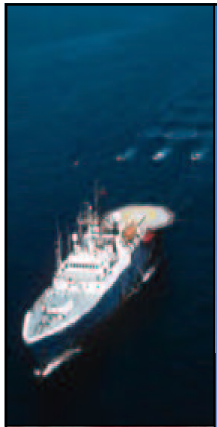
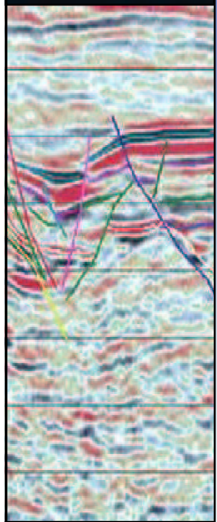
Refraction seismics





- **Two-layer case**
  - Travel times
  - Travel time curves
  - The inverse problem
- **Three-layer case**
- **Inclined layers**
- **N-layer case**
- **Applications**
- **Seismic Tomography**

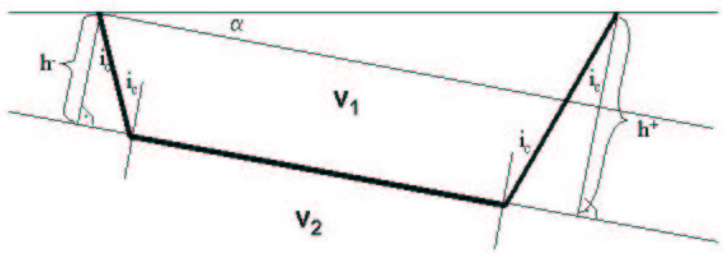
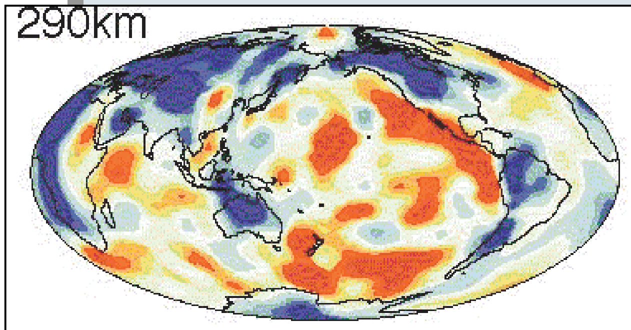
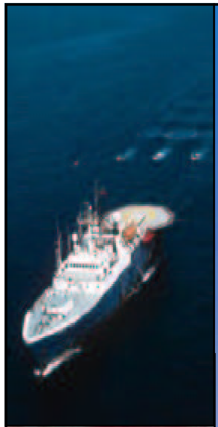
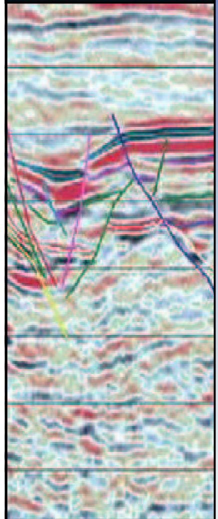


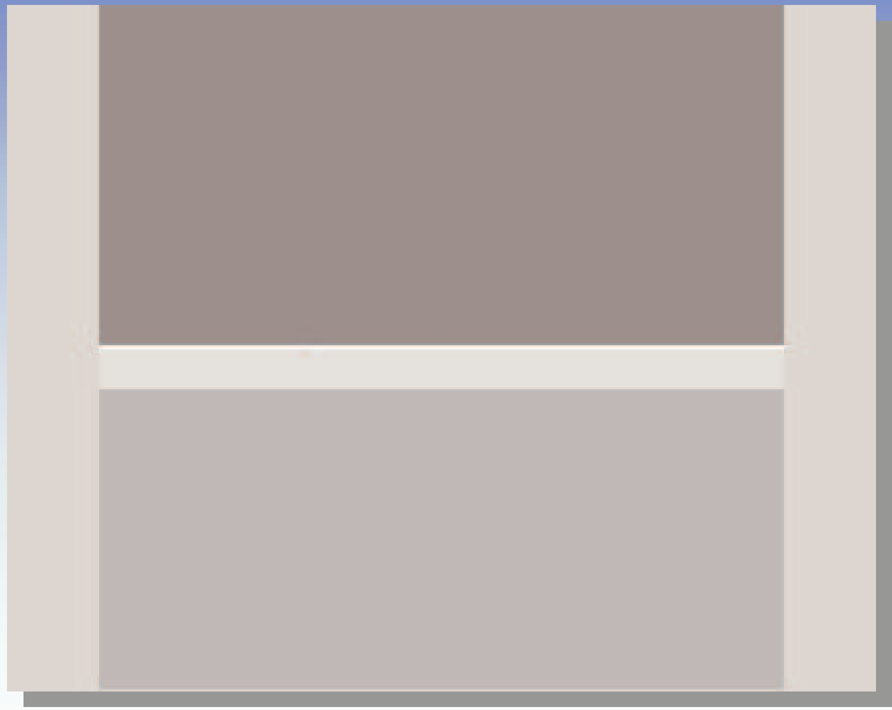
Figure 5: Refracted ray path for the inclined 2-layer case.

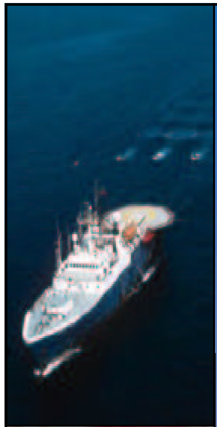


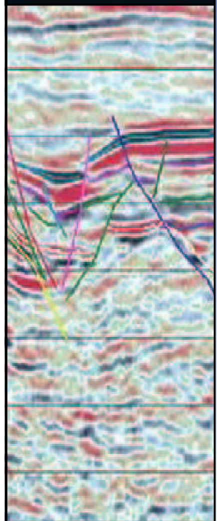
The theoretical part of this lecture is also available as script (www)

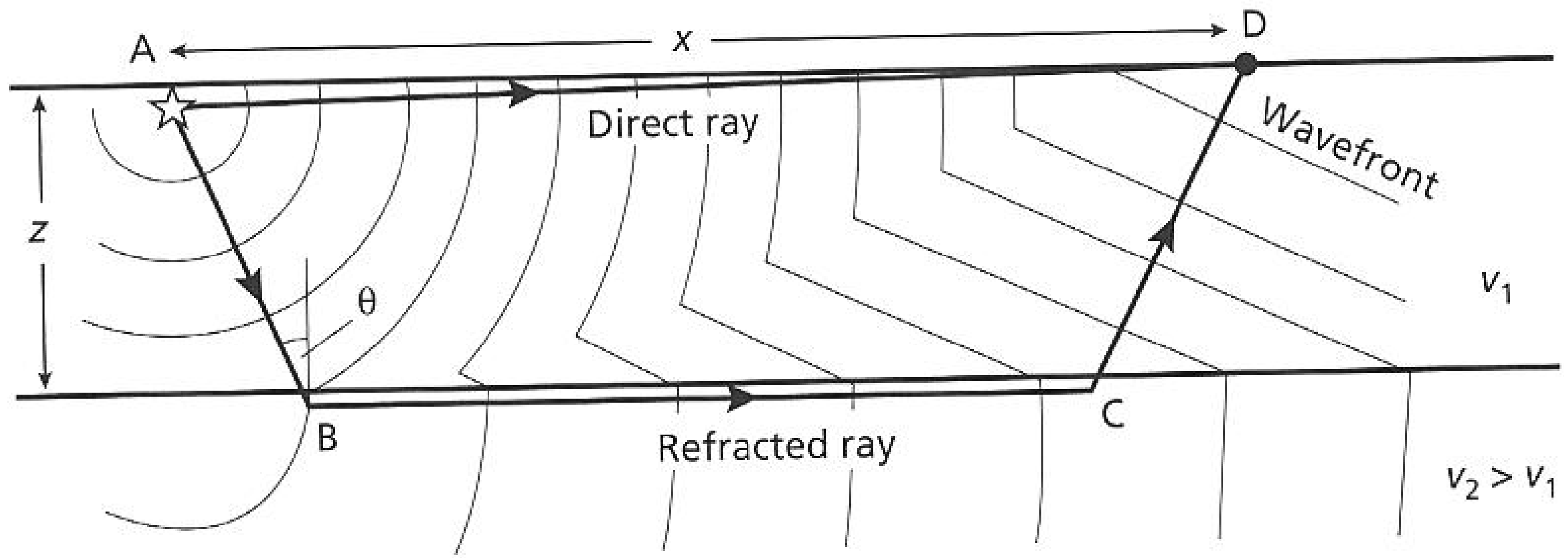

Snapshots and seismograms:
Refracted waves



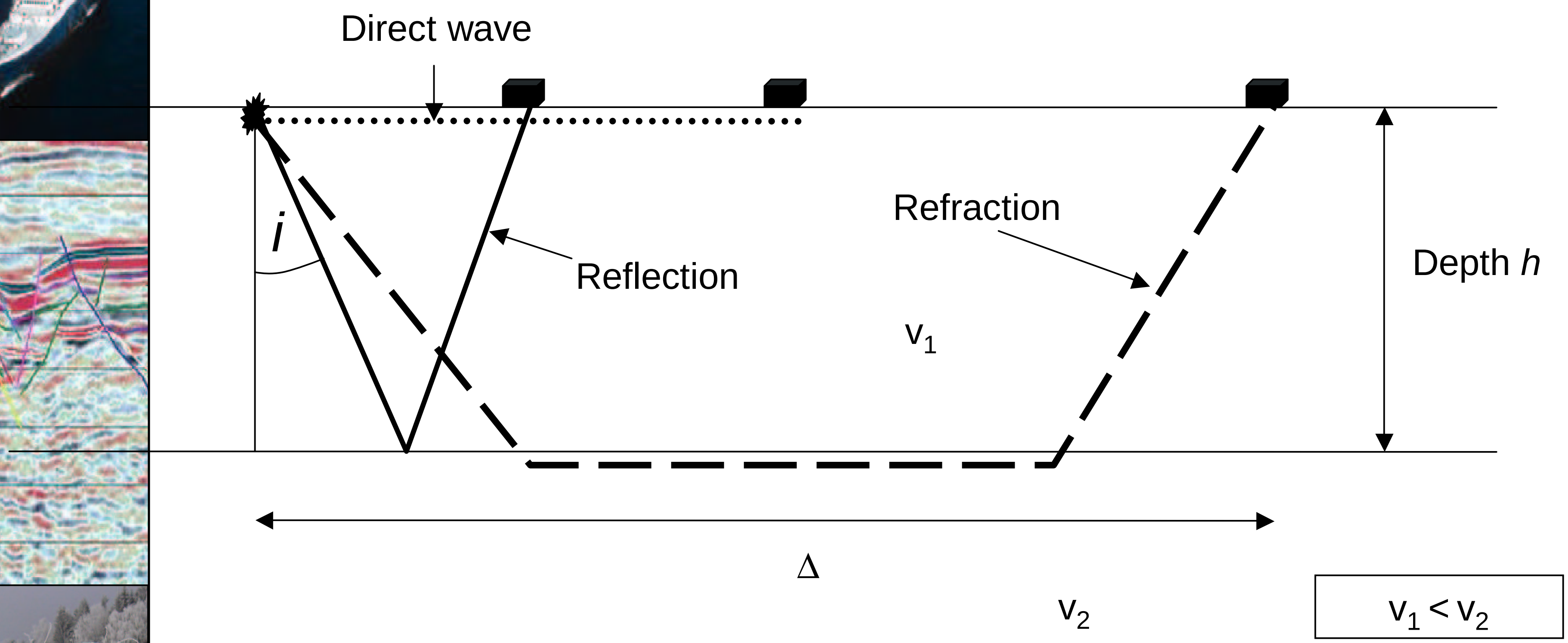




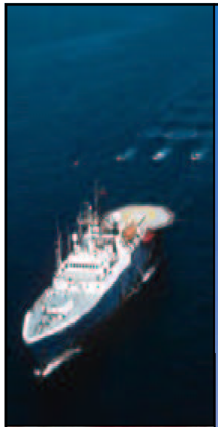
Rays and wavefronts

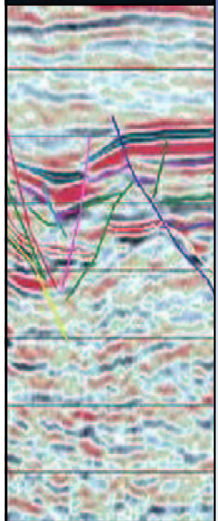


Refraction profile



Geometry of reflection/refraction experiment. There are three arrivals recorded at greater distances: the direct wave, the reflection from the discontinuity at depth h and the refracted wave.





