Scattering and Attenuation

Propagating seismic waves loose energy due to

- geometrical spreading

  e.g. the energy of spherical wavefront emanating from a point source is distributed over a spherical surface of ever increasing size

- intrinsic attenuation

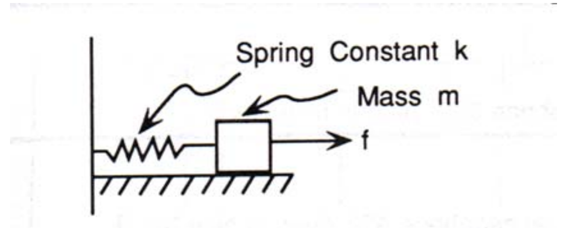
  elastic wave propagation consists of a permanent exchange between potential (displacement) and kinetic (velocity) energy. This process is not completely reversible. There is energy loss due to shear heating at grain boundaries, mineral dislocations etc.

- scattering attenuation

  whenever there are material changes the energy of a wavefield is scattered in different phases. Depending on the material properties this will lead to amplitude decay and dispersive effects.

Intrinsic attenuation

How can we describe intrinsic attenuation?
Let us try a spring model:



The equation of motion for a damped harmonic oscillator is

$$m\ddot{x} + \gamma\dot{x} + kx = 0$$

$$\ddot{x} + \frac{\gamma}{m}\dot{x} + \frac{k}{m}x = 0$$

$$\ddot{x} + \varepsilon\varpi_0\dot{x} + \omega_0^2 x = 0$$

$$\varepsilon = \frac{\gamma}{m\varpi_0}$$

$$\varpi_0 = \left(\frac{k}{m}\right)^{1/2}$$

where ε is the friction coefficient.

Q

The solution to this system is

$$x(t) = A_0 e^{-\varepsilon \varpi_0 t} \sin(\varpi_0 t \sqrt{1-\varepsilon^2})$$

so we have a time-dependent amplitude of

$$A(t) = A_0 e^{-\varepsilon \varpi_0 t} = A_0 e^{-\frac{\varpi_0 t}{2Q}}$$

and defining

$$\varepsilon = \frac{1}{2Q} \qquad \delta = \ln \frac{A_1}{A_2} \qquad Q = \frac{\pi}{\delta}$$

Q is the energy loss per cycle. Intrinsic attenuation in the Earth is in general described by Q.

Energy loss per cycle

The attenuation parameter Q can be interpreted as the energy loss per cycle

$$\frac{1}{Q(\omega)} = -\frac{\Delta E}{2\pi E}$$

For a medium with linear stress-strain relation this can be expressed as

$$\frac{1}{Q(\omega)} = -\frac{\Delta A}{\pi A}$$

Using the fact that A is proportional to $E^{1/2}$.

Dispersion effects

What happens if we have frequency independent Q, i.e. each frequency looses the same amount of energy per cycle?

$$A(x) = A_0 e^{-(f\pi/Qv)x}$$

high frequencies – more oscillations – more attenuation
low frequencies – less oscillations – less attenuation

