Introduction to and State-of-the-Art in Earthquake Source Inversion

– Part 3 –

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Roadmap

Earthquake Source Inversion

(1) Introduction & Theory
- A brief overview
- Fundamentals
- From point-source to extended-fault modeling

(2) Applications & Implications
- Case studies: early developments
- What to learn from these source models?
- What can be extract from them?

(3) Challenges, Developments, Opportunities
- Imaging versus inversion, or combination of both?
- Alternative methods
- Uncertainty quantification

New Developments

So what's next?

- We saw some fundamental methods for finite-fault source inversion, all based on the same equation (representation theorem)

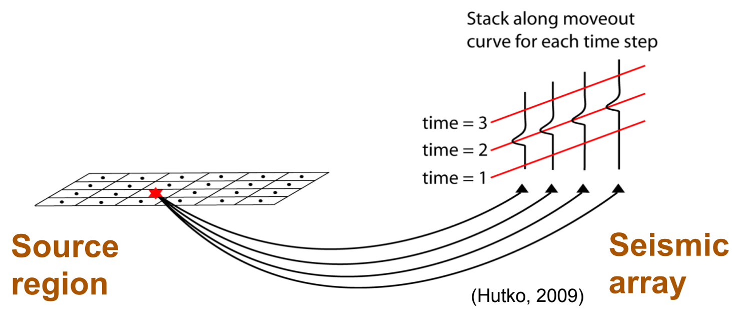
$$u_n(\mathbf{x},t) = \int_{-\infty}^{\infty} d\tau \int\int_{\Sigma} [s_i(\xi,\tau)] \cdot c_{ijpq} \cdot \nu_j \cdot \frac{\partial}{\partial \xi_q} G_{np}(\mathbf{x}, t-\tau; \xi, 0) d\Sigma$$

- We saw the difficulties in solving this inverse problem, not only because of its inherent non-uniqueness, but also because of issues with the data, the prior information, and the how the inversions are carried out
- Can we come up with alternative approaches that shed light onto the rupture process of earthquakes?

New Developments

Rupture-front tracking, or imaging (back projection)

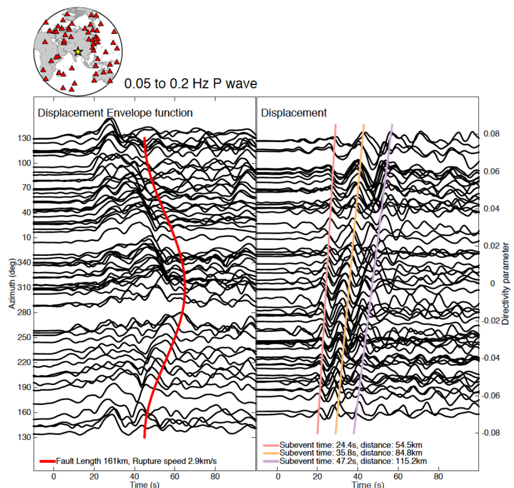
- Waves from different points on the faults will arrive at different times on a spatially dense seismic array; identify coherent arrivals across the array (e.g. Iishi and Shearer, 2005)
- Map arrival-time differences in the array back into the source region to track the rupture front; this can be done for narrow frequency bands



New Developments

Rupture-front tracking, or imaging (back projection)

- Example for April 25, 2015, M 7.8 Nepal earthquake



Fan and Shearer, 2015

New Developments

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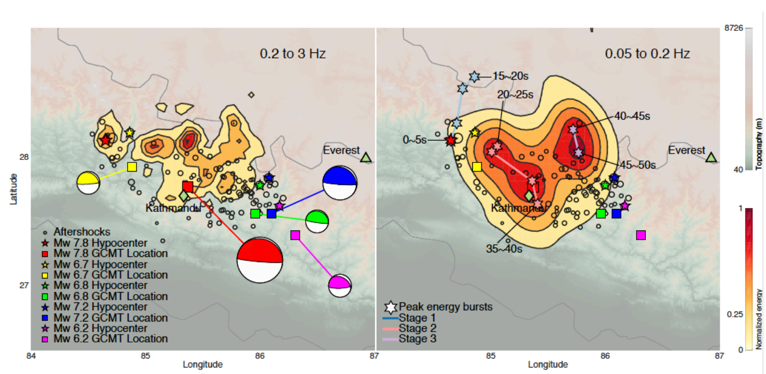

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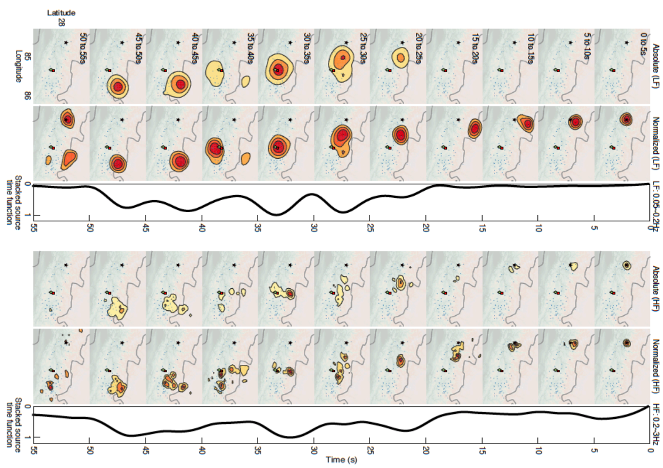

