proportion of multiple events, model (5) being the richest of all in such events (see Figs. 6 and 7), consistent with the fact that reversals and failed reversals last longer for this model than for any of the other models (recall Fig. 5(g) and (h)).

3.4. ADM versus chron and segment lengths

In this section, we now explore whether ADM values during a chron (or segment) could be correlated in some way with the length of the corresponding chron (or segment). To this end, we simply computed the average ADM for each chron (or segment), and plotted it as a function of the chron (or segment) length. We did this for all models and types of filtering, but only report here on models (1), (2) and (5) in the case of filtering (b), for consistency with the results provided so far, and because these are enough to convey the main conclusions of our investigations.

To ease interpretation, all ADM values were normalised to μ as defined in Section 2.3, i.e., to the average absolute value of the ADM during all identified segments of the model. We also plotted for reference the $\mu - \sigma$ and $\mu + \sigma$ levels, where σ is the standard deviation of the ADM over the same segments, again as defined in Section 2.3. Fig. 8(a) and (b) show the corresponding plots when considering chrons and segments in model (1). As can be seen, both plots show that average ADM values in a chron (or segment) display more dispersion for short chrons (or segments) than for long ones. This dispersion is also nicely symmetrical about the average value μ and within the $\mu - \sigma$ and $\mu + \sigma$ levels. Such a behaviour is exactly what one would expect for an ADM fluctuating in the same way about its average value μ , whether the length of the chron (or segment) is short or long. Average ADM values for a given chron (or segment) length are then simply estimates of μ , all the more accurate as the length of the chron (or segment) is long. This shows that model (1) does not lead average ADMs (and therefore individual ADM estimates) to correlate in any way with chron or segment lengths.

Interpretation of our results for model (2) is a little more subtle. Fig. 8(c) and (d) show the corresponding plots, which look essentially the same for both chrons and segments. Compared to the previous plots, however, ADM values appear to be biased, with the scatter being about an average value closer to $\mu - \sigma$ than to μ when chrons (or segments) are short. The reason for this lies in the fact that just before and after a reversal or failed reversal, the ADM value must be close to $\mu - \sigma$. All chrons and segments thus have low ADM transitional periods at their onset and end. As a result, when these transitional periods last a significant fraction of the chron (or segment), the average ADM over the corresponding chron (or segment) is bound to be biased towards $\mu - \sigma$. It may even take lesser values, since the ADM can temporarily explore values less than $\mu - \sigma$, during SLI events. This is typically the case for the shortest chrons and segments produced by model (2), which