Refraction experiment: Arrivals

Direct wave

$$t_{dir} = \Delta / v_1$$

Reflected wave

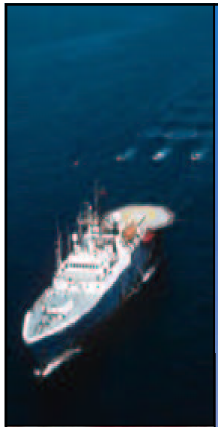
$$t_{refl} = \frac{2}{v_1}\sqrt{(\Delta/2)^2 + h^2}$$

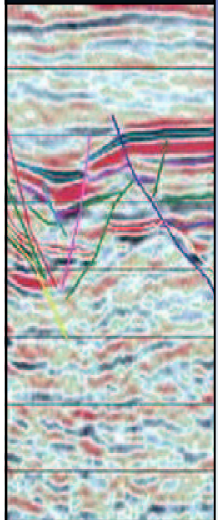
Refracted wave

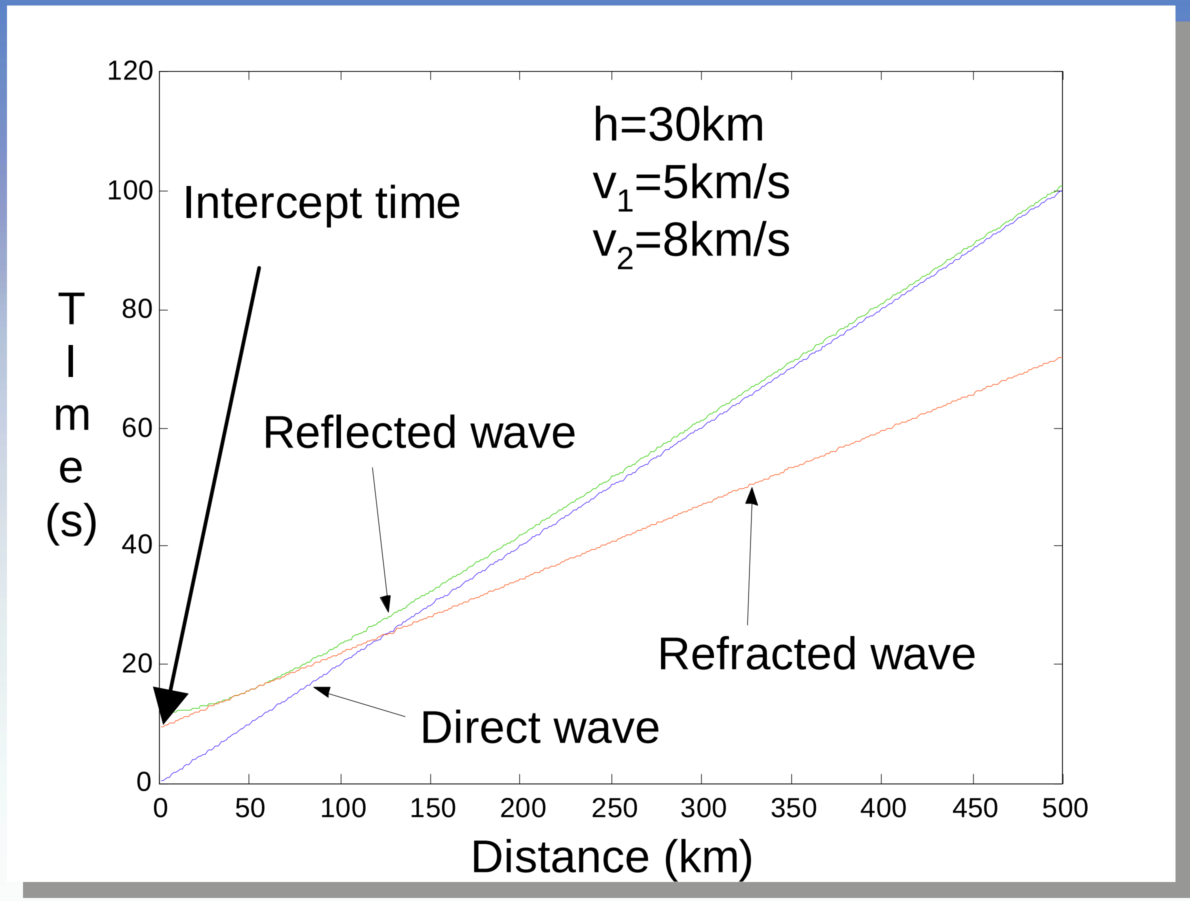
$$t_{refr} = \frac{2h\cos i_c}{v_1} + \frac{\Delta}{v_2} = t^i_{refr} + \frac{\Delta}{v_2}$$

Intercept time

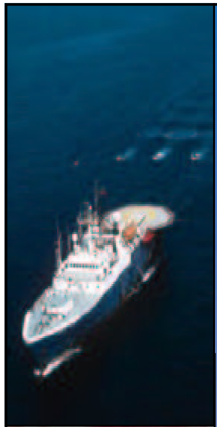
Two-layer case
Travel time diagram

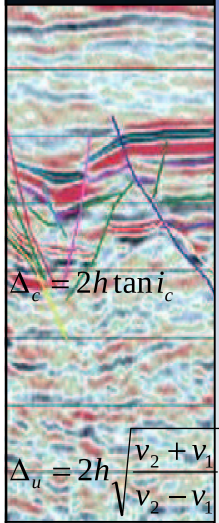






This could correspond to a very simple model of crust and upper mantle and the discontinuity would be the Moho





Critical distance
Overtaking distance

The *critical distance* is the distance at which the refracted wave is first observed according to ray theory (in real life it is observed already at smaller distances, this is due to *finite-frequency* effects which are not taken into account by standard ray theory). The critical distance D_c is from basic geometry

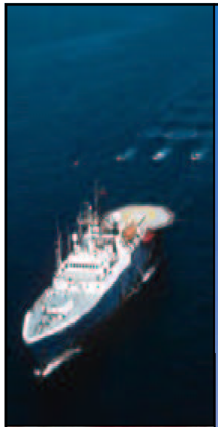
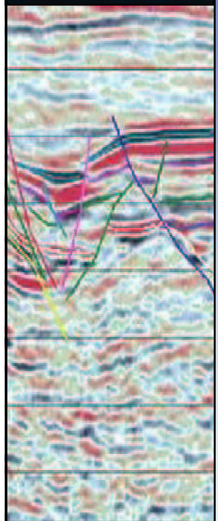
critical distance

$$\Delta_c = 2h \tan i_c$$

where the critical angle i_c is given by Snell's law. If we equate the arrival time of the direct wave and the refracted wave and solve for the distance we obtain the overtaking distance. It is given by

overtaking distance

$$\Delta_{\ddot{u}} = 2h\sqrt{\frac{v_2 + v_1}{v_2 - v_1}}$$

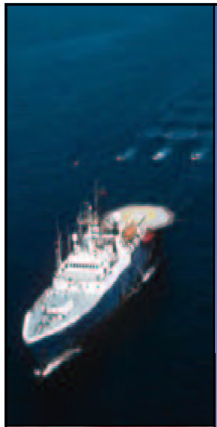

The inverse problem
structure from travel time curves

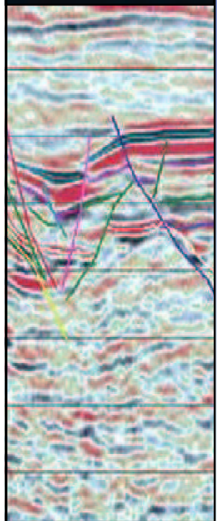
- Determine v_1 from the slope (1/ v_1) of the direct wave.
- Determine v_2 from the slope (1/ v_2) of the refracted wave.
- Calculate the critical angle from v_1 and v_2.
- Read the intercept time t_i from the travel-time diagram.
- Determine the depth z using $$h = \frac{v_1 t_i}{2 \cos i_c}$$

or

- Read the overtaking distance from the travel-time diagram, and calculate z using .

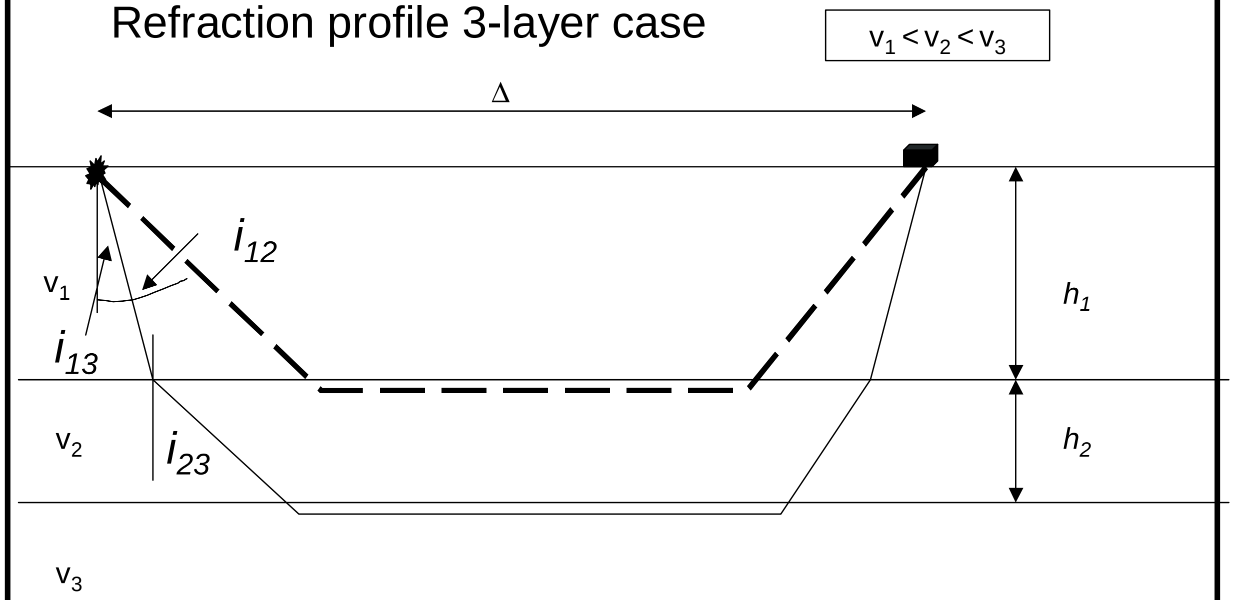
$$\Delta_{ü} = 2h\sqrt{\frac{v_2 + v_1}{v_2 - v_1}}$$





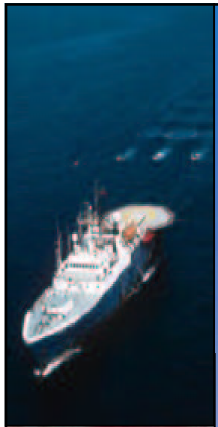
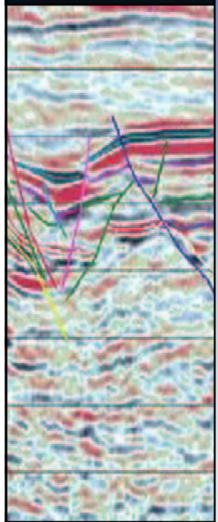
Three-layer case

Refraction profile 3-layer case



We need to find arrival times of

- Direct waves
- Refractions from each interface


Three-layer case
Arrival times

Direct wave

$$t_1 = \Delta / v_1$$

Refraction Layer 2

$$t_2 = \frac{2h_1 \cos i_{12}}{v_1} + \frac{\Delta}{v_2} = t^{i2} + \frac{\Delta}{v_2}$$

