

USMG

NATO ASI

LES HOUCHES

SESSION L

9 Août – 3 Septembre 1988

TOMOGRAPHIE
OCÉANOGRAPHIQUE
ET GÉOPHYSIQUE

OCEANOGRAPHIC
AND GEOPHYSICAL
TOMOGRAPHY

édité par

Y. DESAUBIES, A. TARANTOLA
et J. ZINN-JUSTIN



1990

NORTH-HOLLAND

AMSTERDAM · OXFORD · NEW YORK · TOKYO

© Elsevier Science Publishers B.V., 1990

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, recording or otherwise, without the written permission of the Publisher, Elsevier Science Publishers B.V., P.O. Box 211, 1000 AE Amsterdam, The Netherlands.

Special regulations for readers in the U.S.A.: This publication has been registered with the Copyright Clearance Center Inc. (CCC), Salem, Massachusetts. Information can be obtained from the CCC about conditions under which photocopies of parts of this publication may be made in the U.S.A. All other copyright questions, including photocopying outside the U.S.A., should be referred to the Publisher, unless otherwise specified.

No responsibility is assumed by the Publisher for any injury and/or damage to persons or property as a matter of products liability, negligence or otherwise, or from any use or operation of any methods, products, instructions or ideas contained in the material herein.

ISBN: 0 444 88779 2

Published by:

North-Holland

Elsevier Science Publishers B.V.

P.O. Box 211

1000 AE Amsterdam

The Netherlands

Sole distributors for the U.S.A. and Canada:

Elsevier Science Publishing Company, Inc.

655 Avenue of the Americas

New York, N.Y. 10010

USA

LES HOUCHES
ÉCOLE D'ÉTÉ DE PHYSIQUE THÉORIQUE

ORGANISME D'INTÉRÊT COMMUN DE
L'UNIVERSITÉ JOSEPH FOURIER DE GRENOBLE ET DE
L'INSTITUT NATIONAL POLYTECHNIQUE DE GRENOBLE
AIDÉ PAR
LE COMMISARIAT À L'ÉNERGIE ATOMIQUE

Membres du conseil: M. Nemoz, président, P. Averbuch, R. Balian,
N. Boccara, C. DeWitt, J.P. Hansen, S. Haroche, M. Jacob,
J.L. Lacoume, J.P. Laheurte, G. Lespinard, R. Maynard, A. Neveu,
A. Omont, Y. Rocard, R. Romestain, R. Stora, D. Thoulouze,
N. Vinh Mau, G. Weill

Directeur: J. Zinn-Justin

SESSION L
INSTITUT D'ÉTUDES AVANCÉES DE L'OTAN
NATO ADVANCED STUDY INSTITUTE

9 Août – 3 Septembre 1988

Directeurs scientifiques de la session: Y. Desaubies
et A. Tarantola

LECTURERS

- Cornuelle, Bruce*, Scripps Institution of Oceanography, Univ. of California San-Diego, A-030, La Jolla, CA 92093, USA.
- Desaubies, Yves*, Institut Français de Recherche pour l'Exploitation de la Mer, B.P.70, 29280 Plouzané, France.
- Duckworth, Greg*, Massachusetts Institute of Technology, 54-1324, Cambridge, MA 02139, USA.
- Frisk, George* Woods Hole Oceanographic Institution, Woods Hole, Mass 02543, USA.
- Jensen, Finn*, Saclant Undersea Research Centre, viale San Bartolomeo, La Spezia, I-19026, Italy.
- Kosloff, Dan*, Department of Geophysics, Tel Aviv University, Tel-Aviv, 69978, Israel.
- Madden, Theodore*, Massachusetts Institute of Technology, 54-614, Cambridge, MA 02139, USA.
- Mora, Peter*, Thinking Machines Corporation, 245 First Street, Cambridge, MA 02142, USA.
- Richards, Paul*, Lamont-Doherty Geological Observatory, Columbia University, Palisades, NY 10964, USA.
- Romanowicz, Barbara*, Inst. de Physique du Globe, Université de Paris VI, 4 Place Jussieu, 75252 Paris Cedex 05, France.
- Stewart, Robert*, University of Calgary, Dept. of Geology-Geophysics, 2500 University Drive NW, Calgary, Alberta, T2NIN4, Canada.
- Tarantola, Albert*, Inst. de Physique du Globe, Université de Paris VI, 4 Place Jussieu, 75252 Paris Cedex 05, France.
- Woodhouse, John*, Harvard University, Hoffman Laboratory, 29, Oxford Street, Cambridge, MA 02138, USA.
- Wunsch, Carl*, Massachusetts Institute of Technology, 54-1324, Cambridge, MA 02139, USA.