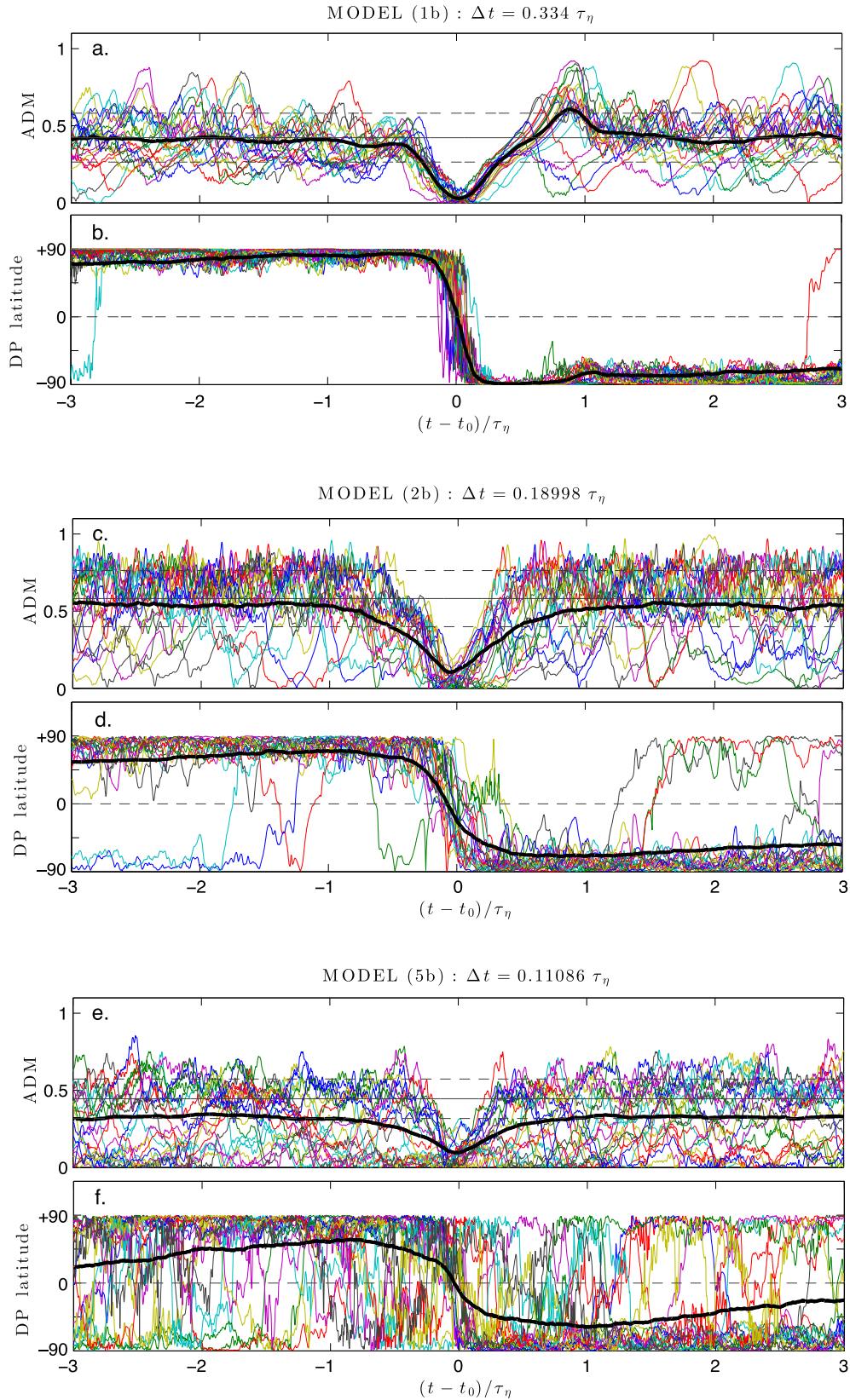


Fig. 6. ADM and DP latitude behaviour during reversals for models (1), (2) and (5), using filtering (b). In each plot, twenty examples of ADM (or DP latitude) curves are shown over an interval of $6\tau_\eta$ centred on the central time t_0 of the event (colour curves), together with a stack of the ADM (or DP latitude), computed from all available events (thick black curve); see text for more details. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

can last as little as a fraction of τ_η , i.e., only a few times the secular-variation timescale $\tau_{SV} = 3.05 \times 10^{-2} \tau_\eta$ (recall Table 1), and com-

parable to the average length of SLI events ($m_{SLI} = 0.16\tau_\eta$, recall Table 2). The correlation this effect produces between average ADMs

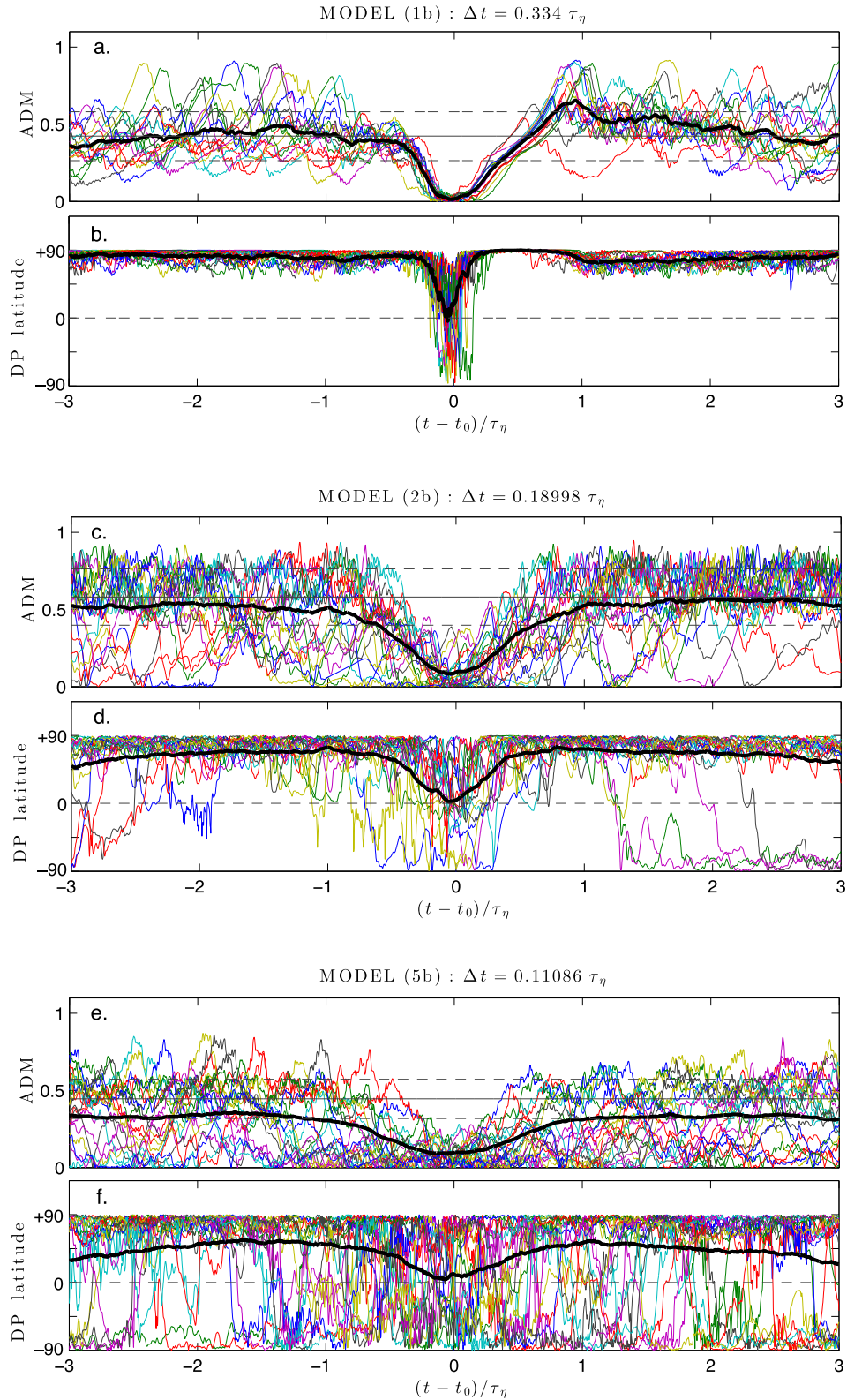


Fig. 7. Same as Fig. 6 but for failed reversals; see text for more details.

and chron or segment lengths is however very weak, and would be very difficult to detect even if quite a few individual ADM estimates were available. It was not observed for model (1), which shares similar secular-variation timescale ($\tau_{SV} = 2.90 \times 10^{-2} \tau_\eta$) and average length of SLI events ($m_{SLI} = 0.14 \tau_\eta$), because model (1) produces much less short chrons and segments.