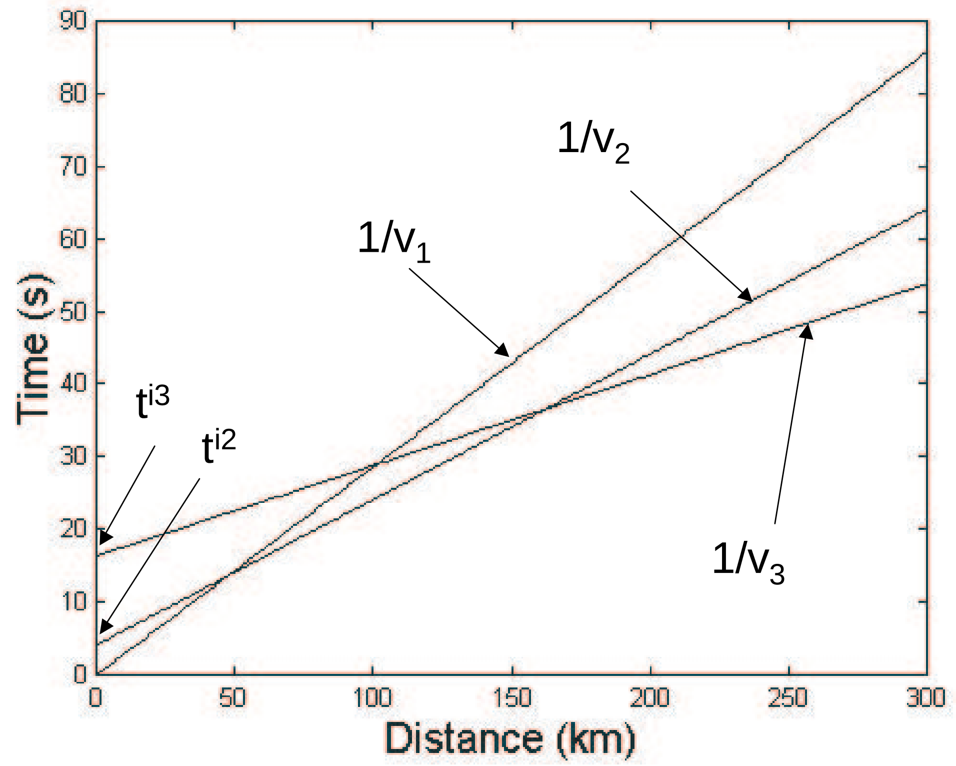
Refraction Layer 3

$$t_3 = \underbrace{\frac{2h_1 \cos i_{13}}{v_1} + \frac{2h_2 \cos i_{23}}{v_2}}_{t^{i3}} + \frac{\Delta}{v_3} = t^{i3} + \frac{\Delta}{v_3}$$

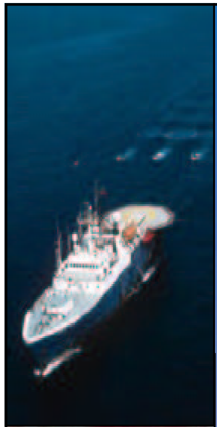
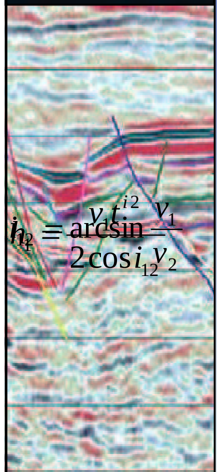
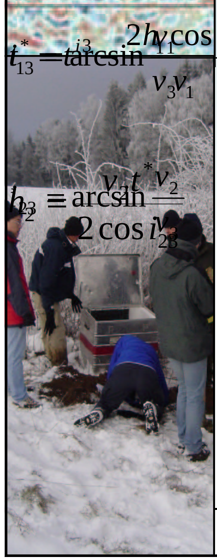
using ...

$$\frac{\sin i_{13}}{v_1} = \frac{\sin i_{23}}{v_2} = \frac{\sin i_{33}}{v_3} = \frac{1}{v_3}$$

Three-layer case
Travel time curves



Travel-time diagram for the 3-layer case

The inverse problem
three-layer case

- Determine the velocities $v_{1\text{-}3}$ from the slopes ($1/v_{1\text{-}3}$) in the travel-time diagram.
- Read the intercept time t^{i2} for the refraction from layer 2.
- Determine thickness h_1 using the equation for t_2 such that ,

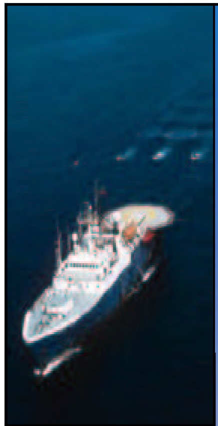
$$h_1 = \frac{v_1 t^{i2}}{2\cos i_{12}} \qquad \text{where} \qquad i_{12} = \arcsin\frac{v_1}{v_2}$$

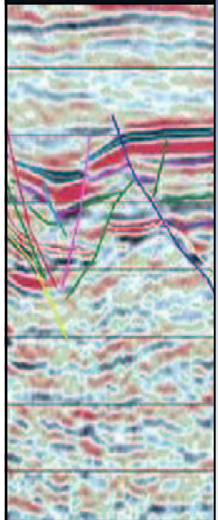
- Read the intercept time t^{i3} for the refraction from layer 3.
- Calculate with the already determined values h_1 an intermediate intercept time t^*

$$t^* = t^{i3} - \frac{2h_1 \cos i_{13}}{v_1} \qquad \text{where} \qquad i_{13} = \arcsin\frac{v_1}{v_3}$$

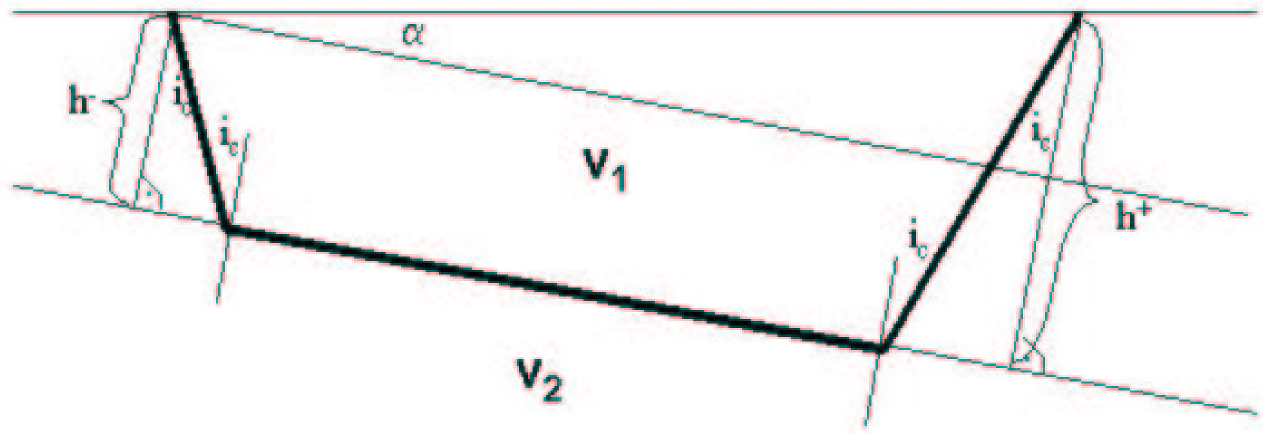
- Using t^* calculate the thickness h_2 of layer 2

$$h_2 = \frac{v_2 t^*}{2\cos i_{23}} \qquad \text{where} \qquad i_{23} = \arcsin\frac{v_2}{v_3}$$



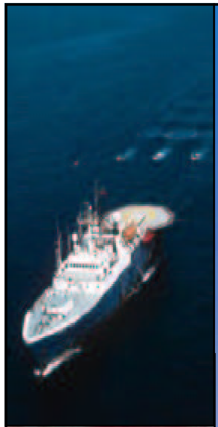
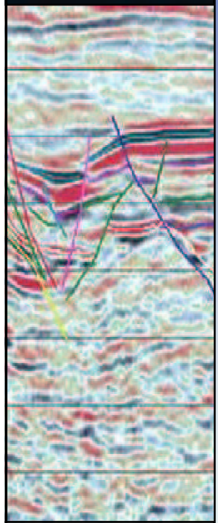


Inclined layers

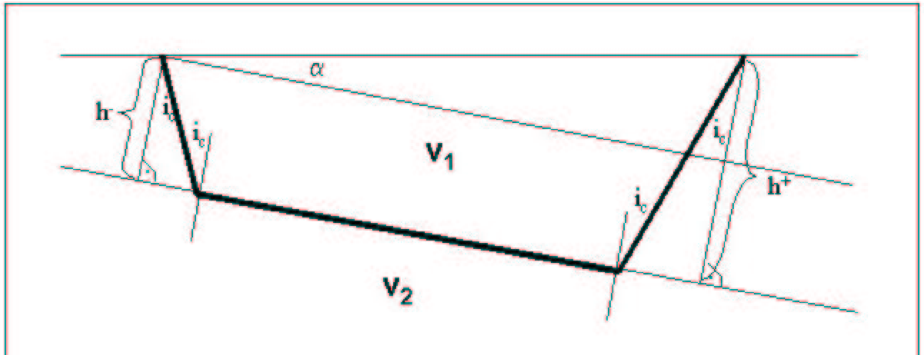


Refracted ray path for the inclined 2-layer case.

In this asymmetric situation we need to shoot from both sides in order to get h^+ and h^- and reconstruct the model.

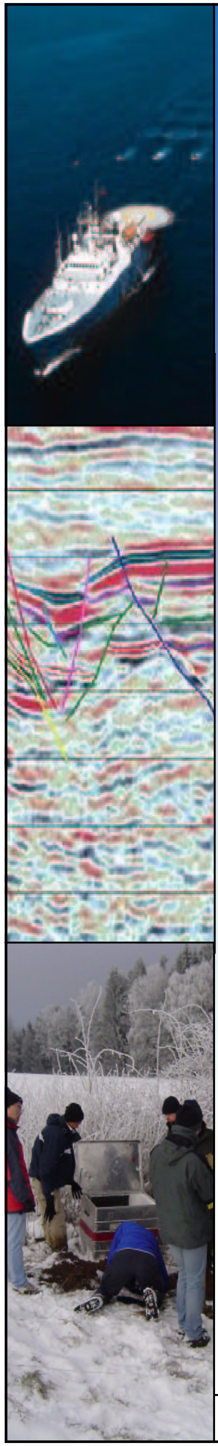

Inclined layers
travel times



Refracted ray path for the inclined 2-layer case.

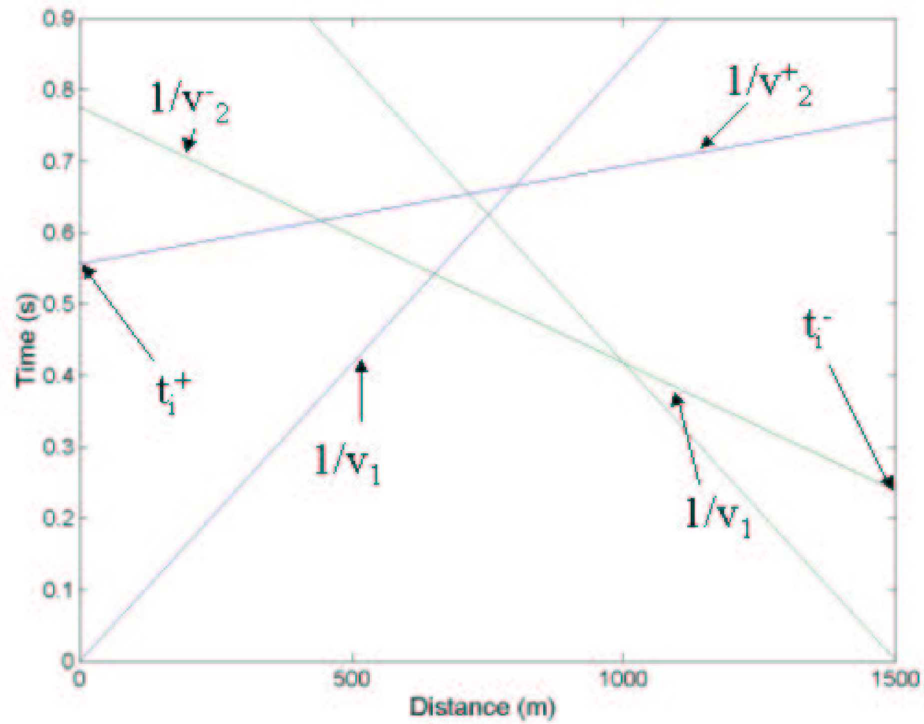
$$t^-_{refr} = \frac{2h^- \cos i_c}{v_1} + \frac{\sin(i_c + \alpha)}{v_1}\Delta = t_i^- + \frac{1}{v_2^-}\Delta$$

$$t^+_{refr} = \frac{2h^+ \cos i_c}{v_1} + \frac{\sin(i_c - \alpha)}{v_1}\Delta = t_i^+ + \frac{1}{v_2^+}\Delta$$

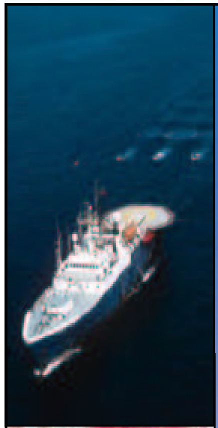


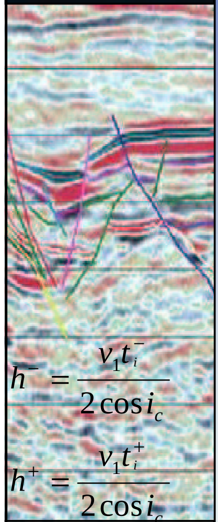
Inclined layer

Travel time curves



Travel-time diagram for the inclined-layer case for the model in the previous figure.