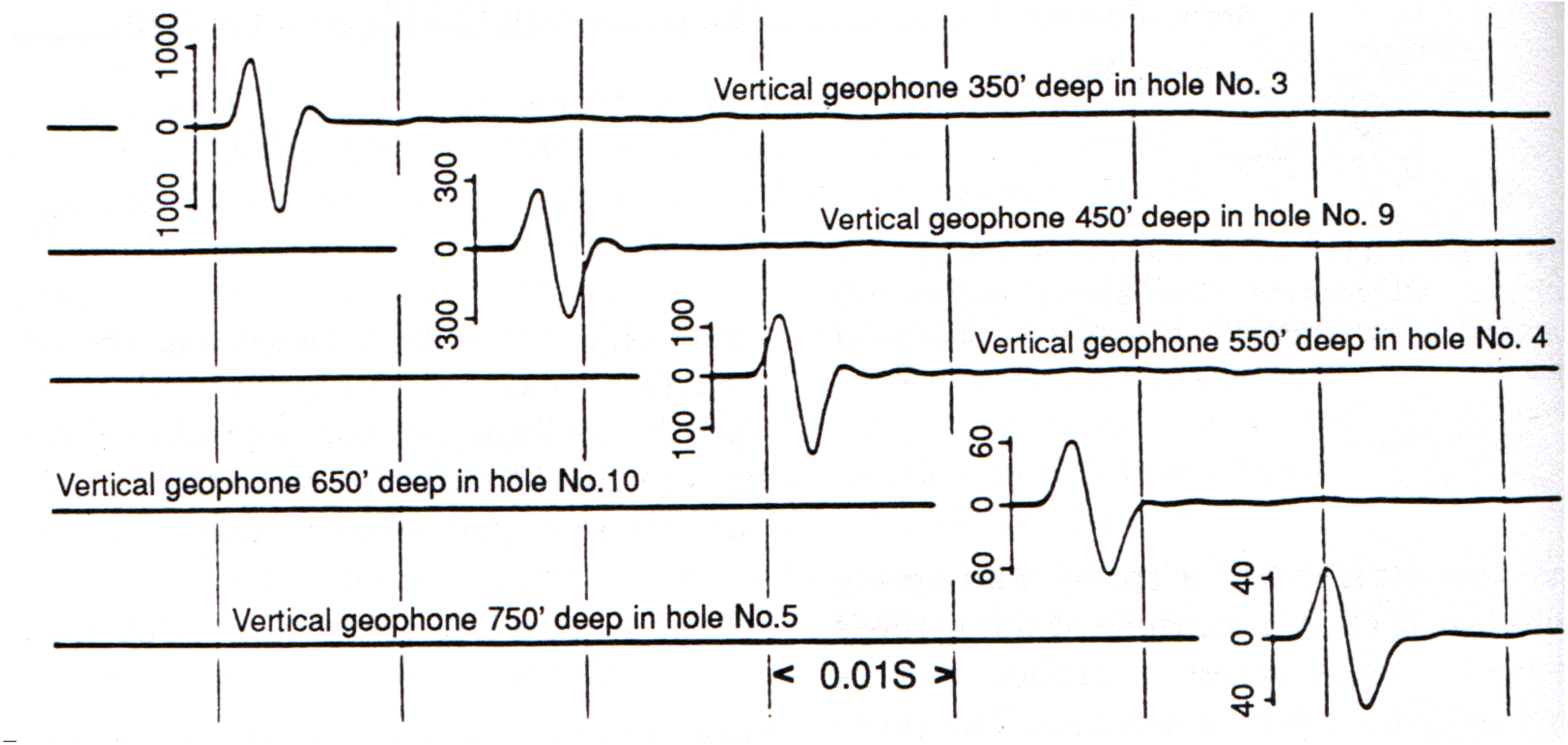
Consequences:

- high frequencies decay very rapidly
- pulse broadening

In the Earth we observe that Q_p is larger than Q_S. This is due to the fact that intrinsic attenuation is predominantly caused by shear lattice effects at grain boundaries.

Pulse Broadening

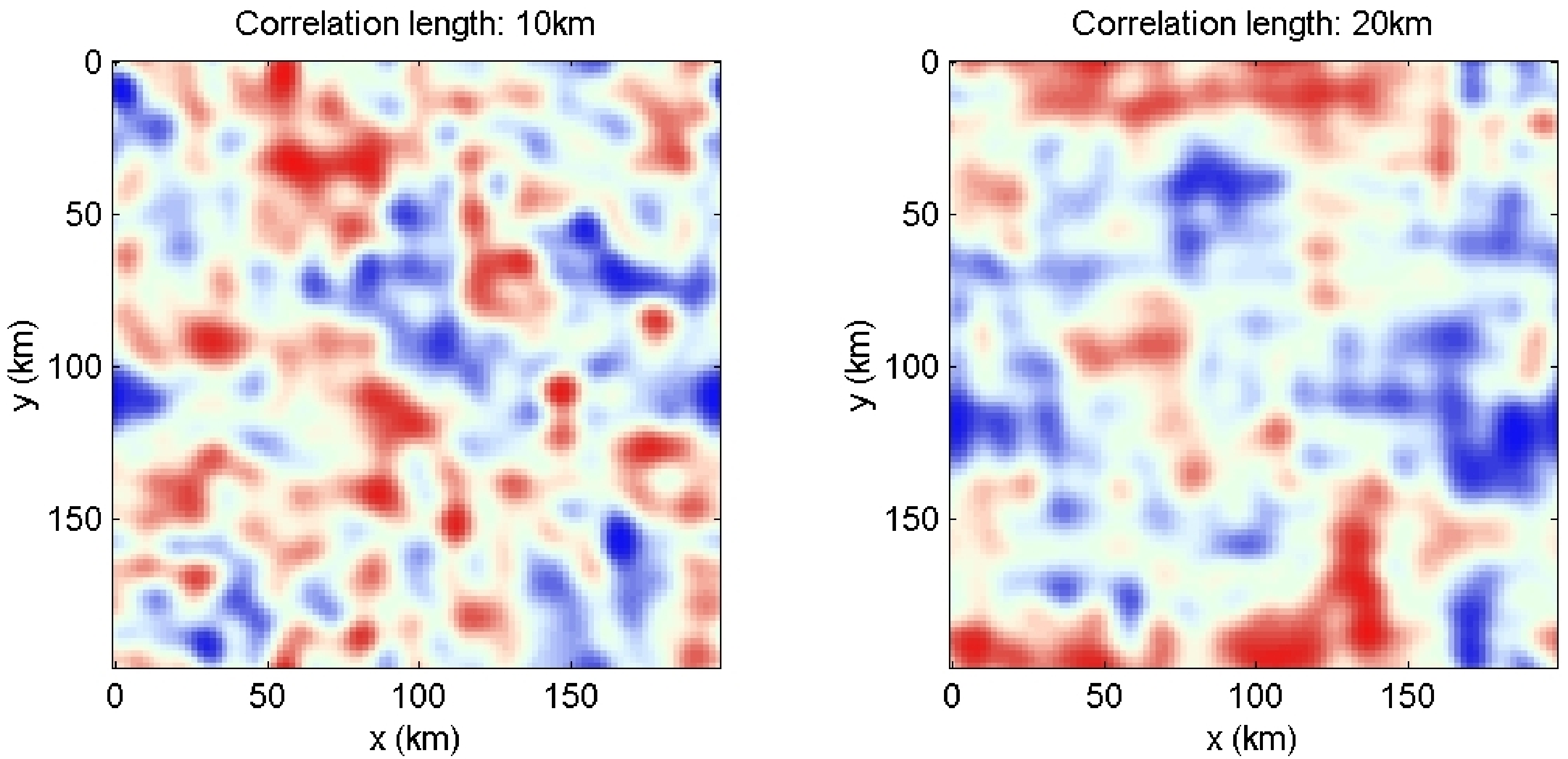
The effects of a constant Q on a propagating pulse:



Q in the Earth

Rock Type	Q_p	Q_S
Shale	30	10
Sandstone	58	31
Granite	250	70-250
Peridotite Midmantle Lowermantle Outer Core	650 360 1200 8000	280 200 520 0

Scattering in random media

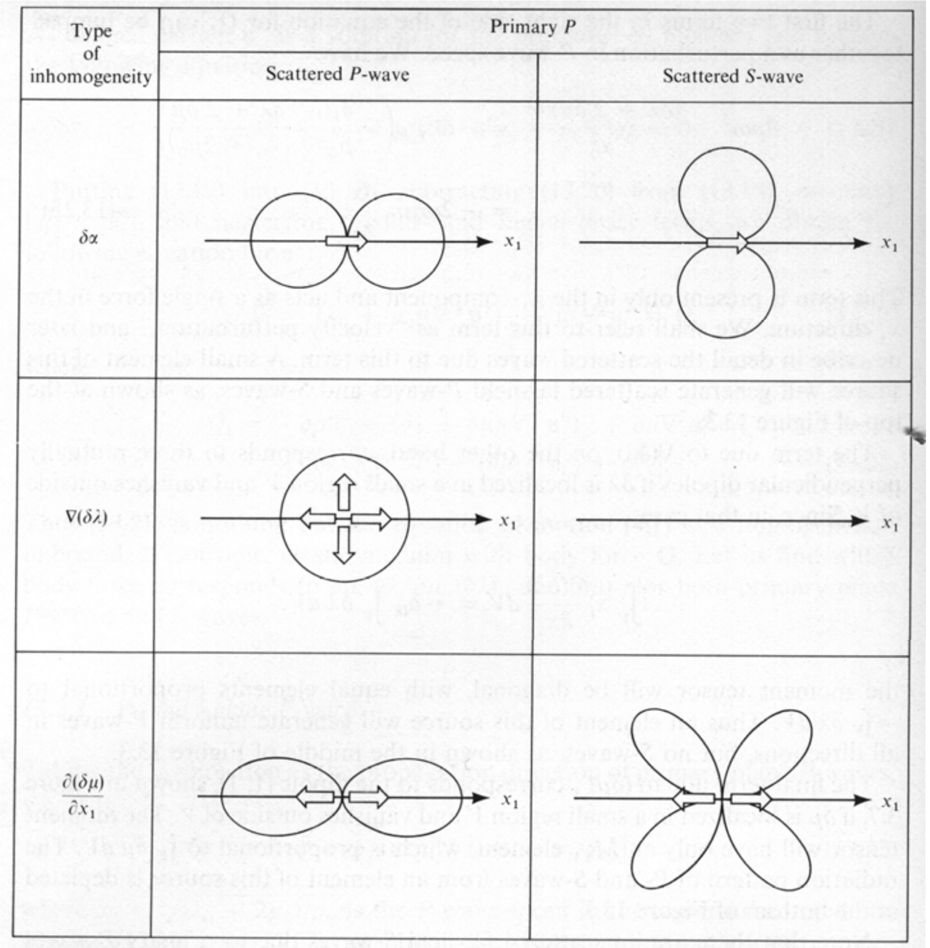


How is a propagating wavefield affected by random heterogeneities?