Model (5) further illustrates how difficult the interpretation of ADM versus chron length plots can be (Fig. 8(e) and (f)). For this model, only short chrons and segments occur. As a result, significant dispersion is always found. As in model (2), a bias towards $\mu - \sigma$ is nevertheless observed for the shortest of all chrons and segments (Fig. 8(f)). But chrons and segments now plot quite

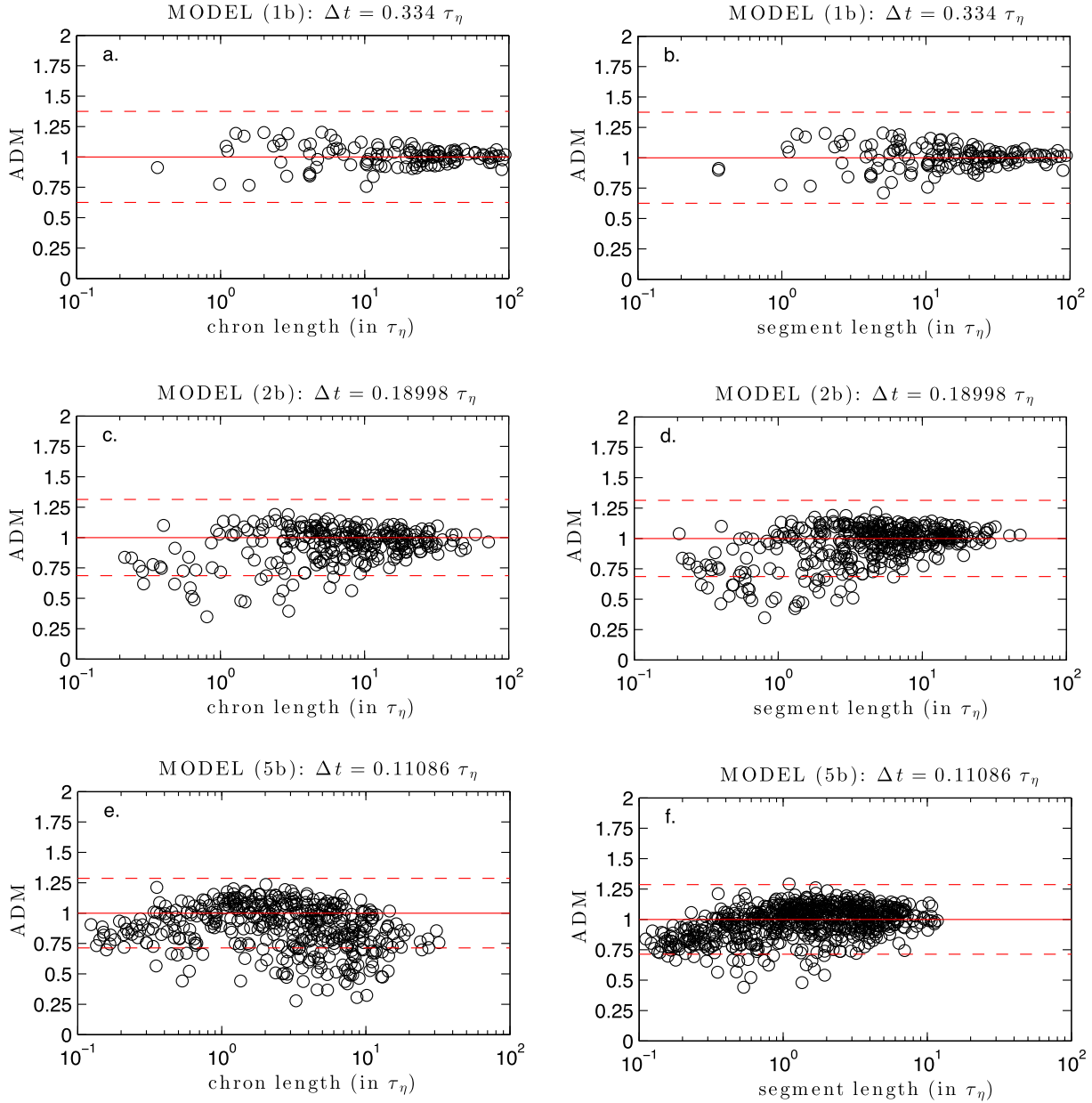


Fig. 8. Correlations between ADM and chron/segment lengths for models (1), (2) and (5), using filtering (b). ADMs are normalised with respect to their average value inferred from the algorithm described in Section 2.3 (i.e. average value over all segments). Also shown in red dashed lines are the limits defined by the σ , also inferred from this algorithm. (For interpretation of the references to colour in this figure caption, the reader is referred to the web version of this article.)

differently, with ADMs being less intense on average during chrons than during segments. This reflects the fact that, contrary to segments, chrons may include time spent in failed reversals, when the ADM is low. This time is not significant for models (1) and (2), but for model (5) it does make a significant fraction of a chron, leading the ADM to be underestimated on average.

3.5. A phase space perspective

We finally focus on the behaviour of the various models in the phase space described by the first three Gauss coefficients (g_1^0, g_1^1, h_1^1) characterising the dipole field produced by the models. Combined with the empirical distributions of the signed ADM (i.e., g_1^0), of the total dipole moment $\sqrt{(g_1^0)^2 + (g_1^1)^2 + (h_1^1)^2}$ and of the DP latitude, these phase-space plots provide very useful complementary information, enlightening many of our previous findings.