Inverse problem
Inclined layer

- Determine the velocities v_1 and $v_2^{+/-}$ from the slopes in the travel-time diagram.

- Use the following relations to determine α and v_2:

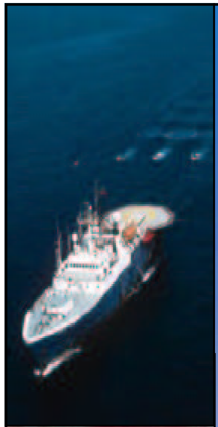
$$\sin(i_c + \alpha) = \frac{v_1}{v_2^-} \Rightarrow i_c + \alpha = \arcsin\frac{v_1}{v_2^-}$$
$$\sin(i_c - \alpha) = \frac{v_1}{v_2^+} \Rightarrow i_c - \alpha = \arcsin\frac{v_1}{v_2^+}$$

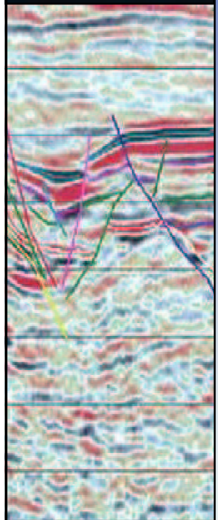
$$\frac{(i+\alpha)+(i-\alpha)}{2} = i \Rightarrow v_2 = \frac{v_1}{\sin i}$$
$$\frac{(i+\alpha)-(i-\alpha)}{2} = \alpha$$

- Read the intercept times t_i^+ and t_i^- from the travel time diagram. Determine the distances from the layer interface as

$$h^- = \frac{v_1 t_i^-}{2\cos i_c}$$
$$h^+ = \frac{v_1 t_i^+}{2\cos i_c}$$

- You can now graphically draw the layer interface by drawing circles around the profile ends with the corresponding heights $h^{+/-}$ and tangentially connecting the circles at depth.





The n-layer case

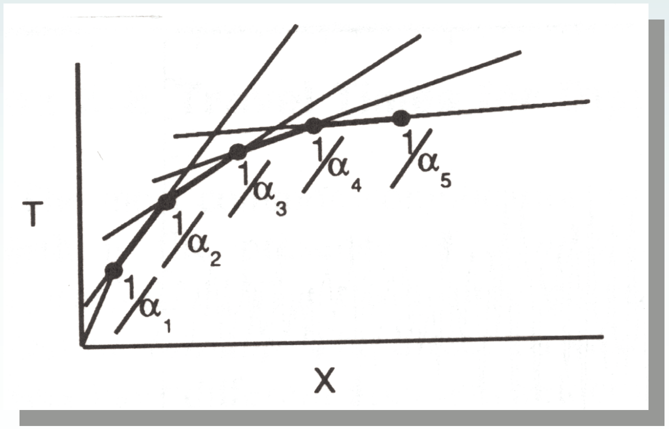
$$t_2 = \frac{2h_1 \cos i_{12}}{v_1} + \frac{\Delta}{v_2} = t^{i2} + \frac{\Delta}{v_2}$$

$$t_3 = \underbrace{\frac{2h_1 \cos i_{13}}{v_1} + \frac{2h_2 \cos i_{23}}{v_2}}_{t^{i3}} + \frac{\Delta}{v_3} = t^{i3} + \frac{\Delta}{v_3}$$

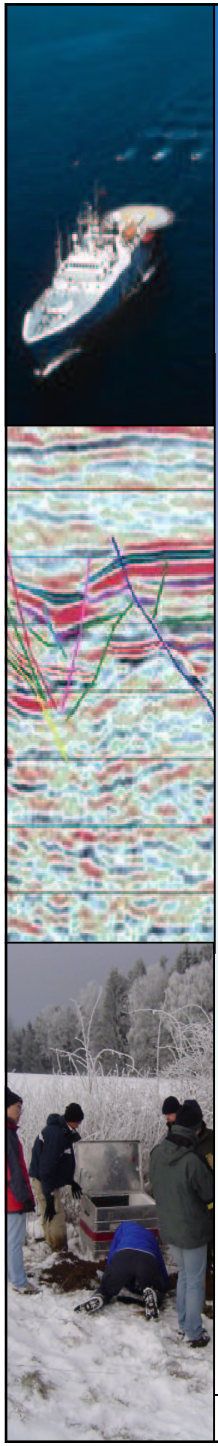
This sequence for the 3-layer case suggests that the arrival time is generalizable to any number of plane layers:

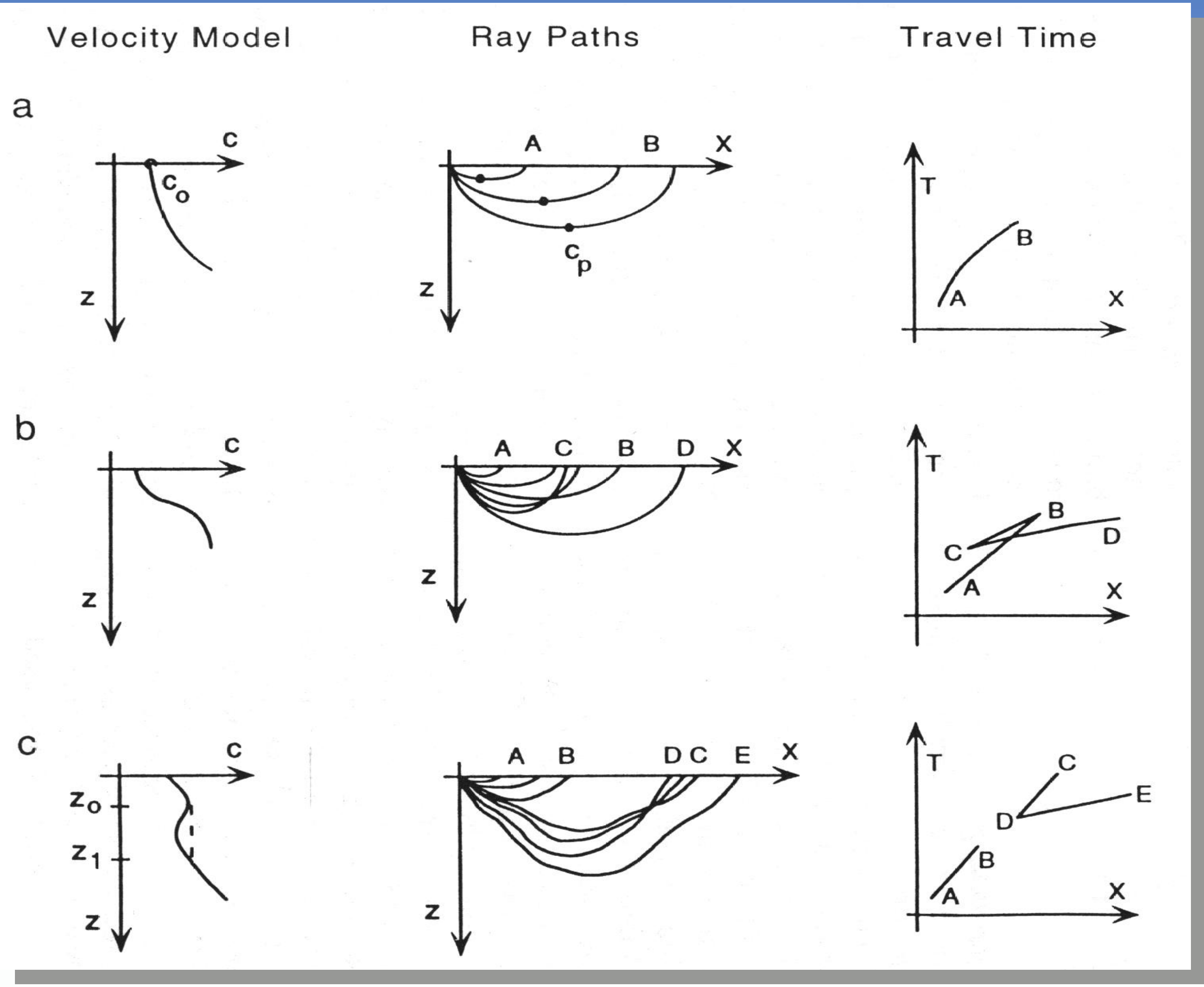
$$t_n = \frac{\Delta}{v_n} + \sum_{i=1}^{n-1} \frac{2h_i \cos i_{in}}{v_i}$$

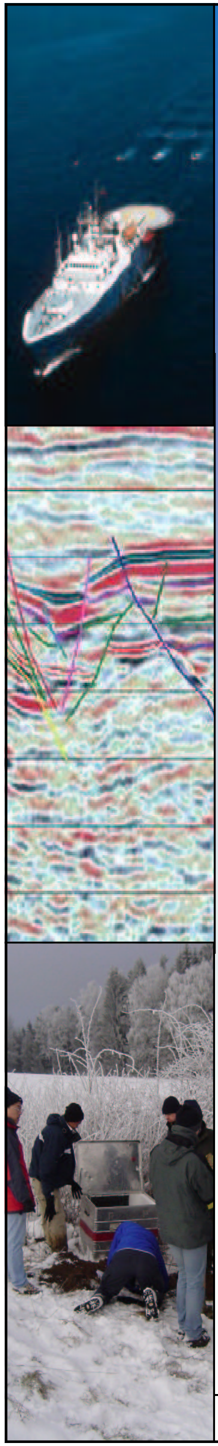
$$i_{in} = \sin^{-1}\left(\frac{v_i}{v_n}\right)$$



