• Travel times in the Earth
• Ray paths, phases and their name
• Wavefields in the Earth: SH waves, P-SV waves
• Seismic Tomography
• Receiver Functions
Long-period transverse displacement for an earthquake at 600km depth recorded at 130° (synthetic).

How can we extract information from seismograms on Earth structure?

- identify phases
- pick travel times
- collect travel times as a function of distance
Travel times for a spherically symmetric Earth model (IASP91) Source at 600km depth

Automatic Picks from real data
• Harrold Jeffreys and Keith Bullen (1940), (J-B) Remarkable accuracy for teleseismic travel times (below 1%)

• Herrin et al. (1968), with well located earthquakes.

• Dziewonski and Anderson (1981), Preliminary Reference Earth Model (PREM)

• Kennett and Engdahl (1991), most accurate radially symmetric model (ia9p91)

• (2000), The first 3-D reference model with travel times?
Particular phases at teleseismic distances are named after the wave types (P or S), regions they pass along their path, and emergence angle at the source (upwards or downwards).
The core-mantle boundary has the most dominant effect on the global wavefield. Multiple reflections from it reveal information on attenuation and the structure near the CMB.
Ray Paths in the Earth - Names

P waves
S waves
depth phases (P)
depth phases (S)
Reflection from CMB
wave inside core
Reflection from Inner core boundary
wave through inner core

Examples:
PcP, pPcS, SKS, PKKKP, PKiKP, PKIKP, sSS, pSSS, sPcS, etc.
**Wavefields in the Earth: SH waves**

Red and yellow color denote positive and negative displacement, respectively.

Wavefield for earthquake at 600km depth.
Wavefields in the Earth: SH waves

Red and yellow color denotes positive and negative displacement, respectively.

Wavefield for earthquake at 600km depth.
Wavefields in the Earth: SH waves

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Wavefield for earthquake at 600km depth.
**Wavefields in the Earth: SH waves**

Red and yellow color denote positive and negative displacement, respectively.

Wavefield for earthquake at 600km depth.
SH waves: seismograms

SH-seismograms for a source at 600 km depth
Wavefields in the Earth: P-SV waves

Red and yellow color denote positive and negative vertical displacement, respectively. Left: homogeneous mantle, right: realistic spherically symmetric model (Preliminary Reference Earth Model, PREM)

Wavefield for explosion at 600km depth.
Wavefields in the Earth: P-SV waves

Red and yellow color denote positive and negative vertical displacement, respectively. Left: homogeneous mantle, right: realistic spherically symmetric model (Preliminary Reference Earth Model, PREM)

Wavefield for explosion at 600km depth.
Wavefields in the Earth: P-SV waves

Red and yellow color denote positive and negative vertical displacement, respectively. Left: homogeneous mantle, right: realistic spherically symmetric model (Preliminary Reference Earth Model, PREM)

Wavefield for explosion at 600km depth.
Red and blue colors denote positive and negative vertical displacement, respectively. Spherically symmetric model (Preliminary Reference Earth Model, PREM).

Wavefield for explosion at surface.

Time: 150s
Red and blue colors denote positive and negative vertical displacement, respectively. Spherically symmetric model (Preliminary Reference Earth Model, PREM).

Wavefield for explosion at surface.

Time: 450s
Red and blue colors denote positive and negative vertical displacement, respectively. Spherically symmetric model (Preliminary Reference Earth Model, PREM).

Wavefield for explosion at surface.

Time: 750s
Wavefields in the whole Earth: P waves

Red and blue colors denote positive and negative vertical displacement, respectively. Spherically symmetric model (Preliminary Reference Earth Model, PREM).

Wavefield for explosion at surface.

Time: 1050s
Wavefields in the whole Earth: P waves

Red and blue colors denote positive and negative vertical displacement, respectively.
Spherically symmetric model (Preliminary Reference Earth Model, PREM)

Wavefield for explosion at surface.

Time: 1350s
Red and yellow colors denote positive and negative vertical displacement, respectively. Spherically symmetric model (Preliminary Reference Earth Model, PREM).

Wavefield for explosion at 600km depth.

Time: 125s
Red and yellow colors denote positive and negative vertical displacement, respectively. Spherically symmetric model (Preliminary Reference Earth Model, PREM).

Wavefield for explosion at 600km depth.

Time: 250s
Red and yellow colors denote positive and negative vertical displacement, respectively. Spherically symmetric model (Preliminary Reference Earth Model, PREM).

Wavefield for explosion at 600km depth.

Time: 320s
Red and yellow colors denote positive and negative vertical displacement, respectively. Spherically symmetric model (Preliminary Reference Earth Model, PREM).

Wavefield for explosion at 600km depth.

Time: 410s
Red and yellow colors denote positive and negative vertical displacement, respectively. Spherically symmetric model (Preliminary Reference Earth Model, PREM).

Wavefield for explosion at 600km depth.
P-wave seismograms for a source at 200km depth, can you identify some phases? Ray-theoretical travel times are added for the direct P wave, the PP and the PKP phase.
PKP phase at 145° distance (source at surface). Note the sudden change of amplitude! Why?
SH-wave seismograms for a source at the surface.
Three characteristic ranges used in seismic studies:

0° - 13° near-field or regional range: crustal phases, spherical geometry can be neglected

13° - 30° upper-mantle distance range. Dominated by upper mantle triplications.

30° - 180° teleseismic range: waves that sample lower mantle, core, upper mantle reverberations.
Three characteristic ranges used in seismic studies:

- **0°-13° near-field**
  complex crustal structure
  seismic reflection and refraction methods

- **13°-30° upper-mantle**
  complex tectonic features,
  high-pressure phase transitions

- **30°-180° teleseismic**
  seismic tomography, 3-D global structure
How to proceed to determine Earth structure from observed seismograms using travel times?