*Front row*

Zoltan Weber
Solofo Rakotonaina
Dimitris
Papanastassiou
Claire Lupton
Jose Simoes
Goetz Boekelmann
Mark Noble
Mohamed Hamoudi
Maria Blanco Sanchez
Rob Van der Hilst
Aletta Zielhuis

Middle row

Jonas Lindgren
Martin Kornig
Svein-Erik Haturan
Tore Johansen
Jennifer Scott
Brijpal Rathor
Isabelle Lecomte
Nathalie Wajeman
Barb Sotirin
Francoise Kornendi
Giovanni Ranieri
Paola Picco

Back row

Monica Hammarstrom
Oyvind Pettersen
Vincent Nimier
Burkhard Buttkus
Francois Guillet
John Van de Car
Bernd Kummer
Hansjoerg Zdarsky
Matt Dzieciuch
Dave Chester
F. Erdeniz Ozel
Durbha Sai Ramesh

Accuracy Disclaimer: C'mon, it's been a year and a half!

PARTICIPANTS

- Blanco Sánchez, María*, Dep. de Geofísica, Facultad de Ciencias Fisicas,
Univ. Complutense, Av. Complutense s/n, 28040, Madrid, Spain.
- Bokelman, Goetz*, Dept. of Geol. and Geophys. Sciences, Princeton Univ.,
Princeton, NJ, 08544, USA.
- Burrascano, Pietro*, Dip. Info. Com., Via Eudossina 18, 00184, Roma, Italy.
- Chester, David*, Woods Hole Oceanographic Institution, Woods Hole, Mass
02543, USA.
- Dietrich, Michel*, Lab. de Géophysique Marine, Univ. de Bretagne Occiden-
tale, 6, av. Le Gorgeu, 29287, Brest, France.
- Dzieciuch, Matthew*, Univ. of Michigan, CSPL, Ann Arbor, MI 48109,
USA.
- Flèche, Jean-Christophe*, Institut Français du Pétrole, ENSPM-CES Ex-
ploration, B.P.311, 92506 Rueil Malmaison, France.
- Gaillard, Fabienne*, IFREMER, Centre de Brest, B.P. 70, 29280 Plouzané,
France.
- Hammarstrom, Monica*, FOA, Division of Hydroacoustics-Seimology, Box
27322, S-10254 Stockholm, Sweden.
- Hamoudi, Mohamed*, CRAAG, B.P.63, Bouzarera 16340, Alger, Algeria.
- Hamran, Svein-Erik*, Royal Norwegian Research Council, NTNF-PFM,
Boks 25, 2007 Kjeller, Norway.
- Jech, Jiri*, Geophys. Inst. of Czechoslovakia, Bočni II, 14131 Praha-4, Spor-
ilov, Czechoslovakia.
- Johansen, Tore*, Nansen Remote Sensing Cent., EDV. Griegsvei 3A, N-5017
Solheimsviken, Norway.
- Kormendi, Françoise*, Lab. Géophysique, 6, av. Le Gorgeu, 29287 Brest,
France.
- Kornig, Martin*, Institut für Meteorol. und Geophys., Feldbergstrasse 47,
6000 Frankfurt, FRG.
- Kummer, Bernd*, Instit. Geophys., Univ. Hamburg, Bundesstr. 55, 2000
Hamburg 13 , FRG.
- Lecomte, Isabelle*, IFREMER, B.P.70, 29280 Plouzané, France.

