Seismometer - The basic Principles

- $u$: ground displacement
- $x_r$: displacement of seismometer mass
- $x_0$: mass equilibrium position
The motion of the seismometer mass as a function of the ground displacement is given through a differential equation resulting from the equilibrium of forces (in rest):

\[ F_{\text{spring}} + F_{\text{friction}} + F_{\text{gravity}} = 0 \]

for example

\[ F_{\text{spring}} = -kx, \ k \text{ spring constant} \]

\[ F_{\text{friction}} = -D\dot{x}, \ D \text{ friction coefficient} \]

\[ F_{\text{gravity}} = -mu, \ m \text{ seismometer mass} \]
using the notation introduced the equation of motion for the mass is

$$\ddot{x}_r(t) + 2\varepsilon \dot{x}_r(t) + \omega_0^2 x_r(t) = -i\ddot{u}_g(t)$$

$$\varepsilon = \frac{D}{2m} = \frac{h\omega_0}{\omega_0^2}$$, \hspace{1cm} $$\omega_0^2 = \frac{k}{m}$$

From this we learn that:

- for slow movements the acceleration and velocity becomes negligible, the seismometer records ground acceleration

- for fast movements the acceleration of the mass dominates and the seismometer records ground displacement
Seismometer - examples

F0=1 Hz, SRC=0.1 Hz, h=1; blue: accel., red: seismometer
Seismometer - examples

F0=1 Hz, SRC=0.4 Hz, h=0

F0=1 Hz, SRC=0.4 Hz, h=0.2

F0=1 Hz, SRC=0.4 Hz, h=1

F0=1 Hz, SRC=0.4 Hz, h=5

Displacement vs. Time (s)
Seismometer - examples

F0=1 Hz, SRC=0.8 Hz, h=0

F0=1 Hz, SRC=0.8 Hz, h=0.2

F0=1 Hz, SRC=0.8 Hz, h=1

F0=1 Hz, SRC=0.8 Hz, h=5
Seismometer - examples

Graphs showing the displacement over time for different values of h:
- F0=1Hz, SRC=1 Hz, h=0
- F0=1Hz, SRC=1 Hz, h=0.2
- F0=1Hz, SRC=1 Hz, h=1
- F0=1Hz, SRC=1 Hz, h=5
Seismometer - examples

- F0=1 Hz, SRC=2 Hz, h=0
- F0=1 Hz, SRC=2 Hz, h=0.2
- F0=1 Hz, SRC=2 Hz, h=1
- F0=1 Hz, SRC=2 Hz, h=5

Seismology and the Earth's Deep Interior

Seismometry
Seismometry

Seismometer - examples

- F_0=1Hz, SRC=5 Hz, h=0
- F_0=1Hz, SRC=5 Hz, h=0.2
- F_0=1Hz, SRC=5 Hz, h=1
- F_0=1Hz, SRC=5 Hz, h=5

Displacement vs. Time (s)
1. How can we determine the damping properties from the observed behavior of the seismometer?

2. How does the seismometer amplify the ground motion? Is this amplification frequency dependent?

We need to answer these question in order to determine what we really want to know: The ground motion.
1. How can we determine the damping properties from the observed behavior of the seismometer?

\[ \ddot{x}_r(t) + h \sigma_0 \dot{x}_r(t) + \sigma_0^2 x_r(t) = 0 \]
\[ x_r(0) = x_0, \quad \dot{x}_r(0) = 0 \]

we release the seismometer mass from a given initial position and let it swing. The behavior depends on the relation between the frequency of the spring and the damping parameter. If the seismometers oscillates, we can determine the damping coefficient \( h \).
Seismometer - Release Test

F0=1Hz, h=0

F0=1Hz, h=0.2

F0=1Hz, h=0.7

F0=1Hz, h=2.5

Displacement vs. Time for different values of h and F0.
The damping coefficients can be determined from the amplitudes of consecutive extrema $a_k$ and $a_{k+1}$.

We need the logarithmic decrement $\Lambda$

$$\Lambda = 2 \ln \left( \frac{a_k}{a_{k+1}} \right)$$

The damping constant $h$ can then be determined through:

$$h = \frac{\Lambda}{\sqrt{4\pi^2 + \Lambda^2}}$$
The period $T$ with which the seismometer mass oscillates depends on $h$ and (for $h<1$) is always larger than the period of the spring $T_0$:

$$T = \frac{T_0}{\sqrt{1 - h^2}}$$
2. How does the seismometer amplify the ground motion? Is this amplification frequency dependent?