1. Determine epicentral distance (from P and S or Rayleigh, then compare with travel time tables)
2. Get travel times for other phases PP, ScS, pP, sS, determine differential travel times (e.g. pP-P, sS-S) to estimate source depth
3. Determine travel time perturbations from spherically symmetric model (e.g. iasp91, PREM)

- the observability of seismic phases depends on the source radiation pattern
- they are also frequency dependent
- all three components of displacement should be used for analysis
We have recorded a set of travel times and we want to determine the structure of the Earth.

In a very general sense we are looking for an Earth model that minimizes the difference between a theoretical prediction and the observed data:

$$\sum_{\text{traveltimes}} T_{\text{obs}} - T_{\text{theory}}(m) = \text{Min!}$$

where $m$ is an Earth model. For spherically symmetric media we can solve the problem analytically:
Previously we derived the travel times for a given layered velocity structure for flat and spherical media: the **forward** problem

\[
T = pX + 2 \int_0^z \sqrt{1/c^2(z) - p^2} \, dz
\]

**Flat**

\[
T = p\Delta + 2 \int_{r_0}^{r_1} \sqrt{r^2/c^2(z) - p^2} \, dr
\]

**Spherical**

The first term depends only on the horizontal distance and the second term only depends on \( r(z) \), the vertical dimension.
The solution to the inverse problem can be obtained after some manipulation of the integral:

\[ T = p\Delta + 2 \int_{r_0}^{r_1} \frac{\sqrt{r^2 / c^2(z) - p^2}}{r^2} \, dr \Leftrightarrow \ln \left( \frac{r_0}{r_1} \right) = \frac{1}{\pi} \int_{0}^{\Delta_1} \cosh^{-1} \left( \frac{p}{\xi_1} \right) \, d\Delta \]

\[ \frac{\text{forward problem}}{\text{inverse problem}} \]

The integral of the inverse problem contains only terms which can be obtained from observed \( T(\Delta) \) plots. The quantity \( \xi_1 = p_1 = (dT/d\Delta)_1 \) is the slope of \( T(\Delta) \) at distance \( \Delta_1 \). The integral is numerically evaluated with discrete values of \( p(\Delta) \) for all \( \Delta \) from 0 to \( \Delta_1 \). We obtain a value for \( r_1 \) and the corresponding velocity at depth \( r_1 \) is obtained through \( \xi_1 = r_1/v_1 \).
A first arrival travel time curve is compatible with an infinite set of structures

\[ \rightarrow \text{non-uniqueness} \]
Structural sensitivity can be improved by using the complete wavefield information and broadband data:

waveform shape can constrain complexity

Improving full wavefield modelling and inversion is one of the most important goals in modern seismology!
The three-dimensional variations in seismic velocities contain crucial information on the Earth’s dynamic behavior!

**Seismic tomography** aims at finding the 3-D velocity perturbations with respect to a spherically symmetric background model from observed seismic travel times (body waves and surface waves, free oscillations).

What are the similarities and differences to medical tomography?
A particular seismic phase has a travel time $T$ which is given by a path integral through the medium as

$$T = \int \frac{ds}{v(s)} = \int u(s) ds$$

where $u(s)$ is the slowness $[1/v(s)]$ along the path $s$. A travel time perturbation can happen anywhere along the path

$$\int \Delta u(s) ds = \Delta T = T_{\text{obs}} - T_{\text{pred}}$$

A medium is discretized into blocks and thus we can calculate the path length $l_j$ in each block to obtain

$$\Delta T = \sum_j l_j \Delta u_j \quad \text{for many observations} \quad \Delta T_i = \sum_j l_{ij} \Delta u_j$$

We want to find $\Delta u_i$ from observed travel times $\rightarrow$ inverse problem
Receiver functions have been used recently to study upper mantle structure.

**FIGURE 7.B1.1** Receiver function analysis of the crustal velocity structure under Death Valley, California. The left panel shows velocity structures obtained by inversions of the observed receiver function, labeled “data” in the upper-right box. The models differ because of different initial models in the inversion, but all produce reasonable fits to the data. To better resolve the structure, short-period surface-wave dispersion observations are modeled as well. (Courtesy of S. Beck and G. Zandt.)
Heterogeneities inside the Earth

(1) Global average; (2,3) lower mantle; (4,5,6) upper mantle from surface waves; (7) asthenosphere; (8) upper mantle from body waves; (9) upper mantle; (11-14) lithosphere; (15,16) crust.
Multiple reflections from the core mantle boundary can be used to infer the attenuation of seismic waves inside the mantle.
The most important information on the 3-D structure of the Earth is contained in the travel times of particular seismic phases (e.g. P, S, ScS, PcP, PKP, PPP, sSS, etc.) travelling through the Earth's interior.

The radial structure of the Earth explains all observed travel times to within 1% accuracy. Several such structures have been determined since the 1940s (e.g. Jeffrey-Bullen, Herrin, PREM, iasp91).

The radial structure of the Earth can be estimated using first-arrival travel times and the Wiechert-Herglotz inversion technique.

The deviations of the observed travel-times from the predicted travel times for spherically symmetric models are used to estimate the Earth's 3-D seismic velocity structure. This processing is called seismic tomography.

Although the travel time data are explained to within 1% by a spherically symmetric structure, the 3-D velocity structure contains crucial information on the dynamic properties of the Earth's mantle (e.g. subducting slabs, plumes, etc.)