Participants

- Lindgren, Jonas, Avd.f.Fasta Jordens Fysik, Box 556, S-75122 Uppsala, Sweden.*
- Lognonné, Philippe, Laboratoire de Sismologie, Inst. de Physique du Globe, 4 Place Jussieu, 75252 Paris Cedex, France.*
- Lupton, Claire, Dept. of Earth Sciences, Univ. Leeds, Leeds LS2 9JT, UK.*
- Martin-Lauzer, François Régis, Serv. Hydrogr. et Océanogr. de la Marine, 12 rue du Chatellier, B.P; 426, 29275 Brest Cedx, France.*
- Nimier, Vincent, CEPHAG-ENSIEG, B.P. 46, 38402 St-Martin d'Heres, France.*
- Noble, Mark, Inst. de Physique du Globe, 4 Place Jussieu, 75252 Paris Cedex 05, France.*
- Ozel, Erdeniz, Dokuz Eylul Univ. Deniz Bizinleri ve Teknolojisi, SSK Bloklari D Blok kat 2, Konak-Izmir, Turkey.*
- Papanastassiou, Dimitris, Seismological Inst.Nat. Observatory of Athens, P.O.Box 20042, Gr-11810 Athens, Greece.*
- Pessoa, Jose Miguel, Departamento Geociencias, Univ. de Aveiro, 3800 Aveiro, Portugal.*
- Pettersen, Øyvind, Univ. of Oslo, Inst.of Geophys., Box 1022, O315 Oslo 3, Norway.*
- Picco, Paola, ENEA, C.R.E.A., C.P. 316, 19100 La Spezia, Italy.*
- Rakotoniaina, Solofoarisoa, Observ. of Antananarivo, B.P. 3843, Antananarivo-101, Madagascar.*
- Ramesh, Durbha, National Geophysical Research Institute, Uppal Rd., Hyderabad-500007 (A.P.), India.*
- Rathor, Brijpal Singh, National Geophysical Research Institute, Uppal Rd., Hyderabad-500007 (A.P.), India.*
- Scott, Jennifer, University of California, San-Diego, A-025, La Jolla, CA 92093, USA.*
- Simoes, Jose, Centro Geofisica, Univ. de Lisboa, R. Escola Politec. 58, P-1294, Lisboa Codex, Portugal.*
- Singh, Satish, Geophys. Lab., Dept. of Physics, University of Toronto, Toronto, MSS IA7, Canada.*
- Sotirin, Barbara, Marine Physical lab., Scripps Institution of Oceanography, Univ. of California San-Diego, La Jolla, CA 92093, USA.*
- Ugolini, Stefania, Dipart. di Fisica, Settore di Geofisica, Univ. di Bologna, viale Berti Pichat 8, 40127, Bologna, Italy.*
- Van Decar, John, Geophysics Program, Univ. of Washington, AK-50, Seattle, WA 98195, USA.*
- Van der Hilst, Rob, Dept. of Theoretical Geophysics, Inst.of Earth sciences, Univ. Utrecht, Budapestlaan 4, 3508 TA, Utrecht, The Netherlands.*

Participants

- Wajeman, Nathalie, GRGS/CNES, 18 av. E.Belin, 31055, Toulouse Cedex,
France.*
- Weber, Zoltan, Geophysical Dept., L.Eotvos Univ., Kun Bela ter 2, H-
1083, Budapest, Hungary.*
- Zdarsky, Hannsjoerg, Inst. für Geowissenschaften der Johannes Gutenberg
Univ., Saarstr. 21, 6500 Mainz, FRG.*
- Zielhuis, Aletta, Dept. of Theoretical Geophysics, Inst.of Earth sciences,
Univ. Utrecht, Budapestlaan 4, 3508 TA, Utrecht, The Netherlands.*

PREFACE

One of the major problems in the study of the Earth and the Oceans is their vast dimensions and the difficulty to measure directly their physical properties. For the solid Earth the only means of in situ measurement is by drilling ; this is particularly expensive and delicate under the oceans, which cover most of the Earth surface. Moreover, drilling provides only a few shallow point measurements, insufficient to describe the deep global structure of the Earth. The use of indirect methods constitutes an attractive solution to this observational problem.

In the ocean the situation is different insofar as in situ measurements are possible and are routinely made. However the spatial density of these observations remains woefully sparse and the significant time variability of the medium would require that they be repeated frequently. This is nearly impossible on a global scale. Here again indirect methods, such as tomography, can contribute to alleviate this problem.

In geophysics tomography is a technique which deduces some physical properties of the medium from the perturbations encountered by waves propagating through it. Thus tomography draws on a variety of disciplines, such as wave propagation in heterogeneous media, statistical estimation and inverse theory, and numerical modeling. These subjects take different forms when applied to solid Earth geophysics or to physical oceanography.

