Numerical methods in the Earth Sciences: 
*seismic wave propagation*

Heiner Igel, LMU Munich

I Waves and supercomputing

II Science with 3D wave propagation and rupture
  - Understanding earthquake *rupture*
  - Prediction of *strong ground motions*
  - The *seismic signature of mantle convection*
  - *Imaging* with 3-D methods - adjoint method
Spatial Scales and Memory
(back of the envelope)

Highest frequency: 0.1 Hz
Shortest wavelength: 20 km (crust)
Shortest wavelength: 50 km (mantle)
Grid points per wavelength: 5
Grid spacing: 2000 m (crust)
Grid spacing: 5000 m (mantle)

Required grid points: $O(10^9)$
Required memory: $O(100 \text{ GBytes})$
Seismology and Geodynamics

Courtesy: G. Jahnke

Courtesy: H.P. Bunge, B. Schuberth
Numerical simulation of seismic wave propagation

Elastic wave equations

\[ \rho \ddot{u}_i = \partial_j (\sigma_{ij} + M_{ij}) + f_i \]

\[ \sigma_{ij} = c_{ijkl} \varepsilon_{kl} \]

\[ \varepsilon_{kl} = 1/2 (\partial_k u_l + \partial_l u_k) \]
The kernel

- Earthquake scenarios
  - Shaking hazard
- Sensitivities
  - Experiment design
- Phenomenological studies
  - Model space studies
- Dynamic rupture
  - Source physics
- Imaging (source and structure)
  - Adjoint methods
Numerical methods

- Finite Differences (high order, optimal operators)
- Pseudospectral methods (Chebyshev, Fourier)
- Finite/spectral elements on hexahedral grids
- Unstructured grids (finite volumes/elements, natural neighbours) or combinations
- Parallelization using MPI (message passing interface)

-> for rupture problems special internal boundary conditions apply
Fundamental concepts

• From the continuous to the discrete world
  - Function approximation
  - Collocation points
  - Stability
  - Numerical dispersion

• Methodologies
  - Finite differences
  - Pseudospectral methods
  - Finite elements
Dynamic rupture
scientific objectives

- Understanding the *earthquake process*
- Understanding the *controlling mechanisms* of earthquakes (frictional properties, strength heterogeneities, material interfaces, etc.)
- **Resolving power** of seismic observations with respect to (dynamic) source parameters
- **Regional conditions** (intraplate, interplate, subduction zones, normal, strike, etc.)

phenomenological studies
Rupture at a bi-material interface

Convergence tests with high-resolution models

- Grid size 500x3200x3200
- 12.5 cm grid spacing
- High-order staggered-grid finite differences
Self-sustained pulse in 3D?

Brietzke, Cochard, Igel, GRL 2007

YES!
Earthquake scenarios
scientific objectives

• Accurate forecasting of hazard and risk scenarios for specific regions and time intervals

• Incorporation of earthquake scenario simulations into probabilistic hazard analysis

Shaking hazard

M5.9 Roermond 1992
Example: Newport-Ingelwood Fault, Los Angeles Basin

Wang, Igel, Cochard, Ewald (2006)
Numerical Green's Functions
Varying slip histories
M7 earthquakes

... while keeping the hypocenter location fixed ...
Variations due to slip history
20 scenarios

Fault || Fault _

PGV

Max Mean
Compatible with Attenuation Relations?
Global and regional seismology
scientific objectives

- High resolution imaging (diffraction tomography) of global earth structure (geodynamics)
- 3D wave effects of structures like plumes, subduction zones, D" → geodynamic issues
- Development of 3D reference models (e.g. European reference model)
Bunge and Schuberth, 2007

Isosurfaces at -0.75% and +1.2%
(SPECFEM3D, Komatitsch and Tromp)

14.5 billion DOF on 1944 procs, down to 5 secs period! 50 h runtime
Study of SS-precursors
Mantle discontinuities

3-D synthetics for Model Earth
Adjoint methods - sensitivities

Quantification of sensitivities with 3D simulation technology

Tromp (2007)

Fichtner et al. (2007)
Conclusions

• Numerical methods are now widely used for the forward problem in many modelling studies
• Young Earth scientists are often not well trained in computational/numerical methods
• Some fundamentals should be known when using community software as black boxes