Fig. 9 (left column) shows such phase-space plots for all five models investigated so far, where values from segments of positive polarity are plotted in red, those from segments of negative polarity in green, those from reversals in blue, and those from failed reversals in grey. These plots immediately reveal a major distinction between model (1) and all other models. Model (1) only has two symmetric stable polarity states showing as red and green balls, whereas all other models clearly display an additional distinct central unstable state, showing as a (mainly) blue and grey ball. Note how each of these three balls appear to be remarkably delineated by the colour coding originally defined by the algorithm we used in Section 2.3 to define segments of stable polarity, reversals and failed reversals. This directly illustrates the fact that reversals and failed reversals occur in these dynamos only once they leave a stable polarity state to reach the central unstable state where the dynamo spends some time (the time of a reversal or a failed reversal) before moving again to one of the two stable

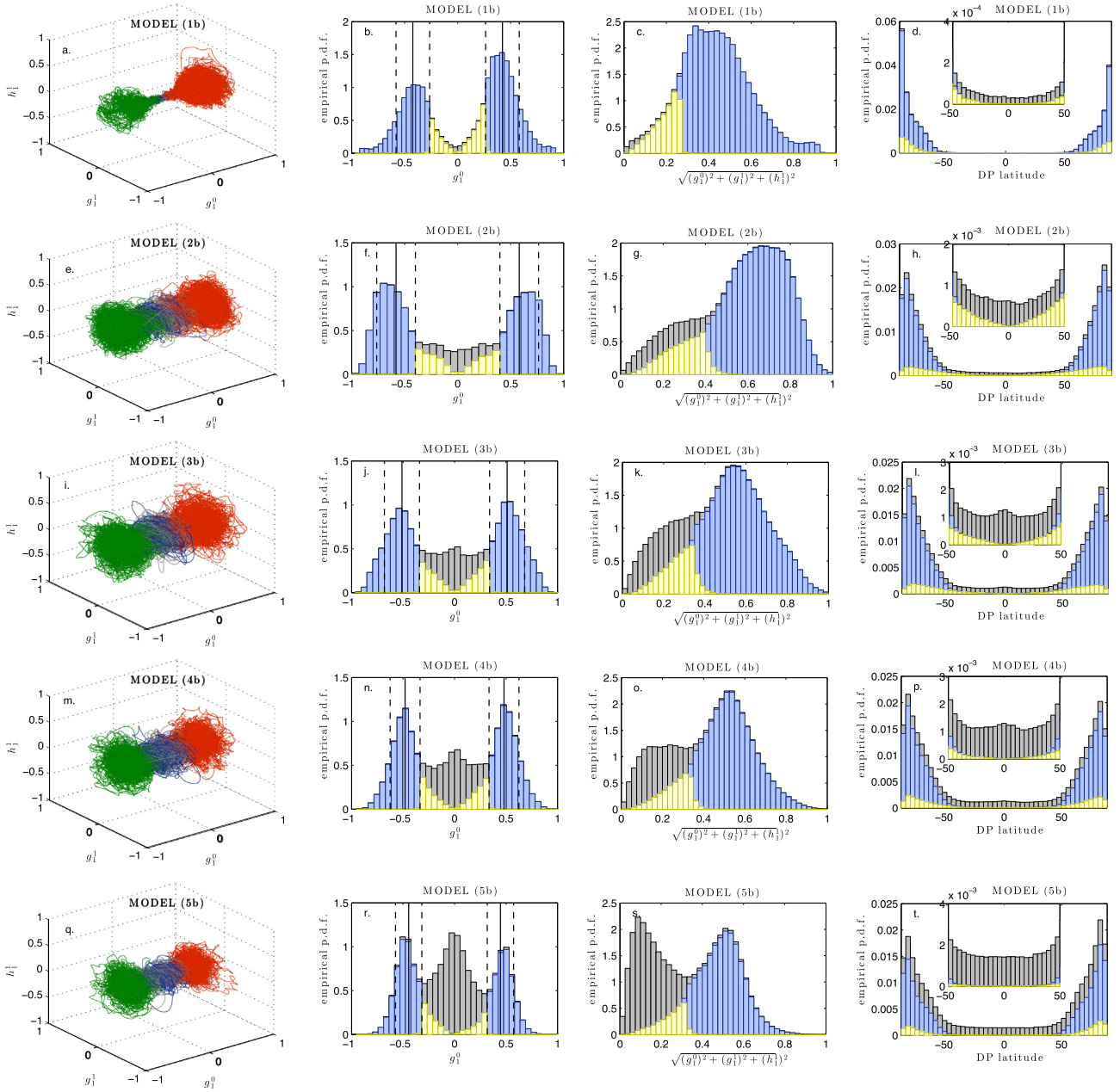


Fig. 9. Phase-space behaviour of models 1 (first line), 2 (second line), 3 (third line), 4 (fourth line), and 5 (fifth line). Left column (panels a, e, i, m, q) shows plots of the dipole behaviour for each of these dynamos in the (g_1^0, g_1^1, h_1^1) space, with colour code as follows: red (resp. green) when the dynamo is evolving in a positive (resp. negative) polarity segment, blue (resp. grey) when the dynamo is experiencing a reversal (resp., a failed reversal). Second column from the left (panels b, f, j, n, r) shows the corresponding empirical distribution for the signed ADM (e.g., g_1^0) where we distinguish contributions from SLI events (yellow), other contributions within positive or negative polarity segments (blue), and contributions at times of reversals or failed reversals (grey); also shown are the locations of the $-\mu$ and μ values (solid line), and $-\mu - \sigma$, $-\mu + \sigma$, $\mu - \sigma$ and $\mu + \sigma$ values (dashed lines). Third column from the left (panels c, g, k, o, s) shows empirical distributions for the full dipole moment, using the same colour coding. Right column (panels d, h, l, p, t) shows empirical distributions for the DP latitude, again using the same colour coding (insets show enlargements for values between -50° and 50°). Criteria for identifying positive (resp. negative) polarity segments, reversals and failed reversals, as well as SLI events, are those defined in Section 2.3, using filtering (b). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

polarity states. This behaviour is very different from that observed in model (1), which displays no central state, in which case reversals (and the few failed reversals) happen as a much more straightforward transition (or failed transition) from one state of stable polarity to the other one.