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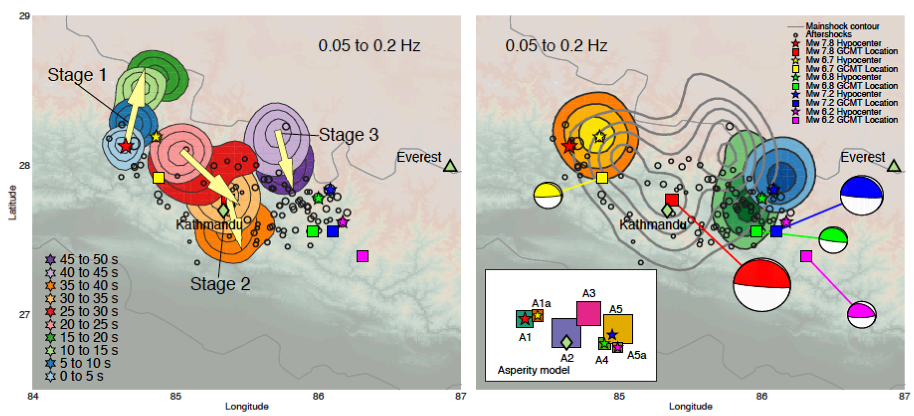
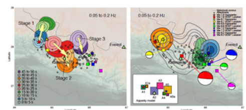


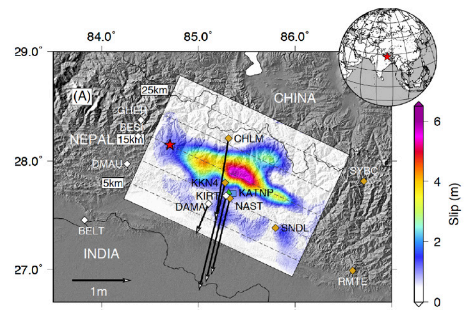
Fan and Shearer, 2015

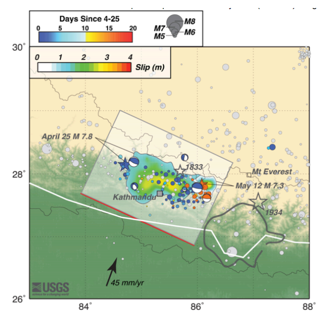
New Developments

Rupture-front tracking, or imaging (back projection)

- Example for April 25, 2015, M 7.8 Nepal earthquake
  - Back-projected high-frequency energy comes from different fault region then the main slip







Avouac et al., 2015

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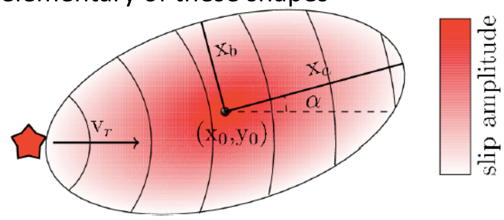
- Map arrival-time differences in the array back into the source region to track the rupture front; this can be done for narrow frequency bands
- **Back-projection shows that the high-frequency radiation comes from different parts of the fault then what is seen in slip inversion !**
- Could use back-projection information as prior information in finite-fault inversion
- "Two-step" inversion of back-projection and finite-fault inversion (Yano and Ji, 2012), or some sort of joint inversion (Yagi, 2012)

New Developments

Low-order source models

- Make simplifying assumptions on the rupture shape (circles, ellipses) and find a few elementary of these shapes



7 parameters per ellipses; 5 to describe the geometry:
- (x_0,y_0): the centre of the ellipse
- (x_a,x_b): length of the semi-major and semi-minor axis
- α : dip of the semi-major axis w.r.t. The horizontal

and 2 to 4 to describe the slip history:
- s_{max}: maximum slip amplitude of the slip distribution
- v_r: constant rupture speed
- τ : rise-time
- λ : the rake

Twardzik et al, 2012

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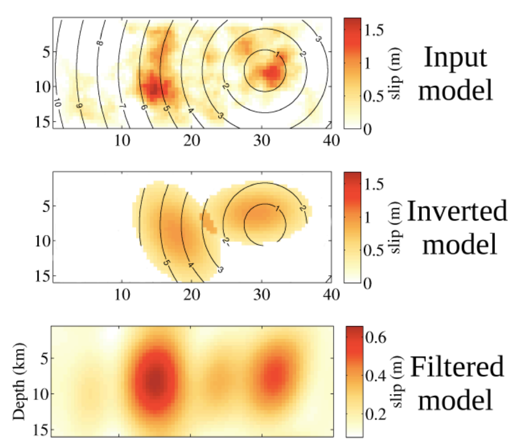
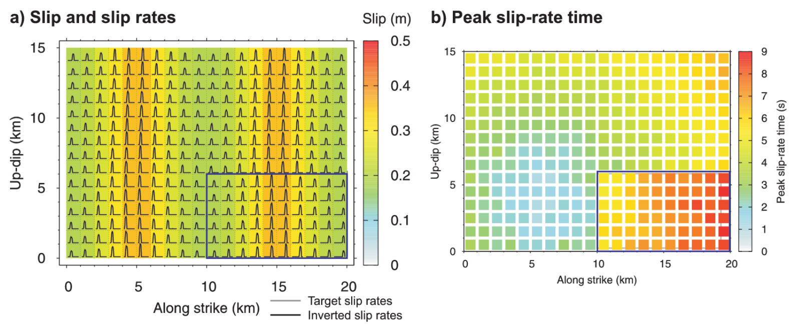


Twardzik et al, 2012

New Developments

New parameterizations

- **Don't prescribe slip function, or even hypocenter, but just a long-enough finely sampled time window** in which rupture is allowed to occur (Gallovic et al., 2015; Fan & Shearer, 2014)
  - Simple, and then more complex test case
  - Inversion is linear, but a large system needs to be solved



New Developments

New parameterizations

- Gallovic et al. (2015)
  - Inversion is linear, but a large system needs to be solved
  - Smoothing constraints are applied, leading to degradation of the estimated local slip rates

$$M(\boldsymbol{m}) = \frac{1}{2}(\boldsymbol{d} - \mathbf{G}\boldsymbol{m})^{\mathrm{T}} \mathrm{C}_{\mathrm{D}}^{-1} (\boldsymbol{d} - \mathbf{G}\boldsymbol{m})$$

