Coda wave interferometry

An interferometer is an instrument that is sensitive to the interference of two or more waves (optical or acoustic). For example, an optical interferometer uses two interfering light beams to measure small length changes.

Coda wave interferometry is a technique for monitoring changes in media over time using acoustic or elastic waves. Sound waves that travel through a medium are scattered multiple times by heterogeneities in the medium and generate slowly decaying (late-arriving) wave trains, called coda waves. Despite their noisy and chaotic appearance, coda waves are highly repeatable such that if no change occurs in the medium over time, the waveforms are identical. If a change occurs, such as a crack in the medium, the change in the multiple scattered waves will result in an observable change in the coda waves. Coda wave interferometry uses this sensitivity to monitor temporal changes in strongly scattering media.

There are many potential applications of coda wave interferometry. In geotechnical applications, the technique can be used to monitor dams or tunnel roofs. In nondestructive testing, the technique can be used to monitor changes due to the formation of cracks or other changes in materials. In hazard monitoring, the technique can be used to monitor volcanoes, fault zones, or landslide areas. In the context of the "intelligent oilfield," coda wave interferometry can be used to monitor changes in hydrocarbon reservoirs during production.

Coda wave interferometry can be used in two different modes. In the warning mode, the technique is used to detect a change in the medium, but this change is not quantified. This mode of operation is used to prompt further action, such as more elaborate diagnostics. In the diagnostic mode, coda wave interferometry is used to quantify the change in the medium.

Volcano monitoring. The use of this technique to detect changes in a medium can be illustrated with seismological data that have been recorded on the Merapi volcano in Java by U. Wegler and coworkers. As seismic (elastic) waves pass through a volcano, they are scattered by heterogeneities (scatters) such as voids, cracks, magma bodies, and faults.

In the experiment, an air gun placed in a small water basin dug in the side of the volcano was used to generate seismic waves. (An air gun is a device that emits a bubble of compressed air in water as a source of seismic waves.) The seismic waves generated by the same source recorded at a fixed receiver at two moments in time (a year apart) are shown in **Fig. 1**.

In the Fig. 1*b* and *c*, the two waveforms are shown superimposed in more detail. For the interval early in the seismogram, these waveforms are similar (Fig. 1*b*). For the later interval (coda), the waveforms are distinctly different (Fig. 1*c*), where one of the waveforms appears to be a time-shifted version of the other, indicating that the interior of the volcano had changed over time.

Note that in coda wave interferometry one needs only a single source and a single receiver, although in practice one may use more receivers to increase the reliability. This means that the hardware requirements of this technique are modest.

Theory. Suppose that a strongly scattering medium is excited by a repeatable source, and that the medium changes with time. Before the change in the medium, the unperturbed wave field $u^{(u)}(t)$ can be written in Eq. (1) as a sum of the waves that

$$u^{(u)}(t) = \sum_T A_T(t) \tag{1}$$

propagate along the multiple scattering trajectories T in the medium, where t denotes time and $A_T(t)$ is the wave that has propagated along trajectory T. When the medium changes over time, the dominant effect is a change τ_T in the arrival times of the waves that propagate along each trajectory T, so that the perturbed wave field is given by Eq. (2).