Plotting the empirical distribution of the signed ADM (i.e., g_1^0) for each model is equally useful. This is done in Fig. 9 (second column from left), where we now distinguish contributions from SLI events within stable segments only (in yellow), other contributions within these stable segments (blue) and contributions at times of

reversals and failed reversals (grey). This colour coding ensures that the distribution of signed ADM when the dynamo is in a stable segment (adding yellow and blue contributions), and the entire distribution of signed ADM at all times (further adding grey contributions) can also be visualised. Comparing these signed ADM empirical distributions for the various models, again highlights the specific case of model (1) (Fig. 9(b)). For this model, most low-value ADMs occur as SLI events (i.e., when the dynamo is in a stable polarity segment). In contrast, for all other models, the fraction of low ADM values associated with a reversal or failed

reversal strongly increases as the ADM values decrease. Low-value ADMs are then clearly associated with an enhanced probability of reversal or failed reversal, reflecting the fact that such events occur once the dynamo has entered the central unstable mode.

Plotting the empirical distribution of the total dipole moment $\sqrt{(g_1^0)^2 + (g_1^1)^2 + (h_1^1)^2}$ for each model further helps to characterise this central mode (Fig. 9, third column from left, where the same colour coding is being used). Not surprisingly, no remarkable signature is found in the case of model (1). In contrast, the central mode shows up in the form of a hump for low total dipole moments, the size of which increases as we move from model (2) to model (5), reflecting the evolution of this central mode from model to model. Interestingly, we also note that, in contrast to what was seen in the signed ADM distributions, this hump is not centred on the zero value, implying that contrary to the signed ADM, the total dipole moment rarely drops to very low values. This indicates that, in the phase space, the central mode is more “spherical-shell” than “ball” shaped, reflecting the fact that during reversals and failed reversals, the total dipole moment always keeps a minimum value.

The “spherical-shell” shape of this unstable mode is also reflected in the DP latitude set of distributions (Fig. 9, right column, where the same colour coding is being used). In models (2) to (5), contributions from reversals and failed reversals (in grey) are almost uniform over all DP latitude values, reflecting a lack of any strong preference for the location of the DP pole while in this “spherical-shell” shaped central unstable state. In model (1), although DP latitudes also appear to be quite evenly distributed between -50° and 50° during reversals and failed reversals (in grey), Fig. 9(d) reveals that the DP latitude distribution has stronger contributions for values close to -90° and 90° , both at times reversals and failed reversals (in grey, as a careful look will reveal), and at times of stable polarity (in blue). This now reflects the dynamics we already observed for the dipole pole in model (1) during and just after reversals and failed reversal (Figs. 6 and 7), with the dipole pole crossing all latitudes before spending some amount of time very close to -90° (or 90° , depending on the final polarity), while the ADM experiences its overshoot.

4. Discussion

One of the most striking findings of the present study is the strong impact of the conducting or non-conducting nature of the inner-core on the properties of the dynamo models we, and previous authors, investigated (see, e.g. Olson, 2007; Olson et al., 2009; Driscoll and Olson, 2009a,b, 2011; Olson et al., 2010, 2011). As can be seen in Fig. 9, model (1) (with an insulating inner-core) misses the central unstable mode found in all other models (with a conducting inner-core). This qualitative difference turns out to be the main cause of all major quantitative differences we otherwise noted between model (1) and all other models.

In particular, reversals or failed reversals occur on much rarer occasions in model (1) (recall Table 2), than even in models (2) and (3), which operate in exactly the same parameter range and only differ by a slightly different Ra^* in the case of model (2), and a slightly different Rm in the case of model (3) (recall Table 1). In the case of model (1), the dynamo simply cannot access the missing central mode and must directly reach the very low ADM and low DP latitudes required for such events to occur. Interestingly, we note that Dharmaraj and Stanley (2012) recently found an opposite effect in their dynamos (thermally-driven, run in a different parameter regime, and at a lower Ekman number, though for much shorter periods of time), these being more stable with, than without a conducting inner-core. Perhaps is this because, in such regimes, the consequences of the electrical nature of the inner-core is the opposite, leading to a transitory central mode only when the

inner-core is insulating (a conjecture we could not check ourselves). In any case, what these results clearly show is that contrary to previous claims (see, e.g., Wicht, 2002, 2005; Amit et al., 2010), the exact electrical nature of the inner core can influence the behaviour of dynamos and in a very radical way, by modifying the nature of the dynamo states that can be reached. This influence, however, is not directly related to the inner-core diffusive timescale, which hardly plays any role in quantifying the frequency of reversals.

The fact that the ADM and DP latitude behave so differently through reversals and failed reversals in model (1) (Figs. 6 and 7), can also be related to a lack of central mode for this model. With no such mode, a common dynamical path must necessarily be found by model (1) to produce each reversal or failed reversal through the “bottleneck” seen in the corresponding phase space plot (Fig. 9(a)). All other models produce much less of a systematic pattern, simply because reversals and failed reversals result from transiting through the “spherical-shell” shaped central mode, within which both the ADM and the DP latitude can fluctuate substantially (possibly producing multiple events) before reaching one of the two stable polarity modes. Note that the specific behaviour of the ADM and DP latitude in model (1) is in fact very similar to the one observed in experimental dynamos and simple models (e.g., Pétrelis et al., 2009), where an analogous ADM overshoot is observed during reversals. These systems too are governed by only two stable fixed points (equivalent to the two stable polarity states we observe here).