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Prior covariance on slip spectrum, to be k^{-2}

$$c_M(f, k_x, k_y) \sim \left(\frac{1}{1 + (k_x L)^2 + (k_y W)^2}\right)^2$$

$$\mathrm{C_M} = \mathrm{U_M}^{\mathrm{T}} \mathrm{U_M}$$

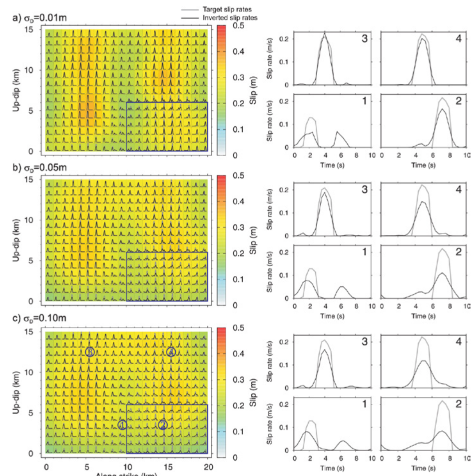
Non-neg. LSQR to solve

$$\begin{pmatrix} \frac{1}{\sigma_D}\mathbf{G} \\ \frac{1}{\sigma_M}\mathbf{U}_{\mathbf{M}}^{-\mathrm{T}} \\ \frac{1}{\sigma_{Mo}}\boldsymbol{E} \end{pmatrix} \boldsymbol{m} = \begin{pmatrix} \frac{1}{\sigma_D}\boldsymbol{d} \\ \mathbf{0} \\ \frac{1}{\sigma_{Mo}}M_0 \end{pmatrix}$$

New Developments

New parameterizations

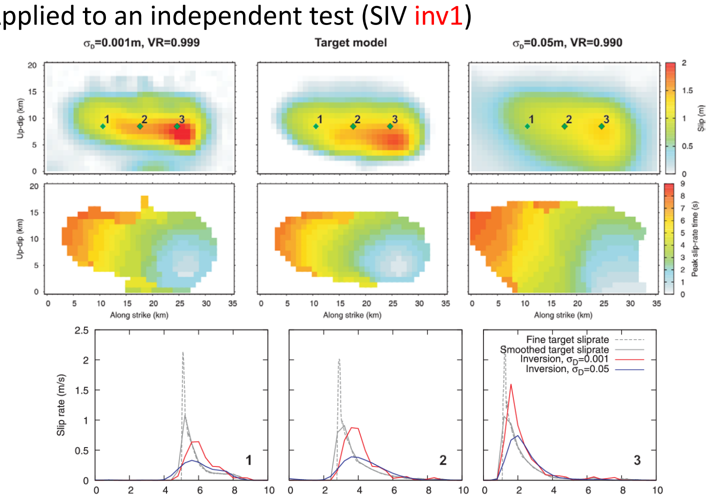
- Gallovic et al., (2015)
  - effects of smoothing on the slip, and local slip-rate



New Developments

New parameterizations

- Gallovic et al., (2015)
  - Applied to an independent test (SIV inv1)



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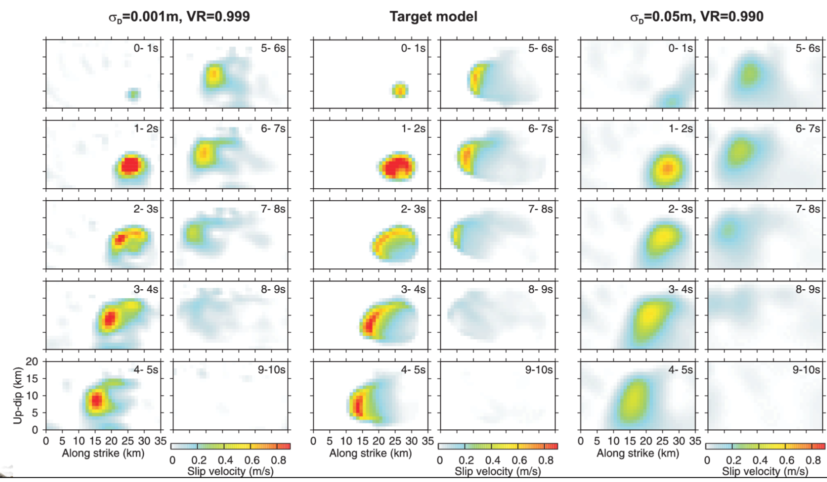


New Developments

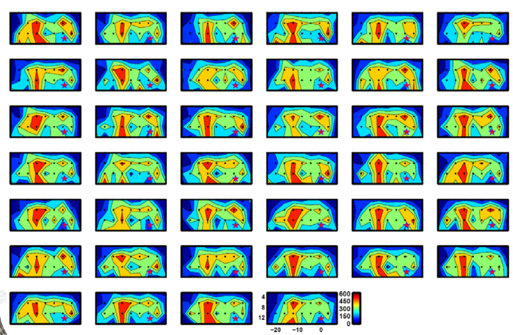
New parameterizations

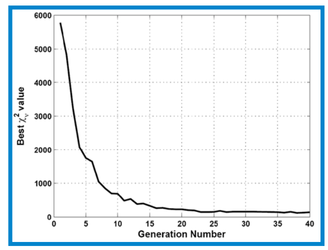
- Fan & Shearer (2014) and Gallovic et al. (2015) try to minimize the prior assumptions regarding the temporal rupture evolution, at the expense of needing to solve a large system with smoothing
- Effects of uncertainty in the Earth structure (Gallovic) or the geometry (e.g. dip) of the fault (Fan & Shearer) are treated separately, as a "secondary branch"