The aim of the 50th session of the Summer School on Theoretical Physics held in Les Houches in the summer of 1988 was to present in detail theoretical and practical aspects of tomography in geophysics and oceanography. One of the goals was to bring together students and practitioners of the two fields to teach, discuss and exchange ideas on all the components of tomography, to point out common approaches, and to contrast various specific applications.

The four-week session included introductory lectures on inverse methods (probabilistic approach to inverse problems, Monte Carlo and least squares methods, discrete and functional inverses, inversions involving rays and waves), theoretical seismology (elastodynamic Green's function, Lamb's

Preface

problem in depth dependent media, excitation and dispersion of surface waves, recent uses of higher modes), ocean acoustic tomography (general concepts, background on physical oceanography, recent experiments). More specialized lectures dealt with various aspects of the subjects, namely : global Earth tomography, seismic networks, time dependent problems in oceanography, numerical modeling in seismology and underwater acoustics, inversion of magnetotelluric data, elastic inversions on massively parallel computers, scattering tomography, and migration techniques in exploration seismology.

A glance at the table of contents of this volume will show that all the subjects are not covered as they were taught at the School. Some editorial choices have been made (because of availability of some topics in existing monographs, for instance), some material has been expanded, other reduced. The level and nature of the courses is diverse, including reviews, theoretical statements, practical considerations, and prospective developments.

Our aim in presenting these lectures on various aspects of the theory and implementation of tomography in geophysics and oceanography is to show the diversity and vitality of this field of research and to underscore the variety of disciplines involved. If this volume can serve as an introduction, a partial reference and a stimulus to further research, it will have reached its goal.

Acknowledgements

The XLth session of Les Houches Summer School and this volume of lectures would not have been possible without:

- the financial support from the Université Joseph Fourier of Grenoble, the NATO Scientific Division, the Commissariat à l'Energie Atomique and the NSF;
- the guidance of the Board of Trustees of the School;
- the typing of manuscripts by Danielle Choupin, Jocelyne Le Gal et Claude Mercier;
- the essential role played by Nicole Leblanc and Anny Glomot in the preparation and administration of the session;

Y. Desaubies
A. Tarantola

CONTENTS

<i>Lecturers</i>	ix
<i>Participants</i>	xi
<i>Préface</i>	xv
<i>Preface</i>	xvii
<i>Contents</i>	xix

Course 1. *Probabilistic foundation of inverse theory, by A. Tarantola*

1

1. Abstract	5
2. Introduction to probability densities and volumetric probabilities	5
3. The notions of capacity element and of volume element	8
4. Information content	13
5. The state of null information	14
6. The state of perfect knowledge	15
7. Combination of informations	15
8. The data space, the model space, and the joint data \times model space	17
9. Information given by physical theories	18
10. The inverse problem as a problem of combination of information	19
11. Example 1: theoretical uncertainties neglected	21
12. Example 2: all uncertainties are Gaussian	22
13. Robust inversion	24
14. Bibliographical comments	25
References	25

Course 2. A short course on theoretical seismology, by P.G. Richards	29
1. A review of underlying concepts	33
1.1. Introduction	33
1.2. Key concepts in the theory of seismic wave propagation: Displacement, body force, traction, stress tensor	37
2. Some important solutions of the wave equation	50
2.1. Introduction	50
2.2. Elastic waves in homogeneous media	51
2.3. Geometrical ray theory	59
3. Attenuation, from intrinsic friction or scattering	63
3.1. Introduction	63
3.2. Attenuation, due to anelasticity	65
3.3. Comparison of pulse shapes with different dispersion	71
3.4. Attenuation, due to scattering	75
4. Seismic waves in media with plane parallel layering	77
4.1. Introduction	77
4.2. The plane wave/cylindrical wave components of a simple spherical wave	80
4.3. The plane wave/cylindrical wave components of a simple scattered wave	85
4.4. Basic analysis of a generalized ray	88
4.5. Cagniard methods	90
4.6. Incorporating anelasticity	91
5. Matrix methods	92
5.1. Introduction	92
5.2. Use of column vectors, to describe a wave solution	93
5.3. An eigenvector-eigenvalue approach to the wave solution	95
5.4. Matrix methods for a medium composed of homogeneous layers	98
5.5. Matrix methods for continuously varying media	101
5.6. Summary of basic choices for $\omega-k$ integration	108
6. Analytic study of seismic waves in 3D structures	110
6.1. Summary	110
6.2. Introduction	110
6.3. The basic relation between travel time and slownesses	113
6.4. Ray calculations without ray tracing	122
6.4.1. One dimensional	122
6.4.2. Three dimensional	125
6.5. A representation for generalized rays	126
6.6. A simple example	128
6.7. Discussion	129
References	132