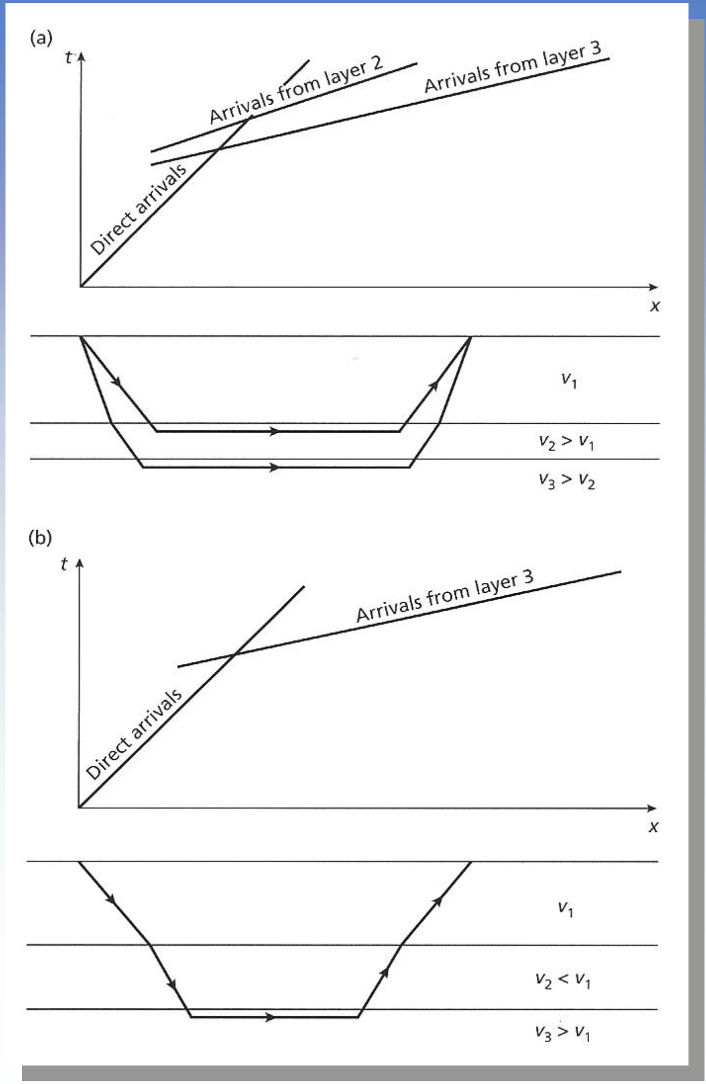
Continuous models

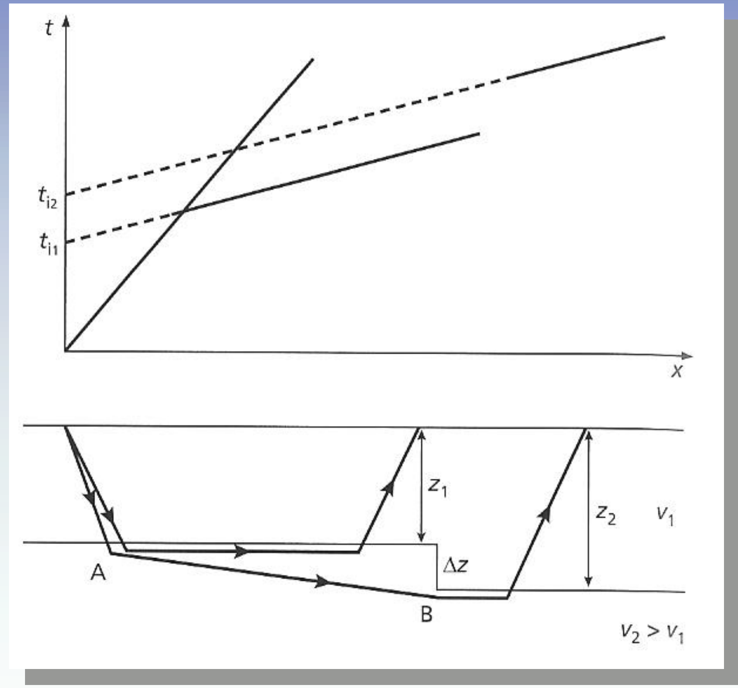


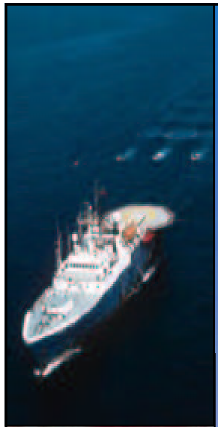


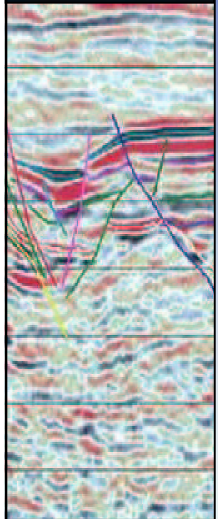


Hidden layers - Faults

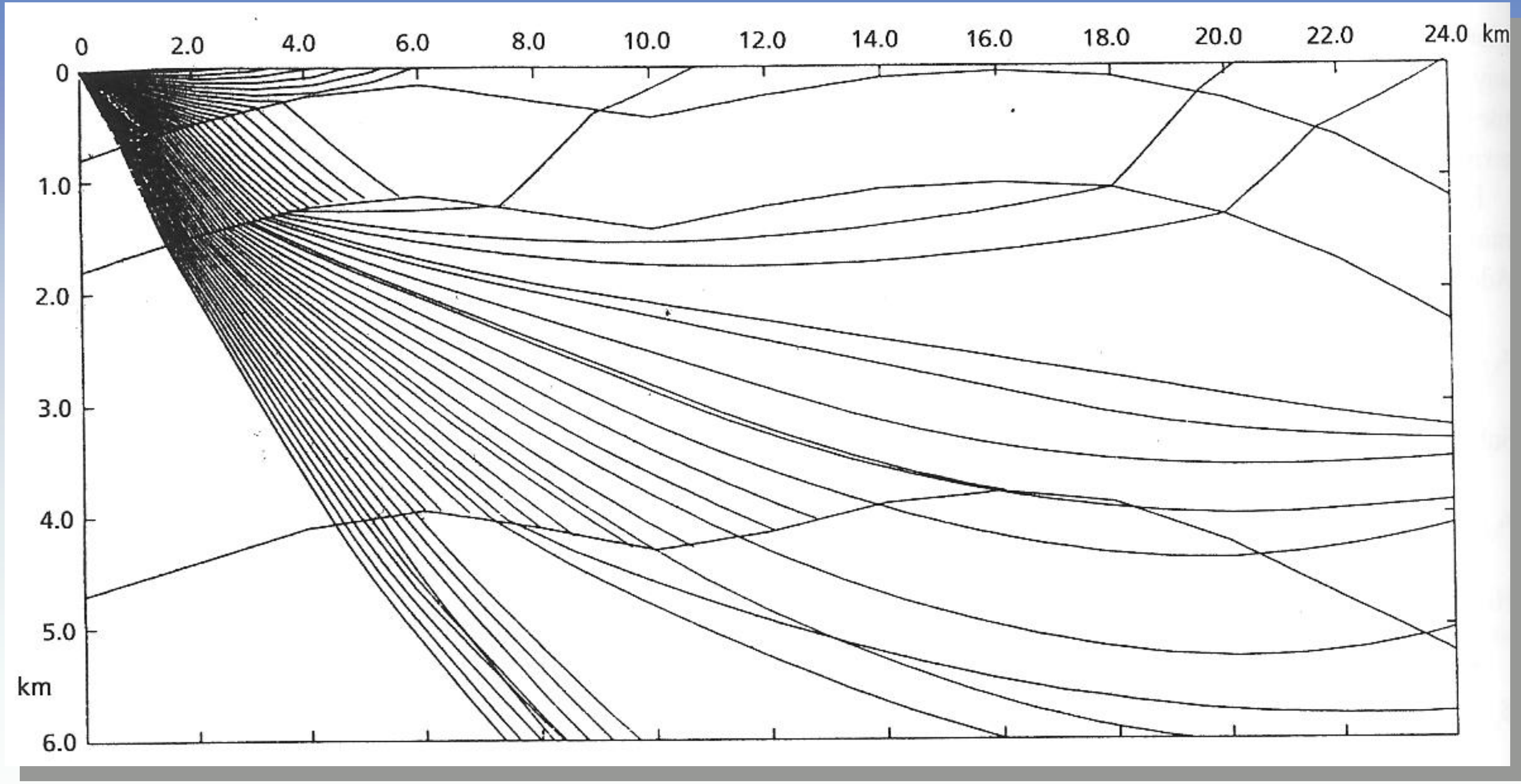


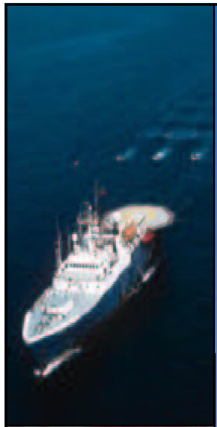


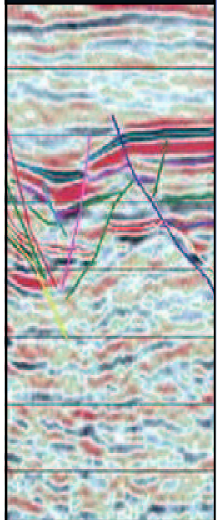




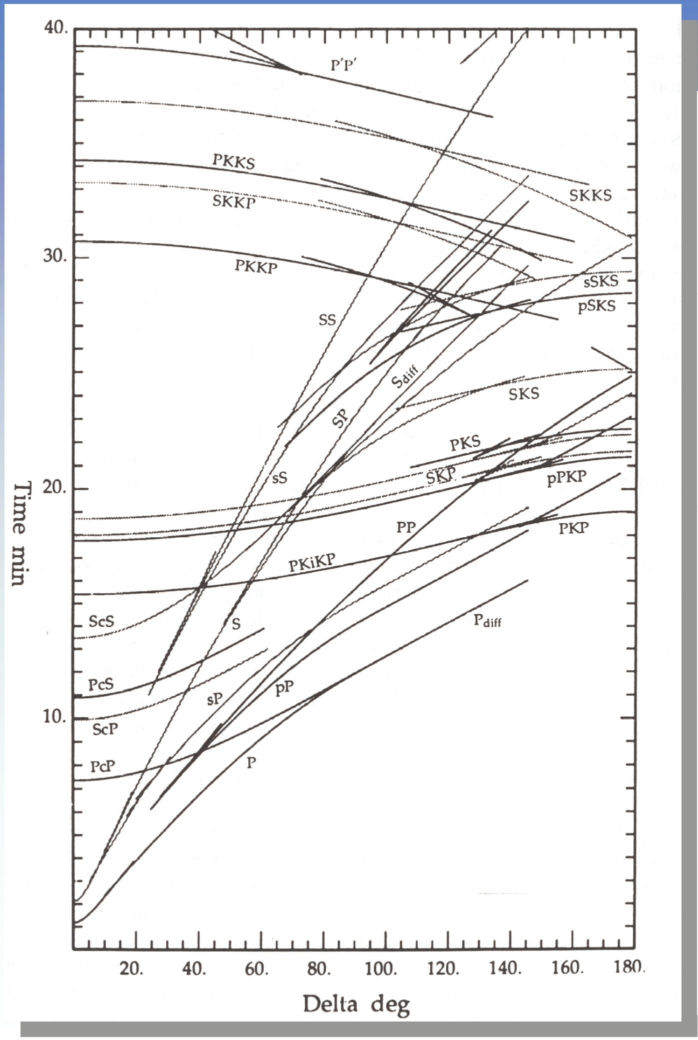
Rays in a complex model

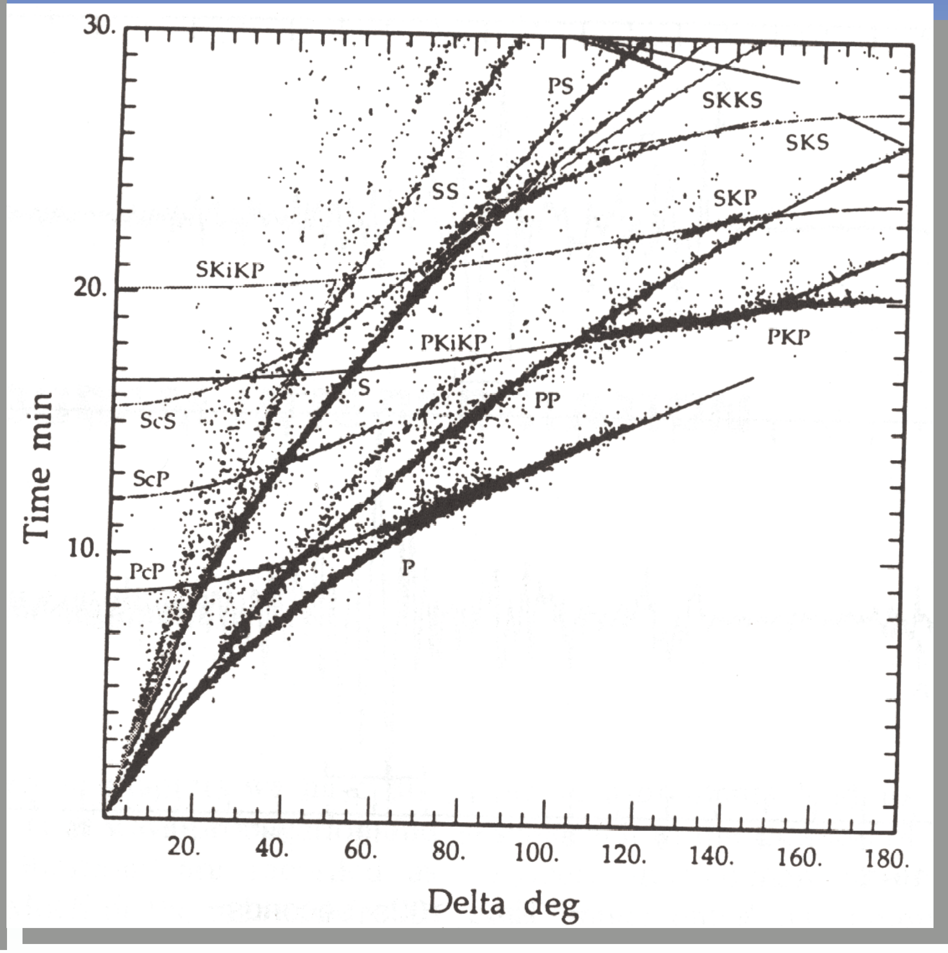


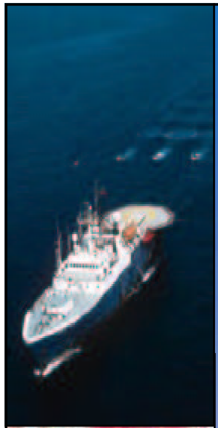
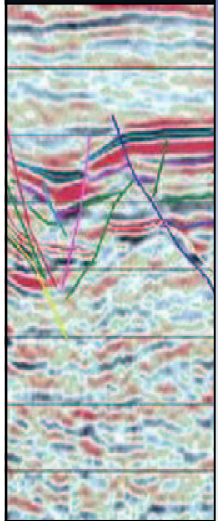