New Developments

Bayesian uncertainty quantification

- Bayesian methods to map the posterior PDFs on the fault
- Non-linear optimization using an Evolutionary Algorithm; the model space is sampled generating ~10^6 earthquake models
- Using this large sample size we perform Bayesian estimation to map the *a posteriori* distribution of the model parameters (MCMC sampler)





Monelli & Mai, 2008

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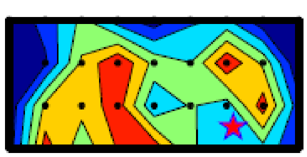

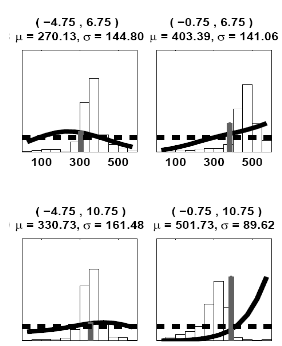

1D-marginals for four nodes on the fault: --
- - prior
--- posterior
hist: 'raw marginal'

Monelli & Mai, 2008

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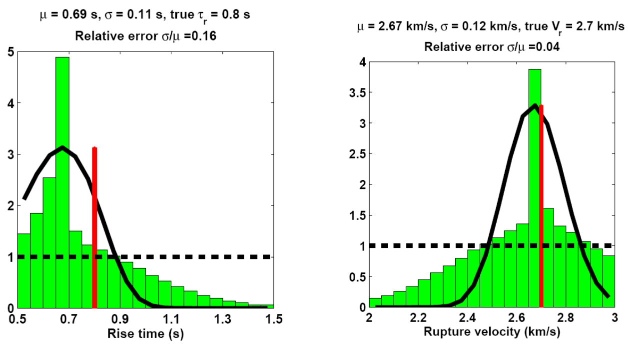
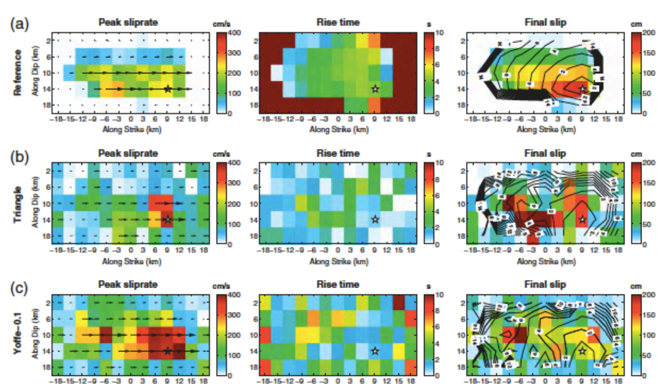


Monelli & Mai, 2008

New Developments

Bayesian uncertainty quantification

- Bayesian methods to map the posterior PDFs on the fault
- Extend this approach to examine effects of different source time functions or Earth models



Razafindrakoto & Mai, 2014

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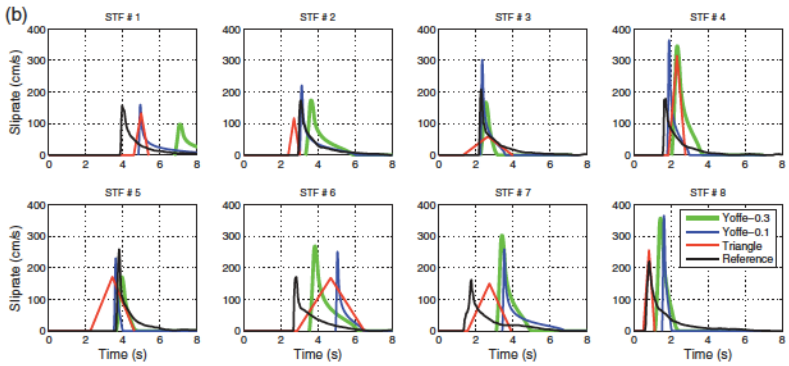

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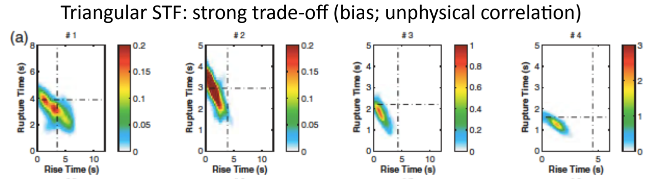
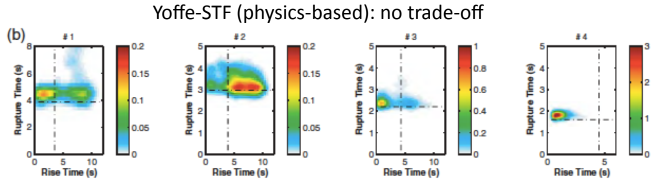
Triangular STF: strong trade-off (bias; unphysical correlation)



Yoffe-STF (physics-based): no trade-off



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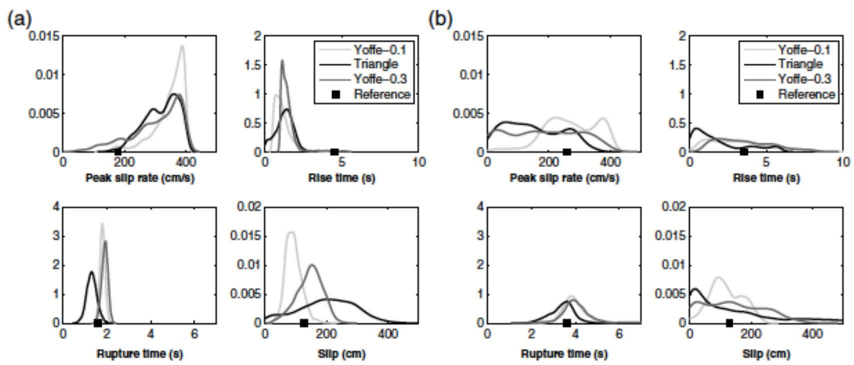