To answer this question we excite our seismometer with a monofrequent signal and record the response of the seismometer:

\[ \ddot{x}_r(t) + h \omega_0 \dot{x}_r(t) + \omega_0^2 x_r(t) = \omega^2 A_0 e^{i\omega t} \]

the amplitude response \( A_r \) of the seismometer depends on the frequency of the seismometer \( \omega_0 \), the frequency of the excitation \( \omega \) and the damping constant \( h \):

\[
\left| \frac{A_r}{A_0} \right| = \frac{1}{\sqrt{\left( \frac{T^2}{T_0^2} - 1 \right)^2 + 4h^2 \frac{T^2}{T_0^2}}}.
\]
Seismometer - Response Function

\[ \left| \frac{A_r}{A_0} \right| = \frac{1}{\sqrt{\left(\frac{T^2}{T_0^2} - 1\right)^2 + 4h^2 \frac{T^2}{T_0^2}}} \]

Curves shown for different damping constants \( h \)
**Sampling rate**

Sampling frequency, sampling rate is the number of sampling points per unit distance or unit time. Examples?
Real numbers are usually described with 4 bytes (single precision) or 8 bytes (double precision). One byte consists of 8 bits. That means we can describe a number with 32 (64) bits. We need one switch (bit) for the sign (+/-)

- 32 bits $\rightarrow 2^{31} = 2.147483648000000e+009$ (Matlab output)
- 64 bits $\rightarrow 2^{63} = 9.223372036854776e+018$ (Matlab output)
(amount of different numbers we can describe)

How much data do we collect in a typical seismic experiment?
Relevant parameters:
- Sampling rate 1000 Hz, 3 components
- Seismogram length 5 seconds
- 200 Seismometers, receivers, 50 profiles
- 50 different source locations
- Single precision accuracy

How much (T/G/M/k-)bytes to we end up with? What about compression?
What is the precision of the sampling of our physical signal in amplitude?

**Dynamic range**: the ratio between largest measurable amplitude $A_{\text{max}}$ to the smallest measurable amplitude $A_{\text{min}}$.

The unit is Decibel (dB) and is defined as the ratio of two power values (and power is proportional to amplitude square)

In terms of amplitudes

**Dynamic range** $= 20 \log_{10}(A_{\text{max}}/A_{\text{min}})$ dB

Example: with 1024 units of amplitude ($A_{\text{min}}=1$, $A_{\text{max}}=1024$)

$20 \log_{10}(1024/1)$ dB $\bowtie$ 60 dB
The frequency half of the sampling rate $dt$ is called the Nyquist frequency $f_N = 1/(2dt)$. The distortion of a physical signal higher than the Nyquist frequency is called aliasing.

The frequency of the physical signal is $> f_N$ is sampled with (+) leading to the erroneous blue oscillation.

What happens in space? How can we avoid aliasing?
A cattle grid
Almost all signals contain noise. The signal-to-noise ratio is an important concept to consider in all geophysical experiments. Can you give examples of noise in the various methods?
**Discrete Convolution**

Convolution is the mathematical description of the change of waveform shape after passage through a filter (system).

There is a special mathematical symbol for convolution (*): 

\[ y(t) = g(t) * f(t) \]

Here the impulse response function \( g \) is convolved with the input signal \( f \). \( g \) is also named the „Green’s function“

\[
\begin{align*}
  y_k &= \sum_{i=0}^{m} g_i f_{k-i} \\
  k &= 0,1,2,\ldots,m+n
\end{align*}
\]

\[
\begin{align*}
  g_i &\quad i = 0,1,2,\ldots,m \\
  f_j &\quad j = 0,1,2,\ldots,n
\end{align*}
\]
Convolution Example (Matlab)

```
>> x
x =
    0   0   1   0
>> y
y =
    1   2   1
>> conv(x,y)
ans =
    0   0   1   2   1   0
```

Impulse response
System input
System output
### Convolution Example (pictorial)

<table>
<thead>
<tr>
<th>x</th>
<th>„Faltung“</th>
<th>y</th>
<th>x*y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>
Deconvolution is the inverse operation to convolution.

When is deconvolution useful?
Digital Filtering

Often a recorded signal contains a lot of information that we are not interested in (noise). To get rid of this noise we can apply a filter in the frequency domain.

The most important filters are:

- **High pass**: cuts out low frequencies
- **Low pass**: cuts out high frequencies
- **Band pass**: cuts out both high and low frequencies and leaves a band of frequencies
- **Band reject**: cuts out certain frequency band and leaves all other frequencies
Digital Filtering

![Graph of Digital Filtering]

- **Seismology and the Earth's Deep Interior**
- **Seismometry**
Low-pass filtering

Seismology and the Earth's Deep Interior
Lowpass filtering

![Graph showing time-domain and frequency-domain representations of seismic data after lowpass filtering.](image-url)
High-pass filter
Band-pass filter

Graph showing a seismogram with time on the x-axis and amplitude on the y-axis. Another graph showing frequency on the x-axis and amplitude on the y-axis.
Observed seismic noise as a function of frequency (power spectrum). Note the peak at 0.2 Hz and decrease as a distant from coast.
Instrument Filters

[Diagram showing magnification versus period for different filters: WWSSN (SP), Benioff (SP), Wood-Anderson (SP), and Press Ewing.]

Seismology and the Earth's Deep Interior
Time Scales in Seismology

- Acceleration
- Displacement
- Gravity
- Strain
- Tilt

Wave propagation
Dynamic rupture
Afterslip, slow rupture
Tides
Unsteady strain
Secular strain

Phenomena of Interest

Frequency (Hz)

10^-8 10^-6 10^-4 10^-2 10^0 10^2