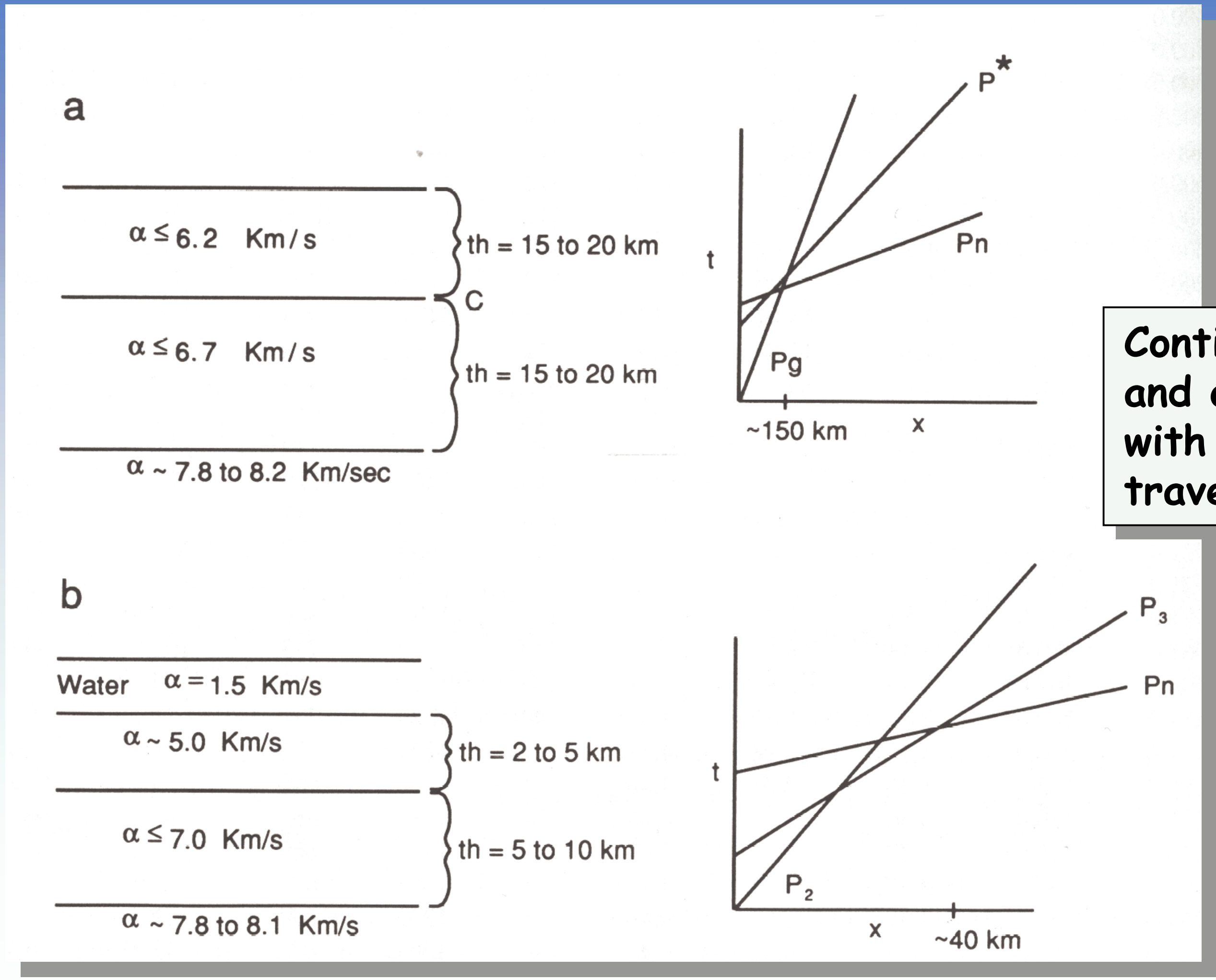
Global seismology
travel times



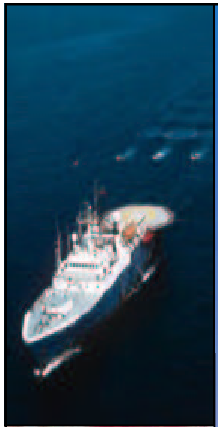
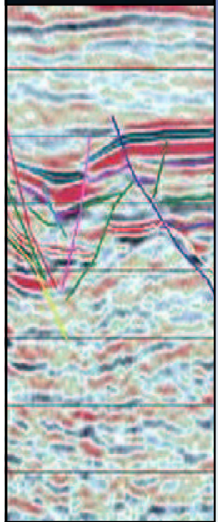



The Earth's Crust: Travel Times



Continental crust (a) and oceanic crust (b) with corresponding travel-time curves

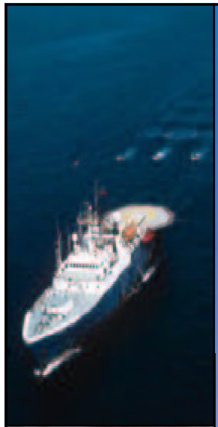

The Earth's Crust: Minerals and Velocities

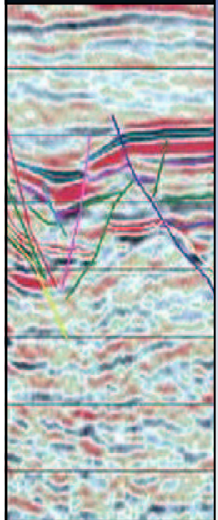
TABLE 3-3
Average Crustal Abundance, Density and Seismic Velocities of Major Crustal Minerals

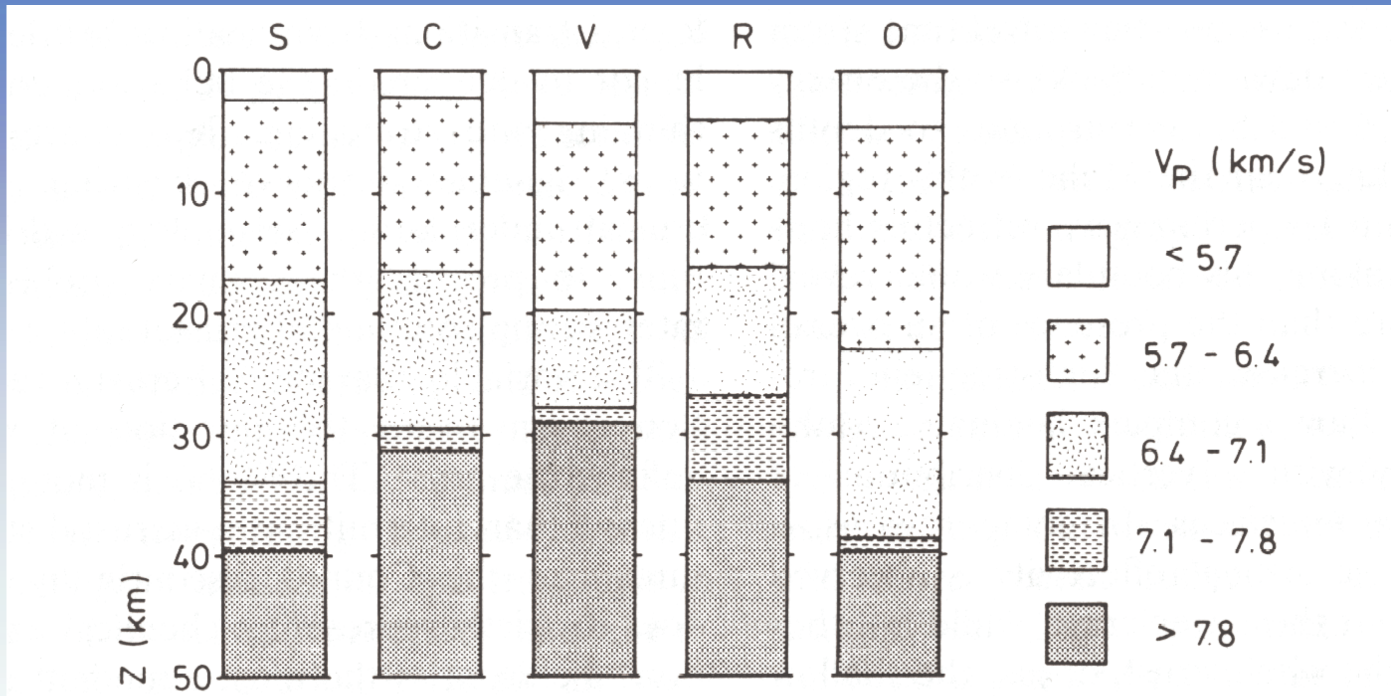
Mineral	Volume percent	ρ (g/cm³)	V_p (km/s)	V_s (km/s)
Quartz	12	2.65	6.05	4.09
K-feldspar	12	2.57	5.88	3.05
Plagioclase	39	2.64	6.30	3.44
Micas	5	2.8	5.6	2.9
Amphiboles	5	3.2	7.0	3.8
Pyroxene	11	3.3	7.8	4.6
Olivine	3	3.3	8.4	4.9

Average crustal abundance, density and seismic velocities of major crustal minerals.