<i>Course 3. Asymptotic theory of normal modes and surface waves, by B. Romanowicz</i>	<i>135</i>
1. Introduction	139
2. Spherical earth	140
2.1. Notations	140
2.2. Correspondence with propagating waves	144
3. Aspherical earth	146
3.1. Introduction	146
3.2. Zeroth order asymptotic theory	147
3.3. Isolated multiplet	148
3.4. Coupling terms included	150
3.5. Higher order asymptotics	153
Appendix 1.	156
References	156
<i>Course 4. Ocean acoustic tomography, by Y. Desaubies</i>	<i>159</i>
1. Introduction	163
1.1. Overview	164
1.2. The inversion	167
2. Sound propagation in the ocean	169
2.1. The acoustic wave equation	169
2.2. Geometric acoustics	171
2.3. The sound speed field	173
2.4. Sound propagation	176
3. Some elements of ocean dynamics	177
3.1. Governing equations	177
3.2. Eddies and mesoscale, waves and turbulence	179
3.3. The vertical and horizontal structure	180
3.4. Mesoscale and tomography	182
4. The first ocean acoustic tomography experiments	184
4.1. Preliminary experiments	184
4.2. The 1981 ocean acoustic tomography experiment (OAT 81)	186
4.2.1. The acoustic analysis	186
4.2.2. A nonlinear constrained inversion	189
4.3. The 1983 reciprocal transmission experiment (RTE83)	192
5. Error analysis	195
5.1. Signal design	195
5.2. Instruments	196
5.3. Internal waves and sound transmission	196
5.4. Ray tracing and linearization	198

6. Conclusions	199
References	199
Course 5. Using data with models; Ill-posed and time-dependent ill-posed problems, by C. Wunsch	203
1. Introduction	207
1.1. Observing the ocean	207
1.2. Technical approaches	209
2. Conventional models	209
3. A model example	211
4. Unorthodox observations	213
4.1. Role of acoustic tomography	213
4.2. The role of altimeters	216
5. Simple computations	217
5.1. A discretization	217
5.2. A whole domain method	218
6. Recursive least squares	223
6.1. Classical least-squares	223
6.2. Improving without re-inversion	225
7. The control formalism	227
7.1. Canonical forms	228
7.2. The forward problem	231
7.3. The state estimation problem	231
7.3.1. The Kalman filter	231
7.3.2. The Kalman smoother	233
7.4. The terminal constraint problem	237
7.5. Reduced order observers and estimators	241
8. Controllability and observability	242
9. Final comments	243
Appendix 1.	245
References	246
Course 6. Seismic numerical modeling, by D. Kosloff and D. Kessler	249
1. One dimensional acoustic wave propagation	254
1.1. The one dimensional acoustic wave equation	254
1.2. The finite difference approximation	254
1.3. Accuracy	255
1.4. Stability and numerical dispersion	256

1.5. Improving spatial accuracy with higher order schemes	258
1.6. Acoustic wave equation with variable density	261
1.7. Staggered grids	262
1.8. The Fourier method	263
1.9. Numerical dispersion and stability	265
1.10. Fourier method without FFT	266
1.11. Finite differences using the FFT	267
1.12. A hybrid method	268
1.13. The finite-element method	269
1.13.1. Notation and spatial discretization	269
1.13.2. Variational principle	272
1.13.3. Remarks	274
2. Two dimensional and three dimensional acoustic forward modeling by the Fourier method	277
2.1. The multi dimensional acoustic wave equation and the Fourier solution method	277
2.2. Stability and numerical dispersion	278
2.3. Problem design	279
2.4. Different types of wave equations	279
2.5. Exploding reflector concept	280
2.6. Free surface	280
2.7. Absorbing boundary conditions	281
2.8. Implementation of a 3D solution scheme	283
2.9. The solution scheme	283
3. Two and three dimensional elastic forward modeling by the Fourier method	286
3.1. Momentum conservation and stress-strain relation for an isotropic elastic solid	286
3.2. Solution algorithm	288
3.3. Source types	289
3.4. Stability and numerical dispersion	290
3.5. Free surface boundary condition	292
3.6. Conservation of energy	292
3.7. Source receiver reciprocity	297
4. Improvement of the time integration	299
4.1. The formal solution	299
4.2. Homogeneous case	300
4.3. Intermediate results	301
4.4. Solution with source term	301
4.5. Nonreflecting boundary condition	302
4.6. Efficiency, comparisons with finite differences	302
5. Forward modeling from an operator view	303
5.1. The formal solution	303
5.2. Rederivation of temporal differencing through a Taylor expansion of the formal solution	305