Razafindrakoto & Mai, 2014

New Developments

The Source Inversion Validation (SIV) project

→ Testing Earthquake Source Inversion methods

- **Cooperative initiative for (code) verification and (inversion) validation**
- **Goal: rigorous uncertainty quantification in earthquake rupture modeling**
- **Develop a series of benchmarks with varying degree of complexity, with and without "noise" in the data (and perhaps in some of the input parameters)**
- **All benchmarks remain accessible for all interested users; only for the most recent test the solution (input model) is not released**
- **Develop metrics to quantitatively compare and "rank" models**

New Developments

The Source Inversion Validation (SIV) project: Benchmarks (http://equake-rc.info/siv)

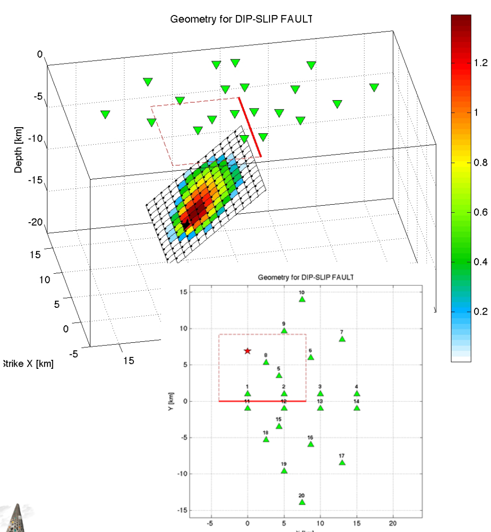
- **Green's function testing**
- **Forward-modeling cases for two "simple" kinematic cases**

- **Inv1: Inversion for "simple" M 6.5 strike-slip dynamic rupture model**
- **Inv2: Inversion for kinematic M 7 normal-faulting scenario, incl. uncertainties in the Green's functions (through 3D scattering)**
- **Inv3: teleseismic case for very large strike-slip rupture in Southern California**

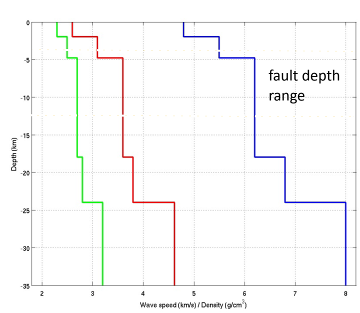
Mai et al., 2016

New Developments

SIV forward exercise: dip-slip extended fault



fault dip = 40° ; fault strike = 270°
Fault dimensions: **12 km along-strike, 12 km down-dip**
Seismic moment: M_0 = 1.824 x 10^{18} Nm (M_W = 6.14)
Hypocenter @ Z = 9.5 km; fault extent Z = [4 – 11.7] km
Distributed slip-rate over the fault plane
Rise time t_r variable over the fault variable
Rupture times imply non-constant rupture speed
Uniform source-time function: boxcar of width t_r

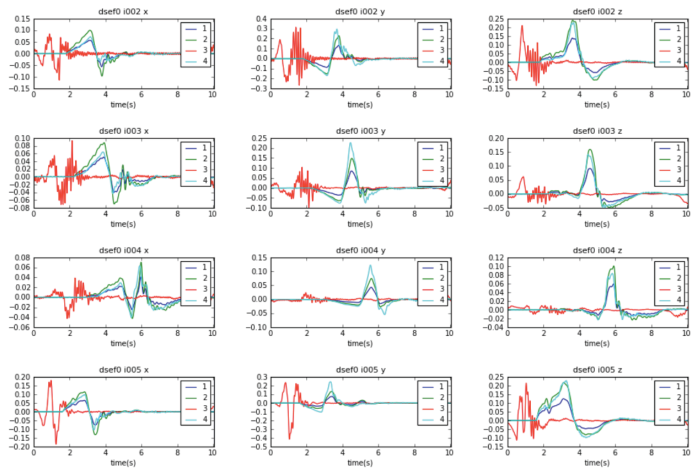


Mai et al., 2016

New Developments

SIV forward exercise: dip-slip extended fault

- **Four solutions: blue AXITRA / green COMPSYN / red TH mod / cyan SORD**

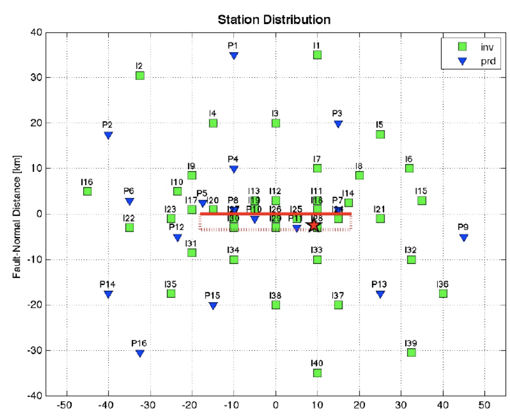


Mai et al., 2016

New Developments

SIV benchmarks: past & current

- **Inv1 – A "simple" dynamic rupture, starting from heterogeneous fault stress**
  - M ~ 6.5, strike-slip, on 80°-dipping fault; geometry & velocity structure given
  - Near-field seismogram at 40 sites; 16 additional site for blind prediction