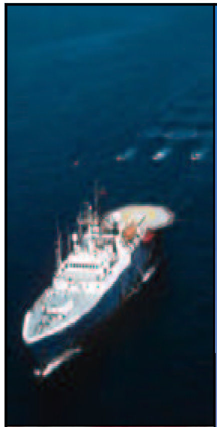
The Earth's Crust: Crustal Types

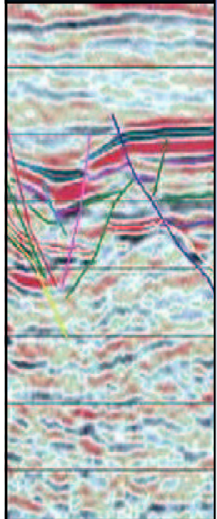




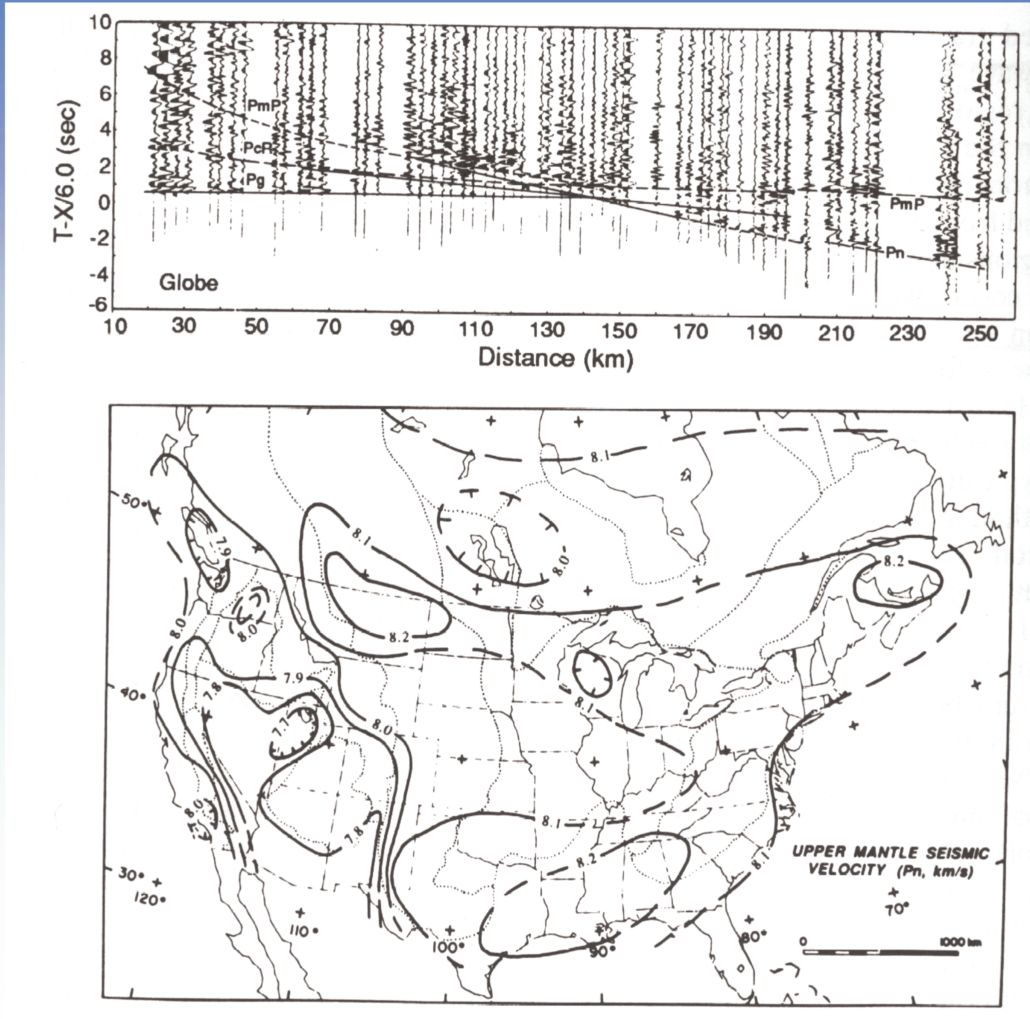


S shields, C Caledonian provinces, V Variscan provinces, R rifts, O orogens



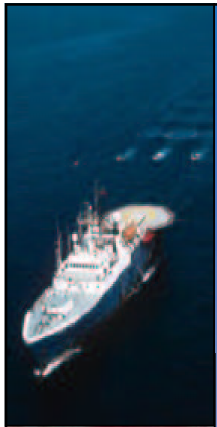


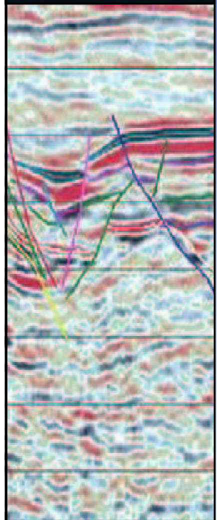
The Earth's Crust: Refraction Studies



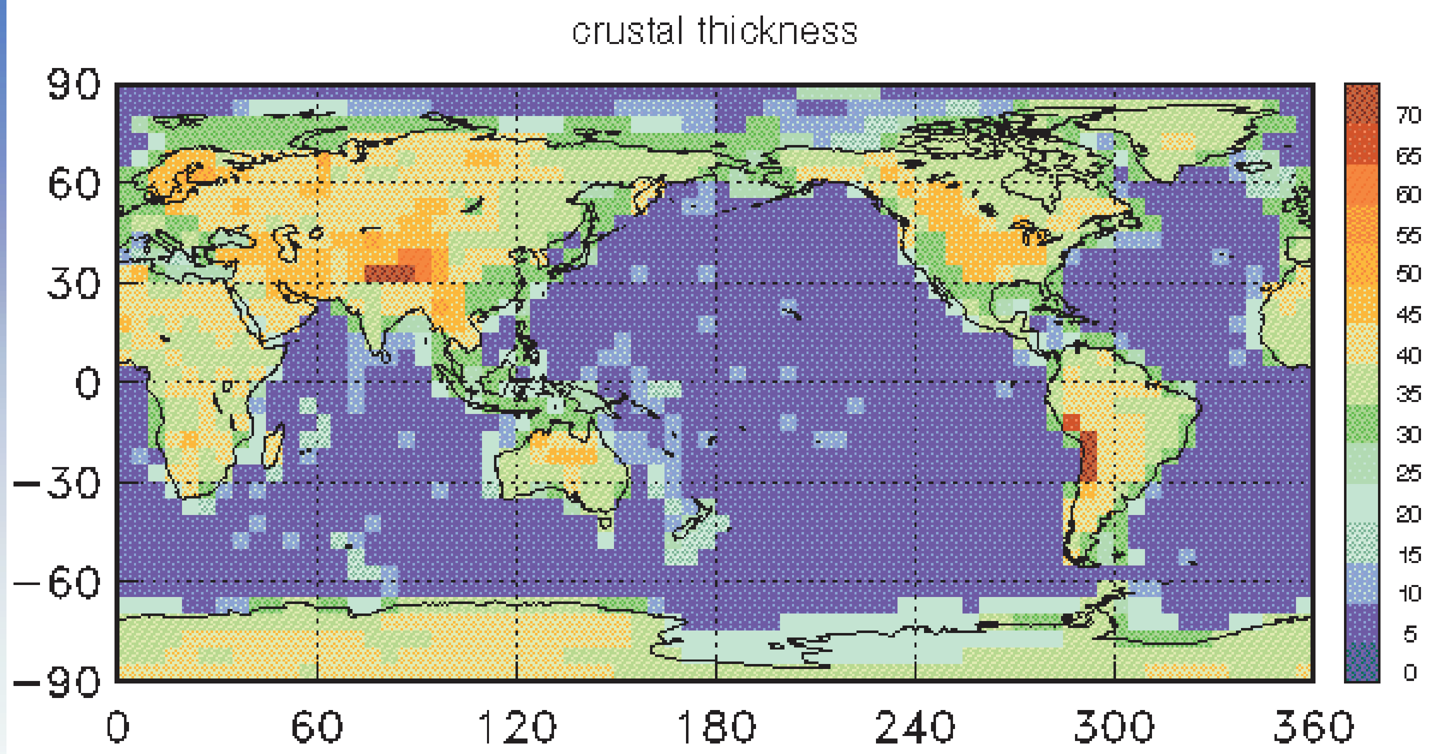
Refraction profiles across North America, (reduction velocity 6km/s) all the determination of lateral velocity variations:

PmP Moho reflection
Pn Moho refraction
Pg direct crustal wave

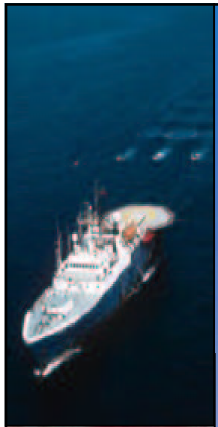


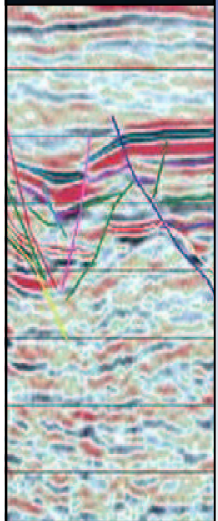


The Earth's crust: Crustal Types

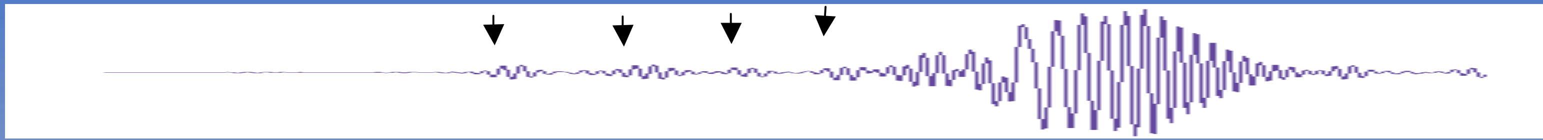


Recently compiled world-wide crustal thickness (km) indicates cratonic areas and mountain ranges with active tectonics. These data are important to correct travel times regionally, i.e. calculate the contribution of crustal thickness to a teleseismic travel-time perturbation.





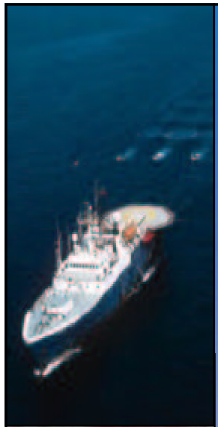
Earth Structure Inversion

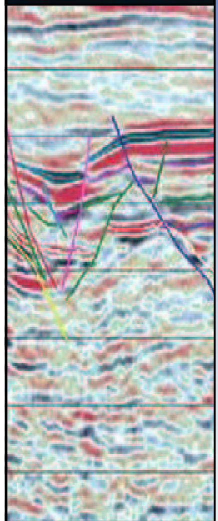


How to proceed to determine Earth structure from observed seismograms using travel times?

1. Determine epicentral distance (from P and S or Rayleigh, then compare with travel time tables)
2. Get travel times for other phases PP, ScS, pP, sS, determine differential travel times (e.g. pP-P, sS-S) to estimate source depth
3. Determine travel time perturbations from spherically symmetric model (e.g. iasp91, PREM)

- the observability of seismic phases depends on the source radiation pattern
- they are also frequency dependent
- all three components of displacement should be used for analysis





Earth Structure Inversion

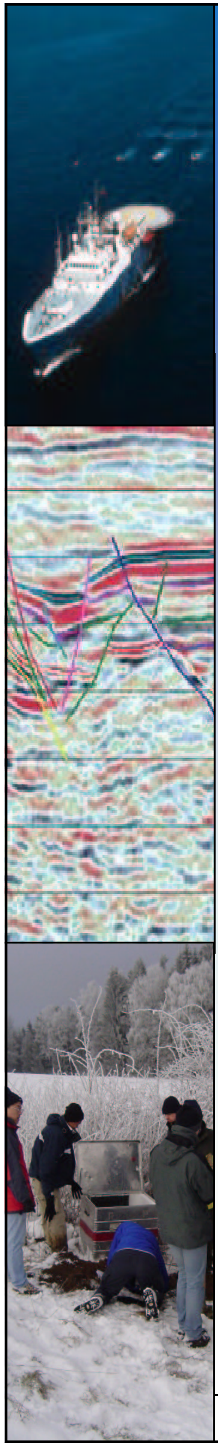
.
We have recorded a set of travel times and we want to determine the structure of the Earth.

In a very general sense we are looking for an Earth model that minimizes the difference between a theoretical prediction and the observed data:

$$\sum_{traveltimes} T_{obs} - T_{theory}(m) = Min!$$

where m is an Earth model. For spherically symmetric media we can solve the problem analytically!

