

- 1) Show that

$$\frac{f(x + \Delta x) - 2f(x) + f(x - \Delta x))}{\Delta x^2}$$

is an approximation for the second derivative. Hint: Use Taylor's theorem.

- 2) A seismometer consists of a spring with constant k , damping parameter D , and mass m . The seismometer is excited by the ground motion $\ddot{u}(t)$. The relative motion of the mass is governed by the following equation

$$m\ddot{x} + \frac{D}{m}\dot{x} + \frac{k}{m}x = \ddot{u}$$

Replace the derivatives on the l.h.s. with finite differences. Solve for $x(t+dt)$.

- 3) Some isotopes (e.g. ^9Be) are washed into the oceans by rivers and are then transported by advection and diffusion. Some of the isotopes are removed of the system (e.g. by sedimentation). These processes can be described by the diffusion-advection-reaction equation. Here, $c(x,t)$ is the tracer concentration, k is the diffusivity, $R(x)$ is the reactivity, $p(x)$ are sources, $v(x)$ is the velocity field. Replace the partial derivatives with finite differences and solve for $c(t+dt)$. How could you realize a circular flow with this model?

$$\frac{\partial c}{\partial t} = k \frac{\partial^2 c}{\partial x^2} + v_x \frac{\partial c}{\partial x} - RC + p$$