$$u^{(p)}(t) = \sum_T A_T(t - \tau_T) \tag{2}$$

The change in the waveforms as shown in Fig. 1*c*

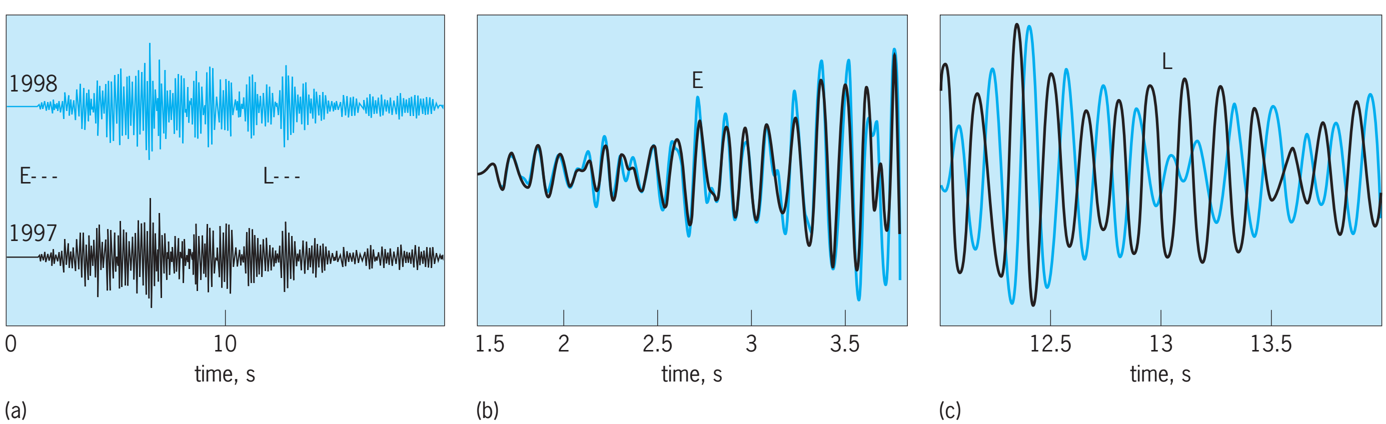

Fig. 1. Waveforms recorded at the Merapi volcano for the same source and receiver on July 1997 (black) and July 1998 (color). (*a*) Complete waveforms and definition of the early (E) and late (L) time interval. (*b*) The recorded waves in the early time interval (E). (c) The recorded waves in the later interval (L). (*Data courtesy of Ulrich Wegler*)

can be quantified by computing in Eq. (3) the time-

$$R(t_s) \equiv \frac{\int_{t-t_w}^{t+t_w} u^{(u)}(t')u^{(p)}(t'+t_s)\,dt'}{\left(\int_{t-t_w}^{t+t_w} u^{(u)2}(t')\,dt' \int_{t-t_w}^{t+t_w} u^{(p)2}(t')\,dt'\right)^{1/2}} \quad (3)$$

shifted cross-correlation over a time window with center time t and width $2t_w$, where t_s is the time shift of the perturbed waveform relative to the unperturbed waveform. Suppose that the waves are not perturbed. In that case, $u^{(p)}(t) = u^{(u)}(t)$ and the time-shifted cross-correlation is equal to unity for a zero lag time $R(t_s = 0) = 1$. When the perturbed wave is a time-shifted version of the original wave, then $u^{(p)}(t) = u^{(u)}(t - \tau)$ and $R(t_s)$ attain its maximum for $t_s = \tau$.

In general, the time-shifted cross-correlation $R(t_s)$ attains its maximum at a time $t_s = t_{\max}$ [Eq. (4)] when

$$t_{\max} = \langle\tau\rangle \quad (4)$$

the shift time is given by the average perturbation of the travel time of the waves that arrive in the employed time window, and the value $R_{\max}$ at its maximum [Eq. (5)] is related to the variance σ_τ of the

$$R_{\max}^{(t,t_w)} = 1 - \frac{1}{2}\omega^2\sigma_\tau^2 \quad (5)$$

travel-time perturbation of the waves that arrive in the time window, where ω is the angular frequency of the waves. Given the recorded waveforms before and after the perturbation, one can readily compute the time-shifted cross-correlation and use Eqs. (4) and (5) to obtain the mean and the variance of the travel-time perturbation in the medium.

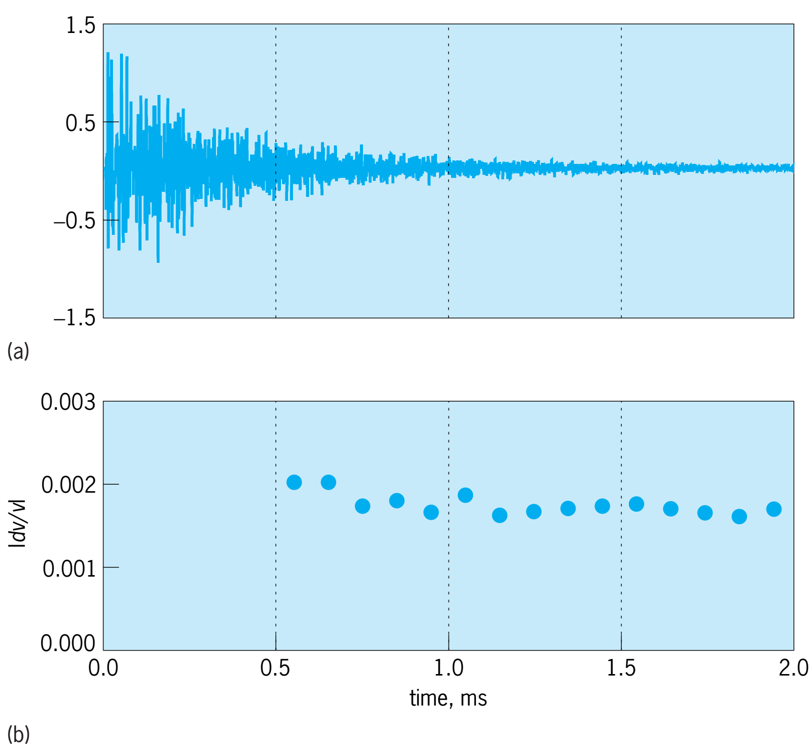

Fig. 2. Measuring velocity change. (*a*) Ultrasound waves were recorded in a small granite sample. (*b*) The relative velocity change for a 5°C (9°F) increase in temperature as a function of the center time of each time window was used to measure the velocity change.