Other dynamo properties also appear to be affected by the electrical nature of the inner-core, and the lack of central mode, though not as strongly. As noted in Section 3.2 (recall Fig. 5), failed reversals appear to last less time in model (1) than in models (2) and (3), while the average duration of successful reversals in model (1) appears to match the trend observed for all other models. But a careful look at Fig. 5(a) and (b) reveals much less dispersion in the duration of these events in model (1) than in all other models, testifying again for the more systematic way the field reverses in model (1). In contrast, models (2) and (3) can have reversals and failed reversals displaying a wider variety of durations, simply because of the wider range of time the dynamo can spend in the central mode. Also worth pointing out, is that in all cases, the basic reference time controlling reversal and failed reversal durations is τ_η (and not τ_{sv} , recall Tables 1 and 2), indicating a clear control of diffusion during reversals in all these dynamos, even when the inner-core is insulating (which further points at diffusion within the entire core, rather than just within the inner-core, being key in defining this basic reference time, contrary to what has been suggested could be the case for the Earth (Gubbins, 1999)).

The two quantities that are least affected by the electrical nature of the inner-core are the secular-variation timescale τ_{sv} and the error-growth timescale τ_e , both of which are dynamical characteristics of the dynamo at times of stable polarity. These appear to be mainly governed by Rm (recall Fig. 2), in agreement with previous results (Hulot et al., 2010a; Lhuillier et al., 2011a,b), and are insensitive to the presence or absence of a central mode.

Analysing Fig. 9 also brings important insight into the nature of the evolution of the dynamos when increasing the strength with which they are driven. As Ra^* increases (from model (2) to model (5), all other parameters being kept unchanged), a change in the relative contributions of the central unstable mode and of the two positive and negative stable modes can be observed. Whereas model (2) displays a relatively weak and diffuse central mode contrasting with strong and well-defined stable modes, models (3) to (5) progressively display a stronger, better defined, and more compact central mode. In the case of model (5), this central mode has essentially become as significant and as often occupied as the two

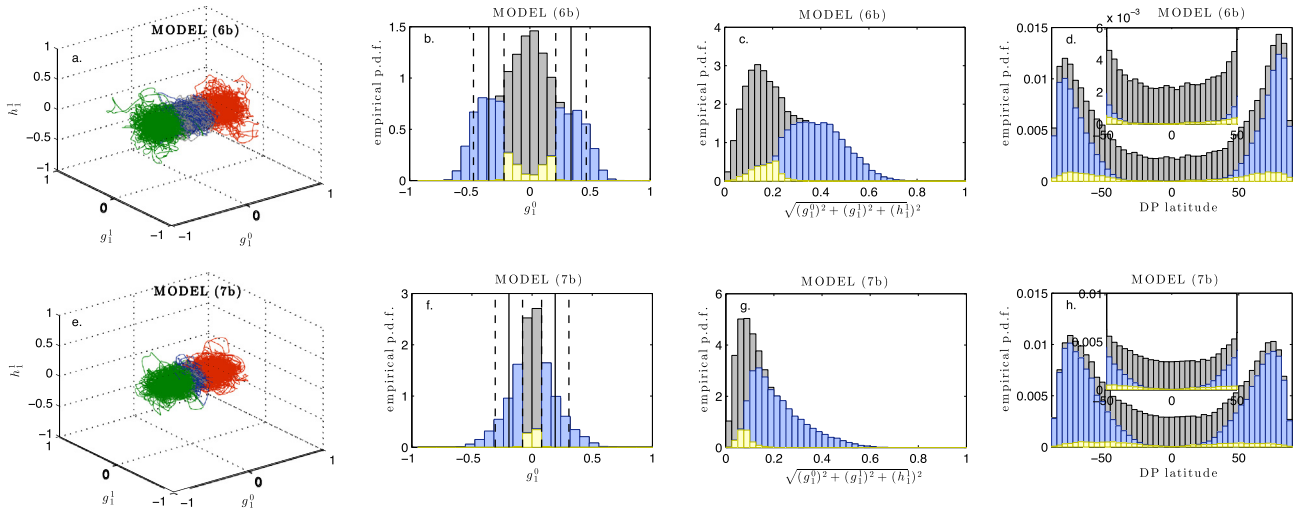


Fig. 10. Phase space behaviour of additional models 6 (first line) and 7 (second line). Same plotting convention as in Fig. 9.

stable modes. This evolution is accompanied by a slight decrease of the dipolarity measure f_{dip}^{13} and a slight increase in the equatorial to axial dipole measure $f_{\text{ax}}^{\text{eq}}$ (recall Table 1). It is also related to the trend observed in Fig. 4 for chrons and segments to last less (and therefore for the reversal frequency to increase) as Ra^* (and therefore Rm) increases, and to the evolution seen in Fig. 5 for the durations of reversals and failed reversals. The non-linear nature of this evolution can now be understood as a consequence of the complex transition from model (2) to (5).

To explore the way the dynamo evolves as Ra^* is further increased, two additional (shorter) runs have been computed (models (6) and (7)), the phase-space behaviours of which have been plotted in Fig. 10, using the same analysis procedure as previously. As can be seen, the central unstable mode now becomes so prominent, that it merges with the two stable modes and that the separation in terms of these modes (and the identification of chrons, segments, reversals, failed reversals and SLI events) becomes irrelevant. Virtually only a central unstable mode remains to be seen in model (7), despite the attempt made by our algorithm to identify stable modes. These models essentially keep on reversing (as had been anticipated based on Figs. 4 and 5), and no stable values can be defined for the various quantities reported in Table 2. Basic parameters for these models are otherwise reported in Table 3. For these two models, and particularly model (7), the field is now much less dipolar, and with a much stronger equatorial to axial dipole ratio.