5.3. Finite difference schemes derived from system	306
5.4. The rem approach (rapid expansion method)	307
5.5. Solution for the source term	309
5.6. Amount of work for REM	310
5.7. Nonreflecting boundary conditions	310
5.8. Concluding remarks	310
References	311
 Course 7. Ocean seismo-acoustic modeling: Numerical methods, by F.B. Jensen	 313
1. Introduction	317
2. The ocean waveguide	318
3. Classification of wave-theory models	324
4. Time-harmonic solutions of separable problems	326
4.1. Fourier integral	326
4.2. Normal modes	327
5. Time-harmonic solutions of non-separable problems	327
5.1. Coupled modes	328
5.2. Parabolic equation	329
6. Pulse solutions by Fourier synthesis	330
7. Numerical results	331
7.1. Time-harmonic modeling of acoustic data	332
7.2. Pulse modeling of interface-wave data	336
8. Summary and conclusions	342
References	342
 Course 8. A unifying view of inversion, by P. Mora	 345
1. Theory of nonlinear inversion	349
1.1. Philosophy of inversion	349
1.2. The forward problem: physics	349
1.3. The inverse problem: inverse physics	350
1.4. Partial inversion theories	353
1.5. Numerical examples	355
2. Unification with partial inversion theories	356
2.1. Analysis of linearized inversion	359
2.2. Nonlinear inversion	363
2.3. Numerical examples	363
2.4. Conclusions	364

3.	Computational physics of the forward problem	365
3.1.	Parallelism in physics	366
3.2.	The Connection Machine	367
3.3.	Solving the anisotropic elastic wave-equation	369
3.4.	Conclusions	372
	References	373
<i>Course 9. Inversion of low-frequency electromagnetic data, by T. Madden</i>		375
1.	Introduction: magnetotellurics, the need for interpretation methods.	379
2.	Magnetotelluric equations: Maxwell's equations in conducting media	382
3.	The maximum likelihood inverse	384
4.	Sensitivity operators	388
5.	The bilinear identity: adjoint operators and reciprocity.	391
6.	Relaxing the inverse problem	397
	References	407
<i>Course 10. Inverse methods in ocean bottom acoustics, by G. Frisk</i>		409
1.	Introduction	413
2.	Specific feature methods	414
2.1.	Caustic range method	414
2.2.	Advantages	417
2.3.	Disadvantages	417
3.	Iteration of forward models methods	417
3.1.	Fitting near-bottom transmission loss	418
3.2.	Advantages	421
3.3.	Disadvantages	421
4.	Perturbative inversion methods	422
4.1.	Reflection coefficient inversion	423
4.2.	Normal mode eigenvalue inversion	425
4.3.	Advantages	428
4.4.	Disadvantages	428
5.	Exact inverse methods	429
5.1.	Gelfand-Levitant method	430
5.2.	Advantages	433
5.3.	Disadvantages	434
6.	Conclusions	434
	References	435

<i>Course 11. Some practical aspects of ocean acoustic tomography, by B. Cornuelle</i>	439
1. Introduction	443
2. Resolution in a vertical slice	444
2.1. Introduction	444
2.2. Loop harmonics	445
2.3. Inverses	451
3. Moving ship tomography	452
3.1. Introduction	452
3.2. The projection-slice theorem	453
3.3. Examples	454
4. Time dependence	457
4.1. Introduction	457
4.2. General problem	458
4.3. The Kalman filter	460
4.4. Examples	461
References	463