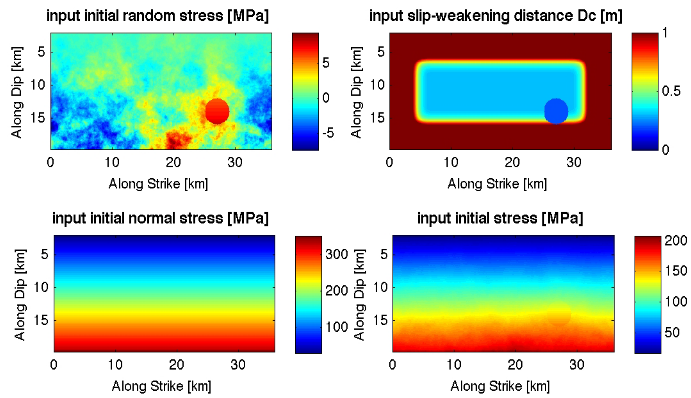


Mai et al., 2016

New Developments

SIV benchmarks: past & current

- Inv1 – A "simple" dynamic rupture, starting from heterogeneous fault stress
  - For dynamic simulations (using G. Ely's SORD code), "constant" Dc under a linear slip-weakening was assumed (Dc increases to fault edges for smooth rupture termination)



Mai et al., 2016

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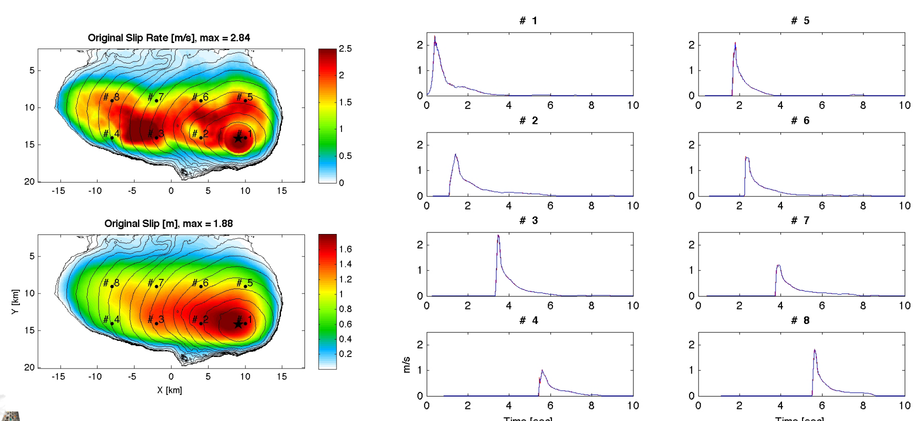
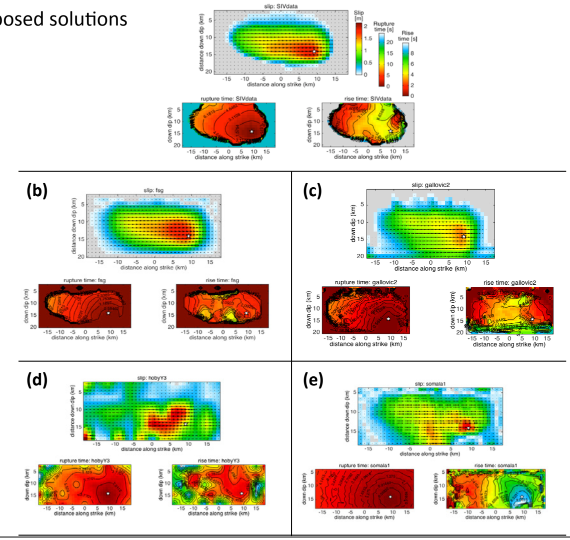


Mai et al., 2016

New Developments

SIV benchmarks: past & current

- Inv1 – A "simple" dynamic rupture, starting from heterogeneous fault stress
  - A few proposed solutions



Mai et al., 2016

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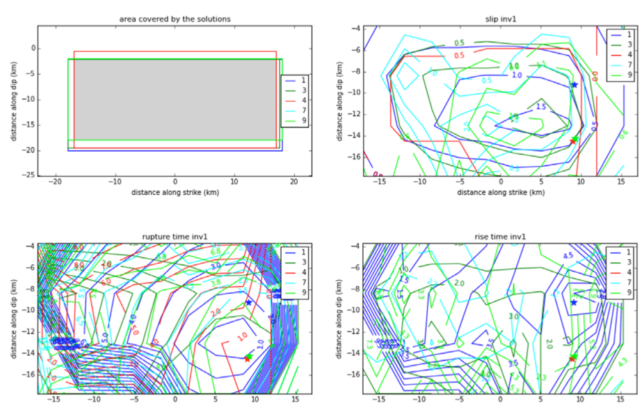

Mai et al., 2016

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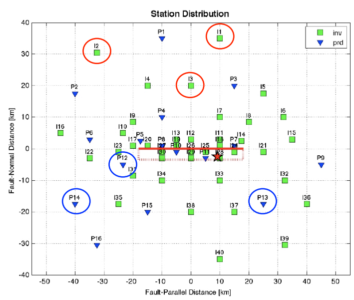
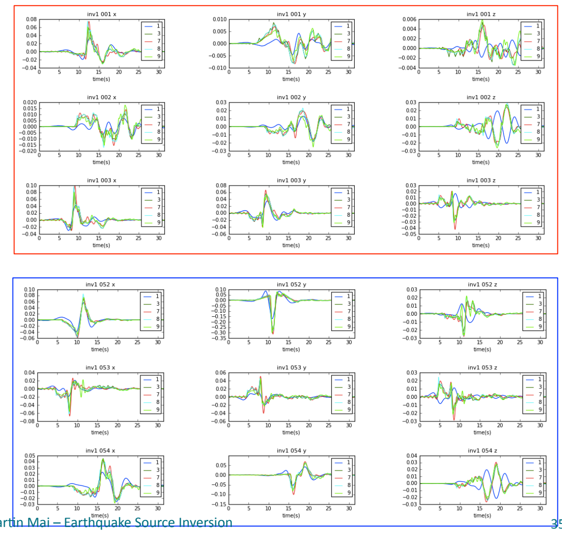
SIV benchmarks: past & current

- **<u>Inv1</u> – A "simple" dynamic rupture, starting from heterogeneous fault stress**