What we observe in terms of dynamo evolution thus complements what Olson et al. (2011) recently observed in an analogous set of dynamo simulations, which they however ran over much shorter periods of time (typically ten to twenty times $\tau_\eta = D^2/\eta$, to be compared to the several thousands of τ_η over which our dynamo models were run, recall Table 1). These dynamos also differ from ours in several other aspects see for details (Olson et al., 2011). But they are in similar regimes, with the same Prandtl number $Pr = 1$, a slightly weaker magnetic Prandtl number of $Pm = 10$, and comparable Ekman numbers. In particular, Olson et al. (2011) investigated the evolution of dynamos with $E = 1 \times 10^{-3}$ as the Rayleigh number is progressively increased. They too noted a major evolution from their case (a), which did not reverse over the course it was run and displayed a single polarity stable mode, to their multipolar permanently reversing case (d), displaying a single central unstable mode, with several intermediate cases (see their Fig. 2). Although these short runs were not analysed in the way we did, they already revealed the way dynamos can evolve from

Table 3

Properties of the two additional models (6) and (7). Same definitions as in Table 1.

Model	Inner-core	T/τ_η	Ra^*	Rm	f_{dip}^{13}	$f_{\text{ax}}^{\text{eq}}$	$\tau_{\text{SV}}/\tau_\eta$
(6)	Conducting	250	8.5×10^{-3}	165	0.44	0.43	2.19×10^{-2}
(7)	Conducting	250	1.1×10^{-2}	202	0.36	0.48	1.91×10^{-2}

being dominantly axial dipolar with two stable polarity modes, to being dominantly multipolar with a single central unstable mode. Our results are in complete agreement with this view, and illustrate the crucial role played by the central unstable mode in controlling most aspects of reversals (and failed reversals) in these dynamos.

Another important outcome of the present study is the property found in all our models that, provided that the finite duration of reversals and failed reversals is properly taken into account, chron and segment lengths (defined as described in Section 2.1) always closely follow a Poisson distribution (Section 3.1). This confirms similar findings by, e.g. Wicht et al. (2009) (who investigated thermally-driven dynamos, with an even higher Ekman number of $E = 1 \times 10^{-2}$). It also implies that these dynamos do not have any memory of their past behaviour beyond the duration of a reversal or failed reversal. This is consistent with the fact that the error-growth timescale τ_e , defining the limit of predictability of these dynamos at times of stable polarity (see Hulot et al., 2010a; Lhuillier et al., 2011b for details), is always found to be an order of magnitude smaller than the duration of reversals and failed reversals (recall Tables 1 and 2). Note that such a small value of τ_e only implies that the time when the dynamo leaves a stable mode cannot be predicted much ahead of time. But it does not preclude some amount of longer-term deterministic behaviour of the field during reversals, as is particularly well illustrated by model (1). In this model, as soon as the dynamics leads the system to enter the “bottleneck” seen in Fig. 9(a), the dynamo almost unavoidably produces either a reversal or a failed reversal (reversals being five times more likely than failed reversals, recall Section 3.1 and Table 2), following the near-systematic pattern shown in Figs. 6 and 7. Even in the case of models (2) to (5), some amount of determinism is also to be found. For these models, once the system has entered the central unstable mode, either a reversal or a failed reversal will almost inevitably occur (with reversals again being slightly more likely, recall Section 3.1 and Table 2), though the

exact way the event will unravel is then much less reproducible. Finally, we note that the previous results are also consistent with the fact that no clear correlation could be found between ADMs and the duration of chrons or segments (Section 3.4).

5. Concluding remarks: implications for paleomagnetism and the geodynamo

Dynamos investigated in this paper belong to a class of dynamos that have been extensively used to understand and tentatively reproduce important features of the Earth's magnetic field (e.g. Driscoll and Olson, 2009a,b, 2011; Olson et al., 2010, 2011, 2013). Although not all concepts introduced in this study exactly compare with equivalent paleomagnetic data-derived concepts, our results do bring important complementary insight.

First, consider the distribution of chron lengths. As discussed above, this distribution is always found to be very nearly Poissonian for this class of dynamos. This is very similar to what is being observed for geomagnetic polarity reversals, the history of which is well known for the past ~160 Myr, thanks to the record of oceanic magnetic anomalies (e.g. Cande and Kent, 1992a; Channell et al., 1995). Indeed, when analysed in terms of a Gamma process (as defined by Eq. (3)), this data leads to the conclusion that contrary to μ , which may vary through geological times, k does not significantly differ from unit (e.g. Lowrie et al., 2004), even though its determination remains limited by our imperfect knowledge of chrons shorter than 30 kyr and by the still discussed issue of the physical origin of the “tiny wiggles” also known as “cryptochrons”, corresponding to sea-floor magnetic anomalies of very short wavelength (e.g. Cande and Kent, 1992b; Bouligand et al., 2006). These observations thus suggest that, just like the dynamos investigated here, the geodynamo produces reversals as a result of a process with no memory beyond the duration of a reversal, a conclusion also consistent with a predictability horizon of a few decades for the geodynamo (Hulot et al., 2010a; Lhuillier et al., 2011b), much shorter than the duration of a reversal.