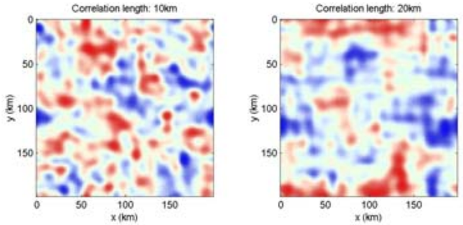
How does a point-like perturbation of the elastic parameters affect the wavefield?

Perturbation of the different elastic parameters produce characteristic radiation patterns. These effects are used in diffraction tomography to recover the perturbations from the recorded wavefield.

(Figure from Aki and Richards, 1980)



Correlation length and wavelength



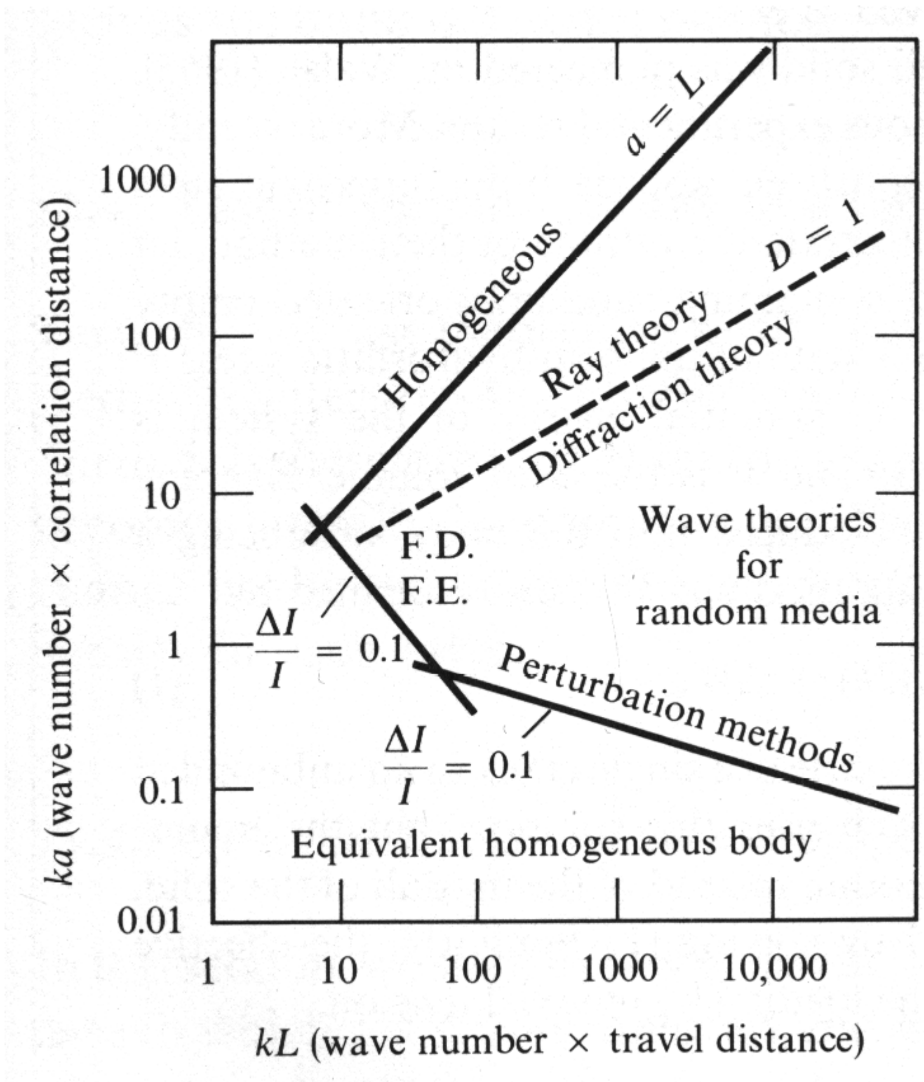
The governing parameters in this problem are:

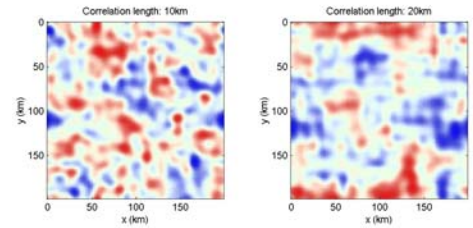
- The wavelength λ of the wavefield (or the wavenumber k)
- The correlation length a
- The propagation distance L

With special cases:

- $a = L$ homogeneous region
- $a \gg \lambda$ ray theory is valid
- $a \approx \lambda$ strong scattering effects

Scattering Classification



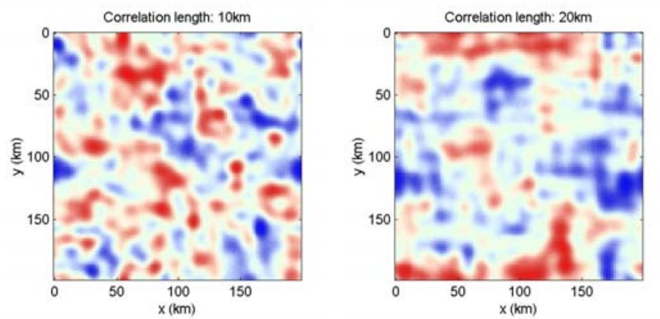


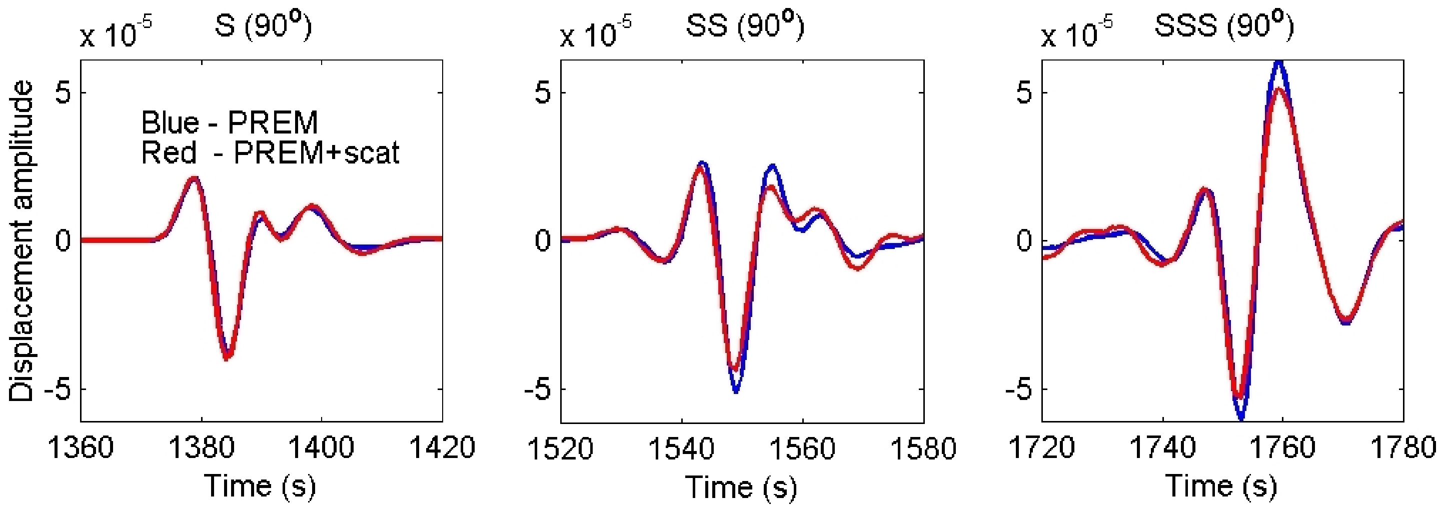
Wave propagation problems can be classified using the parameters just introduced. This classification is crucial for the choice of technique to calculate synthetic seismograms

(Figure from Aki and Richards, 1980)

