Numerical methods in the Earth Sciences: seismic wave propagation

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III The latest developments, outlook

- Grenoble Valley Benchmark
- Waves on unstructured grids
- The SPICE library
3D numerical simulation of seismic wave propagation in the Grenoble valley (M6 earthquake)

Forward modeling benchmark (Chaljub et al., 2006)
3D numerical simulation of seismic wave propagation in the Grenoble valley (M6 earthquake)

$V_s = 3200 \text{m/s}$

$f_{\text{max}} = 3 \text{Hz}$

$\lambda_{\text{min}} = \frac{V_s}{f_{\text{max}}} = 1066.7 \text{m}$
The Courant Criterion

\[ v_P \left( \frac{dt}{dx} \right) \leq \varepsilon \]

Largest velocity

Smallest grid size
Problems …

- ... grid generation is cumbersome with hexahedra, trying to honor complex geometries and material heterogeneities ...

- ... large variations in seismic velocities (i.e. required grid size) lead to very small time steps - overkill in a large part of the model ...
Waves on unstructured grids?

tetrahedral
Arbirtrarily high-or**DER** - 
Discontinuous Galerkin

- Combination of a discontinuous Galerkin method with ADER time integration
- Piecewise polynomial approximation combined with the fluxes across elements (**finite volumes**)
- Time integration as accurate as space derivatives, applicable also to strongly irregular meshes (not so usually for FD, FE, SE)
- Method developed in **aero-acoustics** and computational fluid dynamics
- The scheme is entirely local, not large matrix inversion -> efficient parallelization
- Algorithms on tetrahedral grids slower than spectral element schemes on hexahedra
ADER-DG in *Geophysical Journal International* a.o.


Coming soon: poroelasticity, combined hexahedral and tetrahedral grids, dynamic rupture
Anisotropic Material
Arbitrarily shaped finite sources

Mesh spacing is proportional to P-wave velocity

Käser, Mai, Dumbser, 2007
Local precision

- Use high precision (i.e., high-order polynomials) only where necessary
- High precision where cells are large (high velocities)
- Low precision where cells are small (because of structural heterogeneities)

Käser et al. (2006)
Local time-stepping is possible without losing the accuracy of the scheme.
Mesh Partitioning and Parallel Computing
the problem of load balancing

Same color means same processor
Grenoble Basin Simulation

Time: 0 sec
Mechanical properties

- **Alluvial basin**, $V_s = 300.0 + 19.0 \cdot \sqrt{D}$, $V_p = 1450.0 + 1.2 \cdot D$
  \[ \rho = 2140.0 + 0.125 \cdot D, \quad Q_s = 50, \quad D = \text{depth in meter} \]

- **Bedrock (Depth = 0 - 3 km)**: $V_s = 3.2 \text{ km/s}$, $V_p = 5.6 \text{ km/s}$, $\rho = 2720 \text{ kg/m}^3$

- **Bedrock (Depth = 3 - 7 km)**: $V_s = 3.43 \text{ km/s}$, $V_p = 5.92 \text{ km/s}$, $\rho = 2720 \text{ kg/m}^3$

- Receivers
- Fault strong 1
- Fault strong 2
Seismogram Comparison
Interactive Benchmarking

The SPICE Code Validation

home

To participate in the SPICE Code Validation (that is, calculate your solution for one or more defined models and compare it with solutions submitted by other participants), please follow these steps:

0. go to registration (do it only once for each method)
1. choose and download a model description
2. perform a computation with your code
3. convert your solution into a format appropriate for upload - see solution format
4. upload your solution (your solution will be stored on the server)
5. view/compare solutions

comments and suggestions to spice.cv@trugrake.eu

Moczo et al., 2006

www.spice-rtn.org
SPICE Digital Library

- **Software** for wave propagation problems
- **Training** material - practicals
- **Access to** benchmarking (global tomography, kinematic source inversion, wave propagation and rupture)

www.spice-rtn.org

... more info on the SPICE stand ...
Conclusions - Technical Challenges

• Strongly heterogeneous structures (or complex surfaces) still pose problems particularly when using hexahedral grids (e.g. oversampling, instabilities)

• Unstructured grids (triangles, tetrahedra) have advantages concerning grid generation but numerical operators often are less accurate, or expensive

• Efficient parallelization algorithms with heterogeneous time steps, accuracy and grid density requires substantial interaction with software engineers.
What's missing?

... easy access for data modellers to well tested **simulation tools** ...

... easy (e.g., hidden) access to **HPC infrastructure** (GRIDs, EU-HPC)

... **community codes** for wave propagation problems

... software engineering **support**