If the above conclusion holds, the results of Section 3.4 would then also suggest that no clear-cut correlations should be expected between ADMs and the duration of chrons, beyond the type of effects seen in Fig. 8, with more dispersion and slightly weaker values when chrons are short. Precise relative paleointensity records from pelagic Oligocene sediments (Tauxe and Hartl (1997), see also Constable et al. (1998)), and similar records for the past 2 Myr (Valet et al., 2005), only suggest a weak correlation, which a very recent study on Eocene to Oligocene sediments (Yamazaki et al., 2013) further argues could be due to some lithological contamination of the paleointensity records. Lava flow and submarine basaltic glass samples also only suggest a weak correlation, with very substantial dispersion that could be enhanced when the ADM is larger (e.g. Tauxe, 2006; Tauxe et al., 2007). Although clearer correlations might eventually emerge when considering longer time periods (then possibly reflecting long-term changes in the parameters driving the geodynamo, see, e.g. Hulot et al. (2010b) for a discussion), what our results thus suggest is that recovering more accurate information from such correlations is clearly bound to remain a challenge.

Much more constructive conclusions can be drawn from the analysis of the (g_1^0, g_1^1, h_1^1) phase-space plots shown in Fig. 9. This analysis taught us that it is the occurrence and size of a central unstable mode that controls most of the reversing properties of the investigated dynamos (independently of the reason why such an unstable mode would be found). Unfortunately no analogous plot can directly be built from the data for the geodynamo. But indirect paleomagnetic signatures can be looked for. Comparing the distribution of signed VADM from Sint-2000 (Valet et al.,

2005) to those of the signed ADM in Fig. 9 suggests that the geodynamo is much closer to model (1) (Fig. 9b) than to any other model. The same data also reveals a systematic pattern of the VADM behaviour through reversals, with an overshoot akin to the one we observe for model (1) (compare Fig. 4 of Valet et al., 2005 to Fig. 6). Finally, and as recently pointed out by Valet et al. (2012), the VGP latitude might also display a systematic pattern, again as the DP latitude in model (1), though the details of this systematic pattern are clearly different, and the comparison of VGP and DP latitudes might not be fully appropriate. Such a combination of characteristics would then suggest that the geodynamo currently has no significant central unstable mode.

Other evidence, however, suggests that the situation might not be as simple. Model (1), which produces a systematic pattern in both the ADM and the DP during reversals, also does so during failed reversals. Yet no analysis of paleomagnetic data have revealed such patterns in the VADM and VGP so far. This could be because our dynamo-based concept of failed reversal does not exactly match the more empirical data-based paleomagnetic concept of “excursion” (Laj and Channell, 2007). Indeed, both failed reversals and strong SLI events (based on our definitions in Section 2.3) can lead to events that would qualify as excursions from the paleomagnetic point of view. Since SLI events only are one-sigma-plus fluctuations of the field within the stable modes (as can be checked in Table 2, our dynamos indeed all spend roughly 15% of their stable polarity time in such SLI events), they do not produce any systematic pattern. The pattern of failed reversals could thus be blurred by the occurrence of SLI events qualifying as excursions. In this respect, selecting paleomagnetic data in a way better matching our definition of failed reversals, could be very useful. It might eventually reveal a more systematic pattern and shed even more light on the reversal process. It could also bring new insight into the interpretation of still poorly understood short events, such as microchrons, excursions and cryptochrons (e.g. Cande and Kent, 1992b; Bouligand et al., 2006; Laj and Channell, 2007).

Some paleomagnetic data further show that “complex reversals” (as defined by Coe and Glen, 2004) can also sometimes occur, closer to the type of reversals model (2), rather than model (1), would produce. This has been the case, for instance, for the last known (Matuyama–Brunhes) reversal. Reversals with similar behaviours were investigated recently by Olson et al. (2011), using dynamos similar to ours. Their model (i), that best accounted for this behaviour, displays a small, but clear, central mode signature (in the ADM distribution, see their Fig. 2), similar in this respect to our model (2). The average duration of paleomagnetic chrons (~230 kyr) over the past ~30 Myr (segment C of the geomagnetic polarity timescale defined by Gallet and Hulot (1997) and Hulot and Gallet (2003)), also appears to be closer to the average chron length of ~180 kyr found in model (2) than to the average chron length of ~500 kyr found in model (1), both rescaled with respect to τ_{SV} . This now suggests that the present geodynamo could have a small central mode introducing some amount of complexity in reversals, small enough, however, to not jeopardise the tendency for reversals to usually display some systematic patterns (Valet et al., 2005, 2012).

If such is the case, the geodynamo could thus belong to a dynamo family displaying a small central mode with a strong sensitivity to changes in driving parameters. The relative size and role of this central mode would then evolve in response to such changes as our models evolved in response to changes in the Rayleigh number Ra^* (Fig. 9). In particular, the frequency of reversals would display a strong sensitivity to even minor changes, with this frequency being proportional to some distance from a critical state, as illustrated by τ_{η}/μ_{chr} being proportional to the distance $Rm - Rm_c$, with $Rm_c \approx 90$ in our models (recall Fig. 4). Since this would then also imply a total lack of reversals as soon as one reaches $Rm = Rm_c$,

such sensitivity could then also account for the occasional occurrence of superchrons lasting more than 20–30 Myr (e.g. Pavlov and Gallet, 2005), as envisioned and illustrated by studies such as those of Courtillot and Olson (2007), Driscoll and Olson (2009a,b, 2011) and Olson et al. (2010, 2013). What the present study further suggests is that, if the geodynamo indeed belongs to such a family, no other means of producing superchrons would be as straightforward. In particular, spontaneous transitions to superchrons would be very unlikely, since no such transition was observed in any of the very long runs we investigated. This, we note, would then also imply a particularly high sensitivity of the geodynamo to changes in driving parameters (including heterogeneous boundary conditions), since transitions to and from superchrons have been shown to often occur on very short geological timescales (Hulot and Gallet, 2003; Gallet et al., 2012). Finally, and most importantly, the present study also suggests that, not only the frequency of reversals, but also properties such as the proportion of failed reversals, or the more or less complex way reversals occur, would also be affected by such changes in the dynamo's driving parameters. Searching for such or other evidence would clearly be most useful.

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