Synthetic seismograms

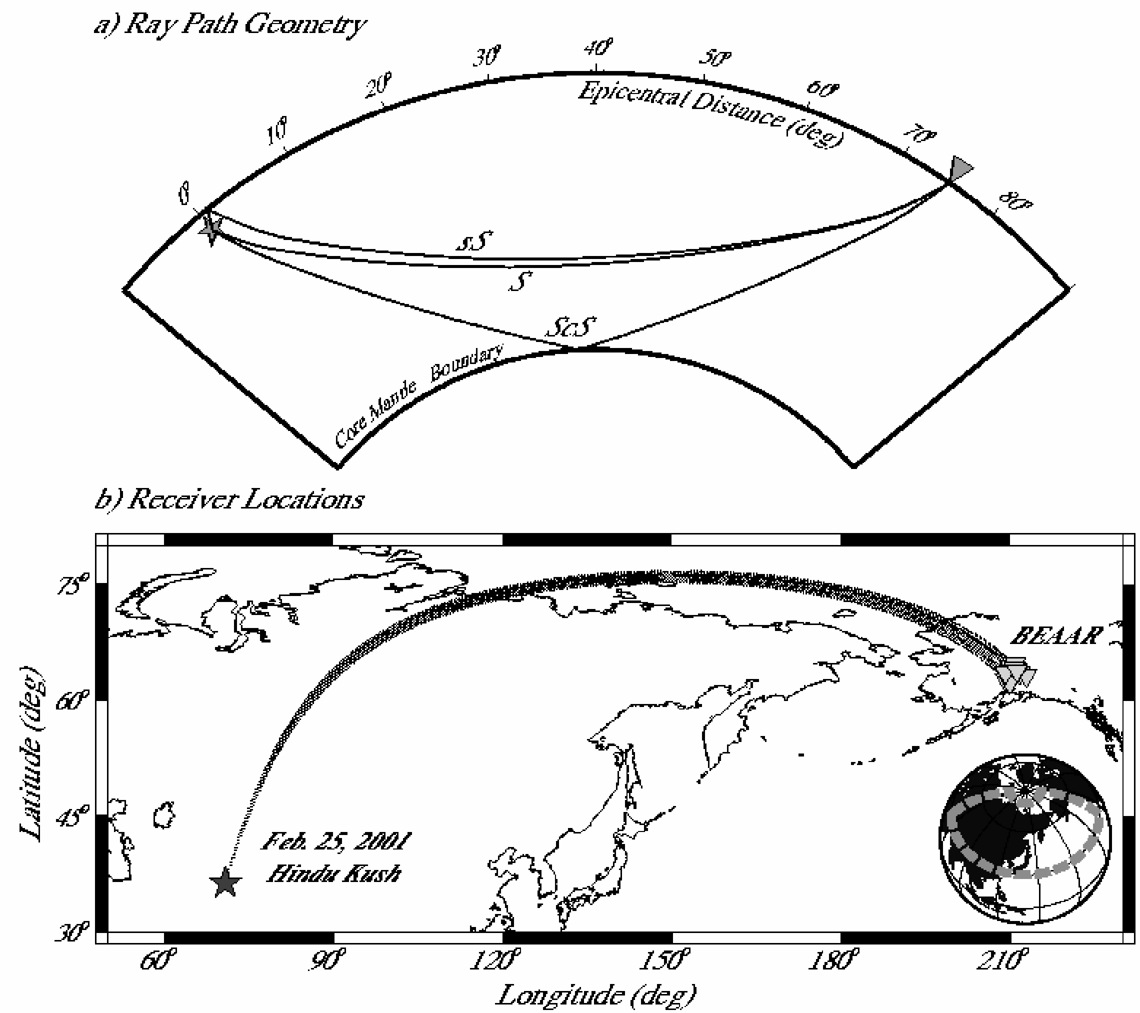
Synthetic seismograms for a global model with random velocity perturbations.