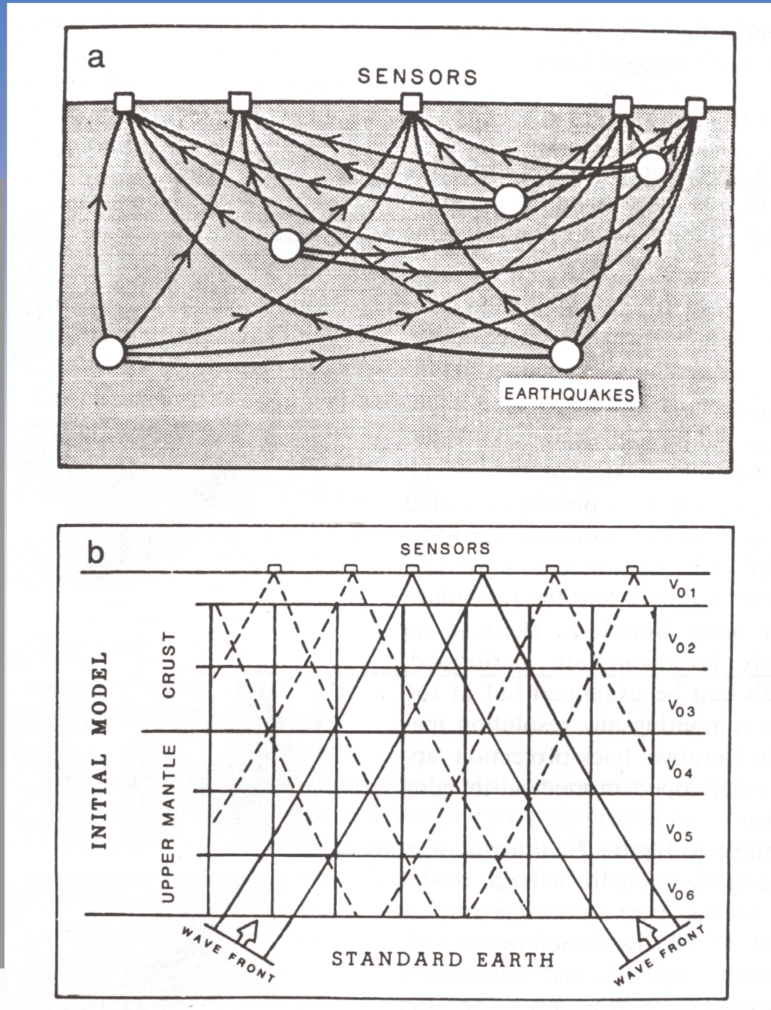
Seismic Tomography

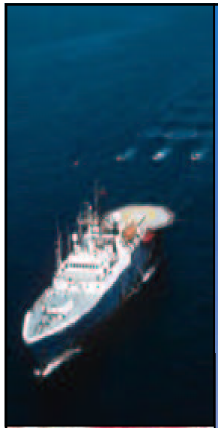
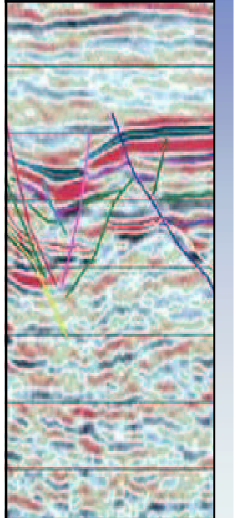


The three-dimensional variations in seismic velocities contain crucial information on the Earth's dynamic behavior!

Seismic tomography aims at finding the 3-D velocity perturbations with respect to a spherically symmetric background model from observed seismic travel times (body waves and surface waves, free oscillations)

What are the similarities and differences to medical tomography?



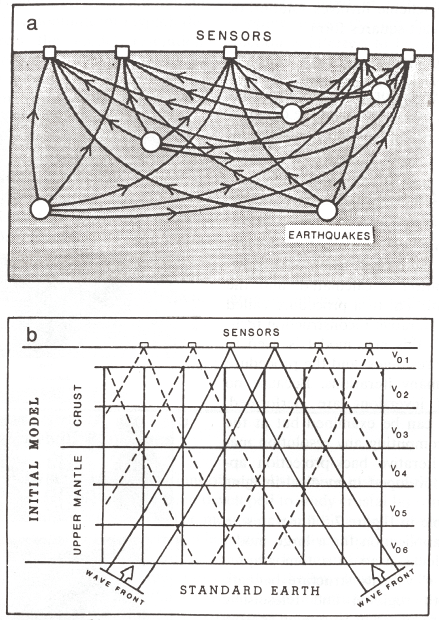

Seismic Tomography - Principles

A particular seismic phase has a travel time T which is given by a path integral through the medium as

$$T = \int_s \frac{ds}{v(s)} = \int_s u(s) ds$$

where u(s) is the slowness [1/v(s)] along the path s. A travel time perturbation can happen anywhere along the path

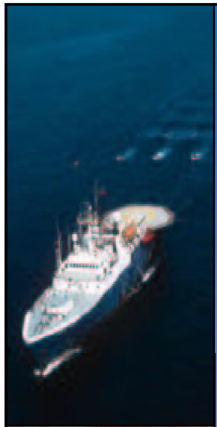
$$\int_s \Delta u(s) ds = \Delta T = T_{obs} - T_{pred}$$

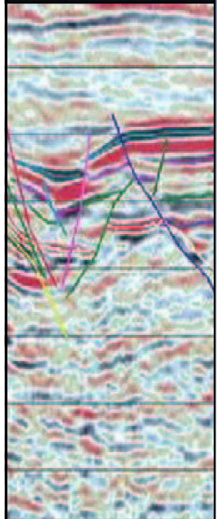


A medium is discretized into blocks and thus we can calulate the path length l_j in each block to obtain

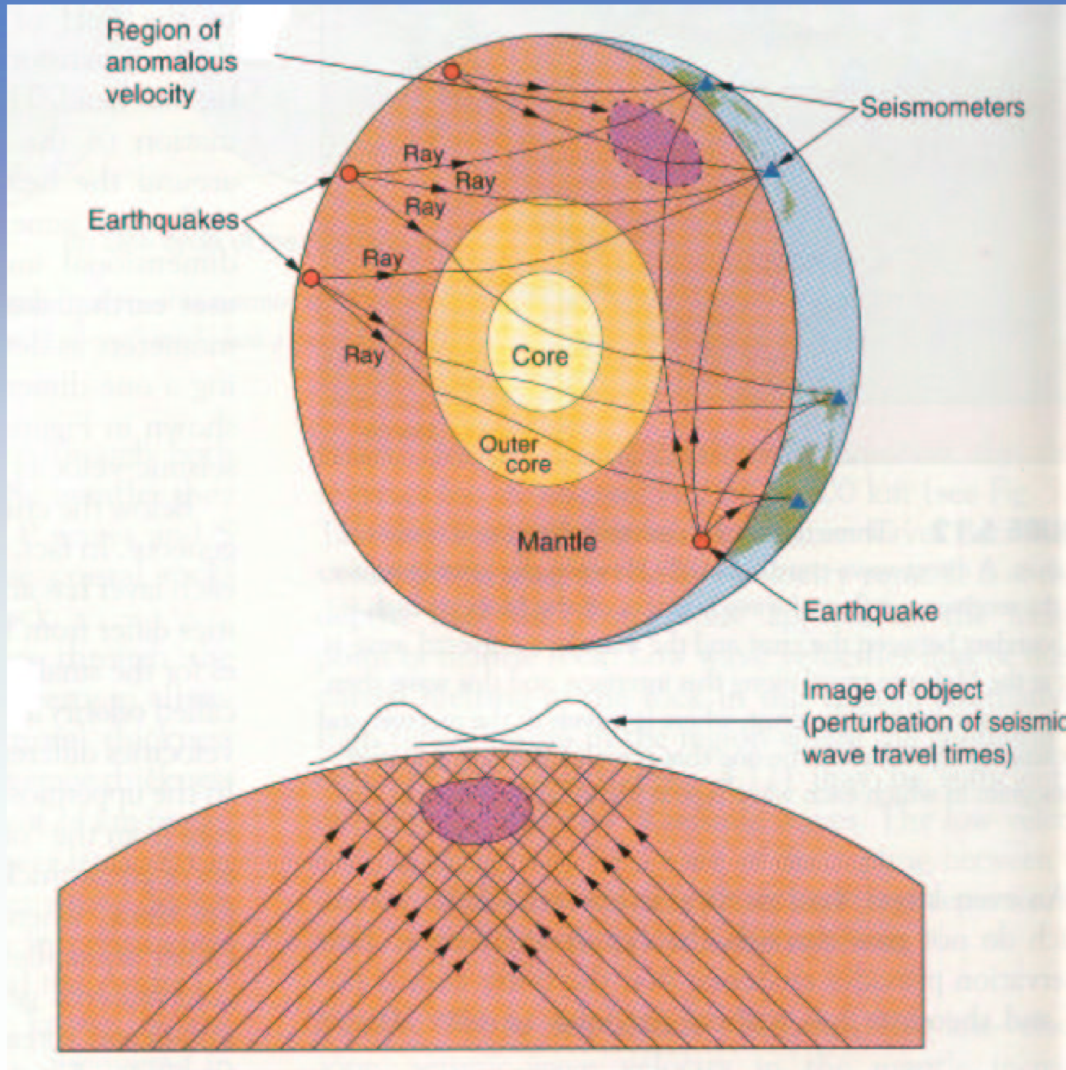
$$\Delta T = \sum_j l_j \Delta u_j \quad \text{for many observations} \quad \Delta T_i = \sum_j l_{ij} \Delta u_j$$

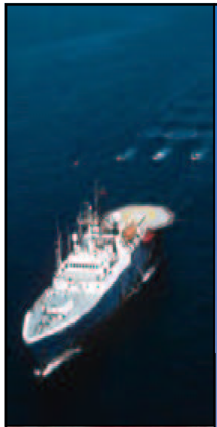
We want to find Δu_j from observed travel times -> **inverse problem**

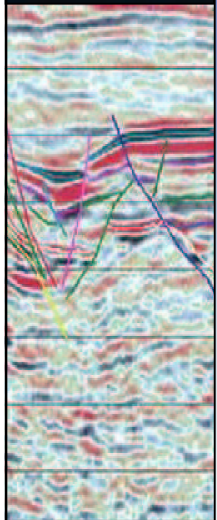




Global Tomography

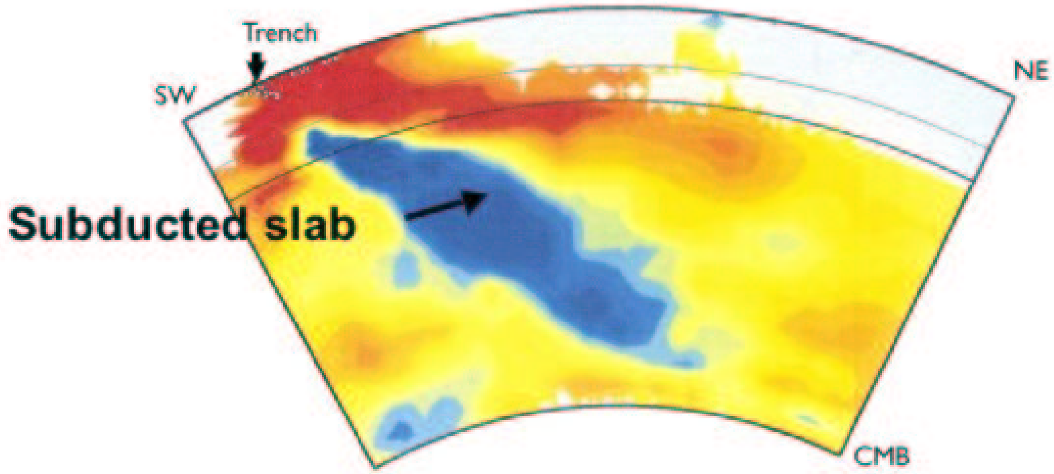






Example

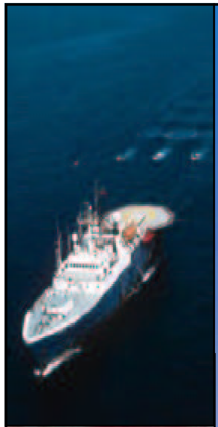
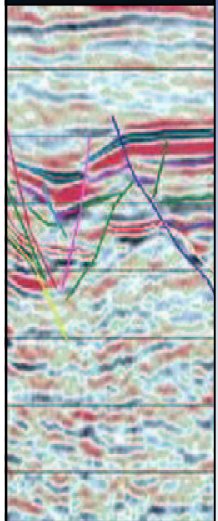
Seismic Tomography Scan of a Section of the Mantle



Seismic tomography records variations in P-wave velocity, which correlate with the temperatures of matter in Earth's interior.

- Slower P waves, indicating warmer-than-average matter
- Average-speed P waves, indicating average-temperature matter
- Faster P waves, indicating cooler-than-average matter
- No data

Fig. 19.9


Summary – Refraction seismics

- When velocities increase with depth in a layered model we observe refractions
- Refractions travel along the layer boundary in the underlying (faster) medium and radiate energy to the surface
- Refracted waves allow the determination of the layered velocity structure
- The generalization of the concept to 3D media is called seismic tomography
- In seismic tomography we seek a seismic velocity model that explains the observed travel times
- This procedure is also called the seismic inverse problem
- Tomographic images may contain large uncertainties due to inappropriate ray coverage or hidden regions (e.g. low velocity zones etc.)