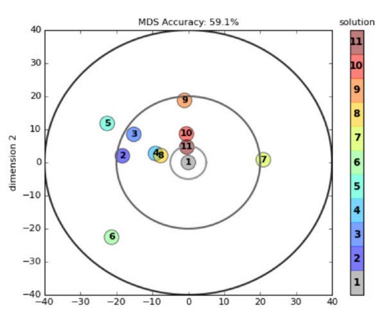
Mai et al., 2016

New Developments

SIV benchmarks: past & current

- **<u>Inv1</u> – A "simple" dynamic rupture, starting from heterogeneous fault stress**

- **Differences in inferred slip; rupture times similar; rise times poorly determined**
- **Overall "good" (reasonable) waveforms fits at ~ 1 Hz**
- **We have developed new quantitative statistical tools that allow to better analyze and rank the resulting solution**



Ranking with respect to the reference

Excellent	Good	Fair	Poor
fsg (11) sivdata (1)	gallovic (3) gallovic2 (4) causse (2) somala1 (8) hobyy3 (10) hobyy1 (9)	gallovic3 (5) hobyt (7) somala (6)	

Mai et al., 2016

New Developments

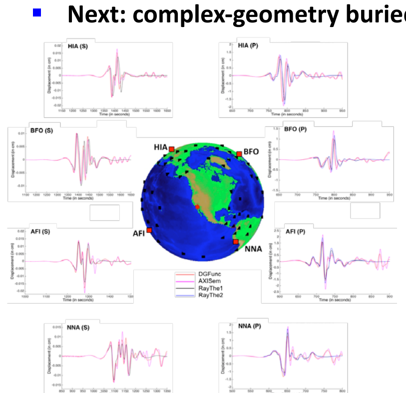
SIV platform (http://equake-rc.info/siv)

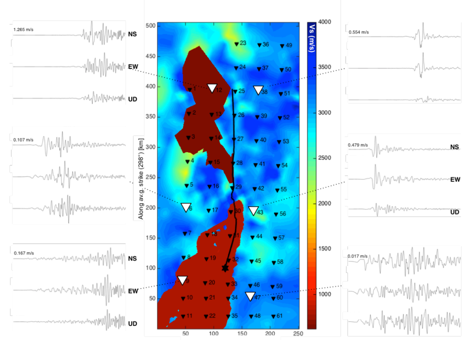
- **Quantitative source-model comparison tools**
  - **Tables of simple scalar metrics and statistical measures available**
  - **Multi-dimensional scaling (MDS) graphical output available for rapid model comparison**
  - **Spatial prediction comparison testing (SPCT) in the implementation phase**

New Developments

SIV platform (http://equake-rc.info/siv)

- **Project continues with more benchmarks**
  - **Latest: large (M 7.8) scenario rupture in southern California, for which several datasets are generated**
  - **Next: complex-geometry buried-faulting scenario in Los Angeles**





Earthquake Source Inversion

Where is the field going? What is next?

- Combine back-projection with inversion in a common framework
- Additional data (rotational motions? Tsunami data!)
- Denser and more dedicated arrays
- Comprehensive uncertainty quantification (Bayesian)
- Constraint-free source inversions
- At the same time: several agencies are driving towards “near-real-time” finite-fault inversions for EEW and rapid damage mitigation (USGS) and rapid dissemination of public information

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Earthquake Source Inversion

Where is the field going? What is next?

- **The earthquake source inversion problem is not solved, there is a lot be done!**
- **Accurate models of earthquake kinematics are important: for earthquake mechanics, rupture dynamics, shaking simulation**

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Thank you

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Additional Slides on
Quantitative Source Model Comparisons

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New Developments

SIV Comparisons: More quantitative

- **Quantitative metrics, based on Spatial Prediction Comparison Test (SPCT, Hering & Genton, 2011; Zhang et al. 2014; Razafindrakoto et al., 2014)**
  - Developed, tested, calibrated for synthetic test cases
  - Application for cases with and without known "true" solution
- **Brief summary of notation**
  - Spatial process Z(**s**) at locations **s** $\{Z(\mathbf{s}) \in \mathbb{R} : \mathbf{s} \in D \subset \mathbb{R}^2\}$
  - General loss function between a realization and a prediction $g[Z(\mathbf{s}_i), \hat{Z}_P(\mathbf{s}_i)]$
  - Squared-error loss (SE) $g[Z(\mathbf{s}_i), \hat{Z}_P(\mathbf{s}_i)] = [Z(\mathbf{s}_i) - \hat{Z}_P(\mathbf{s}_i)]^2$
  - Absolute-error (AE) $g[Z(\mathbf{s}_i), \hat{Z}_P(\mathbf{s}_i)] = \left|Z(\mathbf{s}_i) - \hat{Z}_P(\mathbf{s}_i)\right|$
  - Correlation loss (correlation skill) $g[Z(\mathbf{s}_i), \hat{Z}_P(\mathbf{s}_i)] = \frac{n}{(n-1)\hat{\sigma}_Z \hat{\sigma}_P}[Z(\mathbf{s}_i) - \bar{Z}][\hat{Z}_P(\mathbf{s}_i) - \bar{Z}_P]$