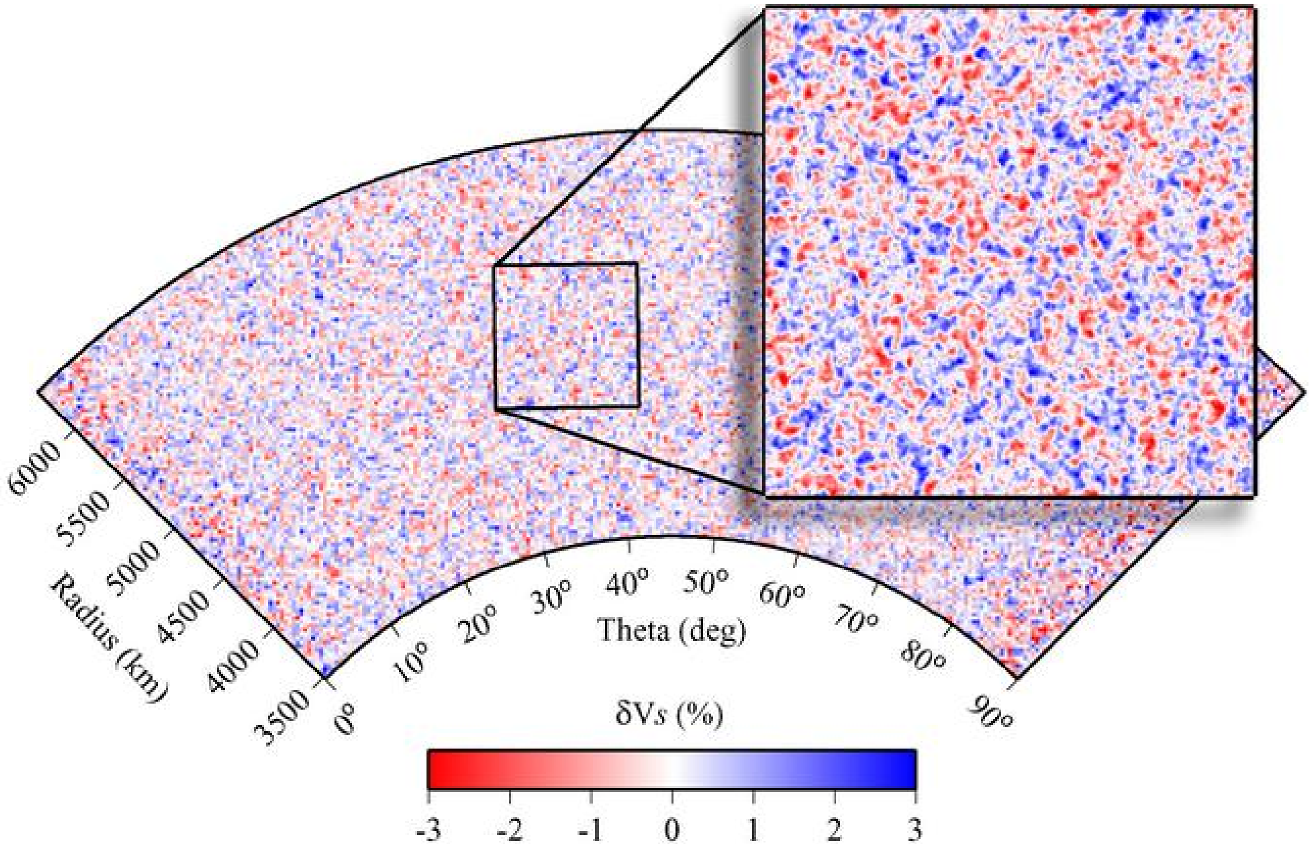




When the wavelength is long compared to the correlation length, scattering effects are difficult to distinguish from intrinsic attenuation.