Measuring velocity change. Figure 2*a* shows an experiment in which ultrasound waves were propagated through a granite cylinder and recorded. The waves are complex due to the reverberations within this cylinder. With a heating coil, the temperature of the cylinder was raised 5°C (9°F). The perturbed waveforms have the same character as the unperturbed waves shown in Fig. 2*a*. The tail of the wave trains was divided in 15 nonoverlapping time intervals. For each time interval, the time shift between the perturbed and unperturbed waves was determined by computing the time-shifted cross-correlation of Eq. (3) and by picking the time for which it attains its maximum $t_{\max}$. The relative velocity change for each time interval is given by $\delta v/v = -t_{\max}/t$. This quantity is shown in Fig. 2*b* as a function of the center time of the employed time windows.

Since the employed time windows are nonoverlapping, the measurements of the velocity change in the different time windows are independent. The scatter in the different estimates of the velocity change is small; this provides a consistency check of coda wave interferometry. The variability in the measurements can be used to estimate the error in the velocity change. Note that the relative velocity change in this example is only about 0.16% with an error of about 0.03%. This extreme sensitivity to changes in the medium is due to the sensitivity of the multiply scattered waves to changes in the granite.

In an elastic medium such as granite, there is no single wave velocity. Compressional (*P*) waves and shear (*S*) waves propagate with different velocities v_P and v_S, respectively. The change in the velocity inferred from coda wave interferometry [Eq. (6)] is a

$$\frac{\delta v}{v} = \frac{v_S^3}{2v_P^3 + v_S^3}\frac{\delta v_P}{v_P} + \frac{2v_P^3}{2v_P^3 + v_S^3}\frac{\delta v_S}{v_S} \quad (6)$$

weighted average of the change in the velocities for the two wave types. For a Poisson medium, an elastic medium, where $v_P = \sqrt{3}v_S$, the relative velocity change is given by $\delta v/v \approx 0.09\delta v_P/v_P + 0.91\delta v_S/v_S$. This means that in practice coda wave interferometry provides a constraint on the change in the shear velocity v_S.

Other applications. Some applications may involve a change in the location of scatterers, or a change in the source position. This can be used to monitor the properties of a turbulent fluid. One can seed the fluid with neutrally buoyant particles that scatter waves. Acoustic waves that propagate through the fluid are scattered by these particles. Over time, the particles are swept along by the turbulent motion. When acoustic waves are sent into the fluid once more from the same source, the waves are scattered by particles that have been displaced by the turbulent flow. The resulting change in the recorded waves can be used to characterize the properties of the turbulent flow.

Coda wave interferometry, to a certain extent, can be used to distinguish between different

Imprint of different type of changes on the mean $\langle\tau\rangle$ and the variance σ_τ^2 of the travel-time perturbation

Type of change	$\langle\tau\rangle$	σ_τ^2
Change in velocity	$\sim t$	0
Movement of scatterers	0	$\sim t$
Displaced source	0	Constant

perturbations. When the velocity changes, the mean travel-time perturbation is nonzero and is proportional to the total travel time. Since the waves that propagate along all paths experience the same travel-time change, the variance of the travel-time perturbation vanishes. When the location of the scatterers is perturbed randomly, the mean travel-time perturbation vanishes, because some paths are longer, while others are shorter. However, the variance of the travel-time perturbation is nonzero and can be shown to grow linearly with time. When the source is displaced, the mean travel-time perturbation vanishes, and the variance of the travel time is constant with time. These results are summarized in the **table**. The mean and the variance of the travel-time change can be computed from the unperturbed and the perturbed waveforms. Since different types of perturbations leave a different imprint on these quantities, coda wave interferometry can help determine the mechanism of the change over time.

For background information *see* ACOUSTIC EMISSION; ACOUSTICS; EARTHQUAKE; INTERFEROMETRY; SEISMOGRAPHIC INSTRUMENTATION; SEISMOLOGY; SOUND; VOLCANO; VOLCANOLOGY in the McGraw-Hill Encyclopedia of Science & Technology.

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