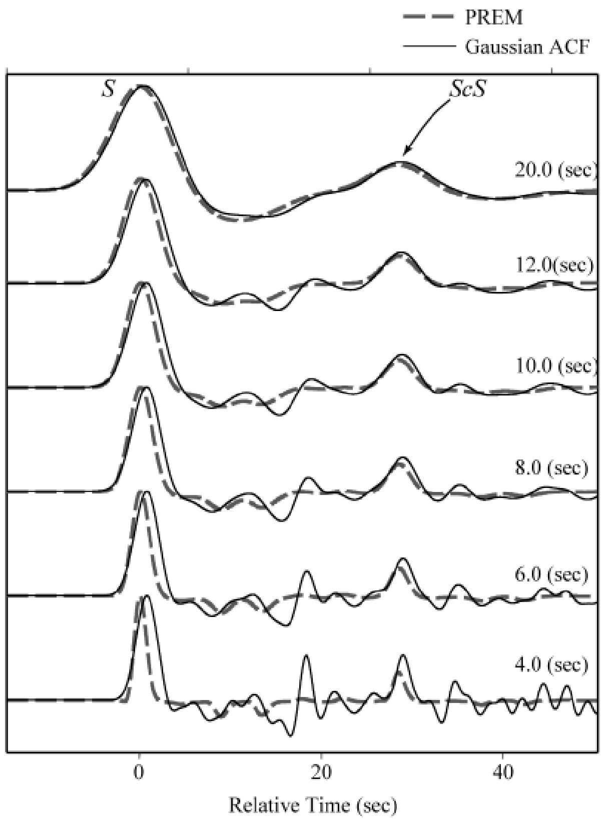
Scattering experiment

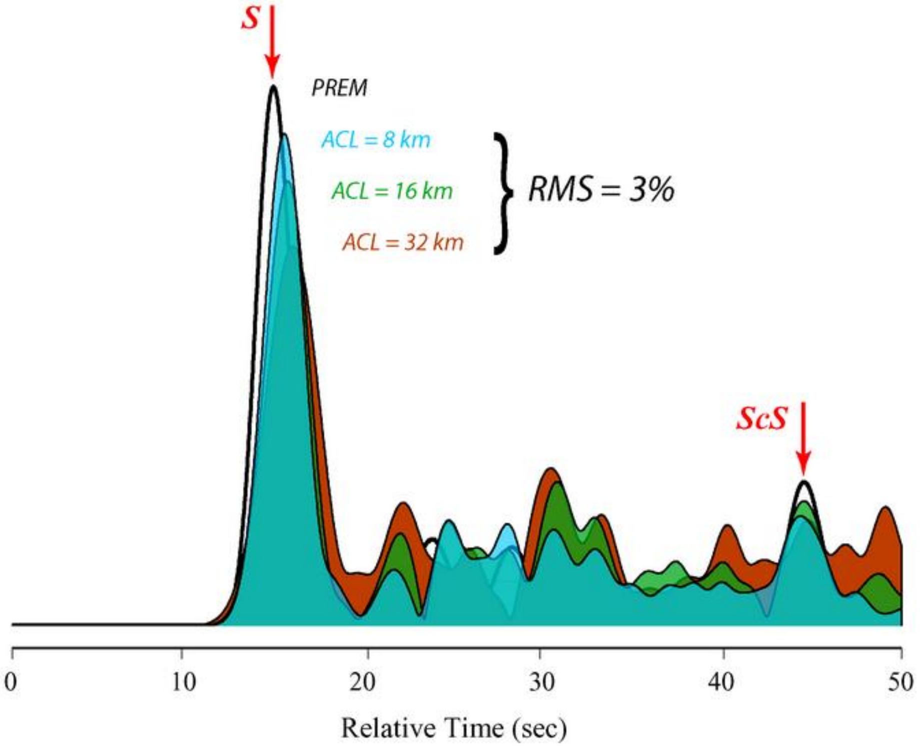




Exponential autocorrelation function. autocorrelation wavelength 32 km.
RMS S-wave velocity perturbation 1%.

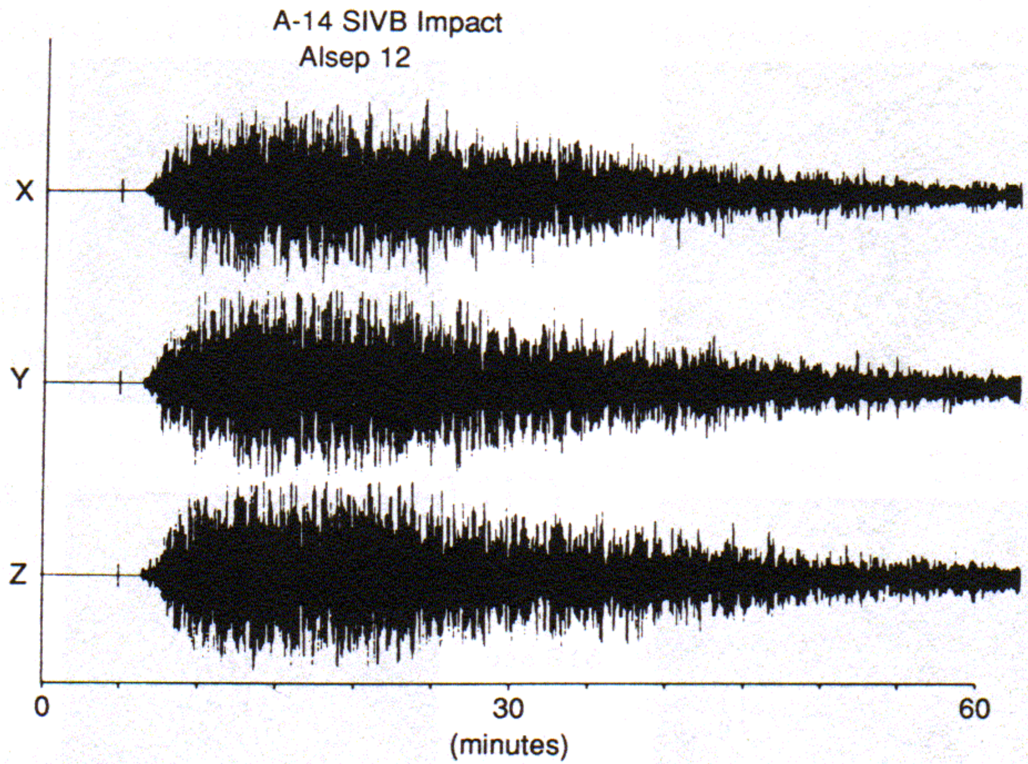
S - Scattering





Scattering on the Moon

The observed wavefield of an impact on the moon looks very different from similar experiments on Earth:



Scattering and Attenuation: Summary

Elastic wavefields not only loose energy through geometrical spreading effects but also through intrinsic and scattering attenuation.

Intrinsic attenuation is described by the frequency-dependent attenuation parameter $Q(\omega)$. Q describes the energy loss per cycle. In the Earth's crust and mantle Q ranges from 10 to 1000.

Any material heterogeneities (point-like, interfaces, etc.) causes a wavefield to be scattered. The parameters governing the kind of scattering are the wavenumber (or wavelength), the correlation length of the scatterers and the propagation distance in the scattering medium.

The classification of scattering is important for the way synthetic seismograms have to be calculated for a particular problem. Ray theory is applicable when the correlation length of the heterogeneities is much larger than the wavelength. Numerical methods have to be used when the correlation length is close to the wavelength.