New Developments

SIV Comparisons: More quantitative

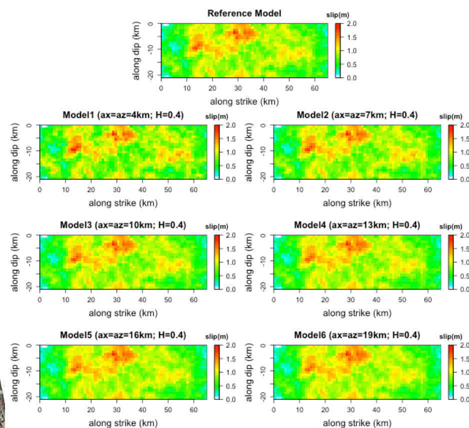
- **Testing uses then**
  - The loss differential $D(\mathbf{s}) = g[Z(\mathbf{s}), \hat{Z}_1(\mathbf{s})] - g[Z(\mathbf{s}), \hat{Z}_2(\mathbf{s})] = f(\mathbf{s}) + \delta(\mathbf{s})$
  - *f(s)* is the mean trend, *δ(**s**)* is a zero-mean stationary process with unknown covariance function *C(**h**)* whose properties are estimated using a semi-variogram analysis
  - Semi-variogram for all N(h_{ij}) points $\hat{\gamma}(h_{ij}) = \frac{1}{2\left|N(h_{ij})\right|} \sum_{N(h_{ij})} [D(\mathbf{s}_i) - D(\mathbf{s}_j)]^2$
  - We test several parametric variogram models; <u>the null-hypothesis is that all models have equal predictive ability</u>. Resulting statistics are given in terms of p-value probability, that is:

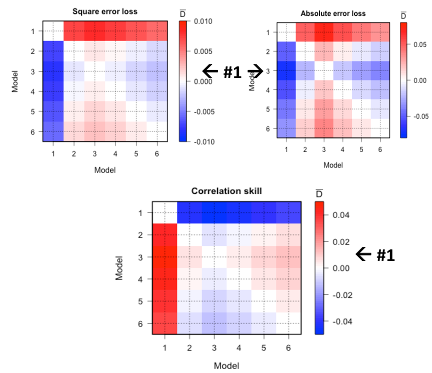
Two competing models have equal predictive ability with respect to the reference model, if the p-value is greater than a chosen statistical significance level (e.g. 5%). Otherwise, the null hypothesis is rejected.

New Developments

SIV Comparisons: More quantitative

- **Quantitative metrics, based on spatial prediction comparison test**
  - Synthetic test with changing correlation length only



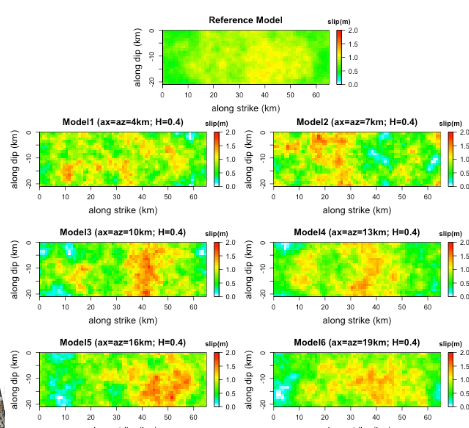


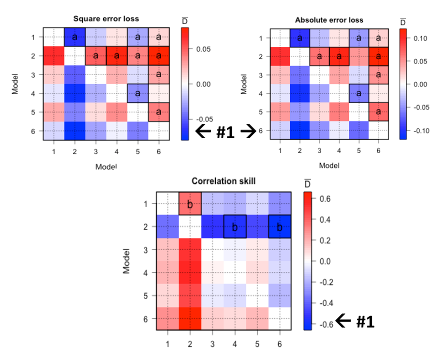
Top (SE, AE): Negative values (blue) indicate that the case named in the corresponding row is the better model
Bottom (CS): values (red) indicate that the case named in the corresponding row is better based

New Developments

SIV Comparisons: More quantitative

- **Quantitative metrics, based on spatial prediction comparison test**
  - Synthetic test with variations in patch location



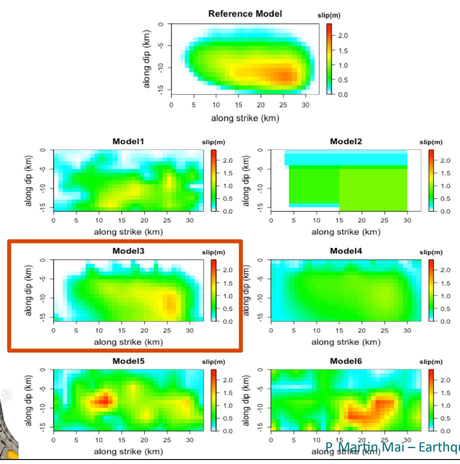


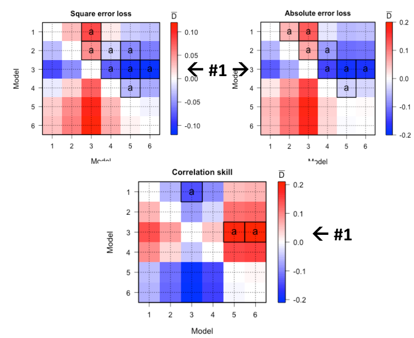
Top (SE, AE): Negative values (blue) indicate that the case named in the corresponding row is the better model
Bottom (CS): values (red) indicate that the case named in the corresponding row is better based. (a) and (b) refers to 5% and 10% testing level

New Developments

SIV Comparisons: More quantitative

- **Quantitative metrics, based on spatial prediction comparison test**
  - SPCT applied to subset of **inv 1**





Top (SE, AE): Negative values (blue) indicate that the case named in the corresponding row is the better model
Bottom (CS): values (red) indicate that the case named in the corresponding row is better based. (a) and (b) refers to 5% and 10% testing level

New Developments

SIV Comparisons: More quantitative

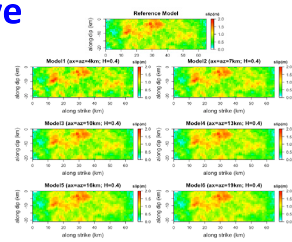
- **Multi-dimensional scaling**
  - Generate an *m*-dimensional configuration in Euclidian space based on (dis-)similarity between pairs of 2D random fields (e.g. slip models)
  - Visualize these point-configurations in a lower-dimensional (2D, 3D) representation

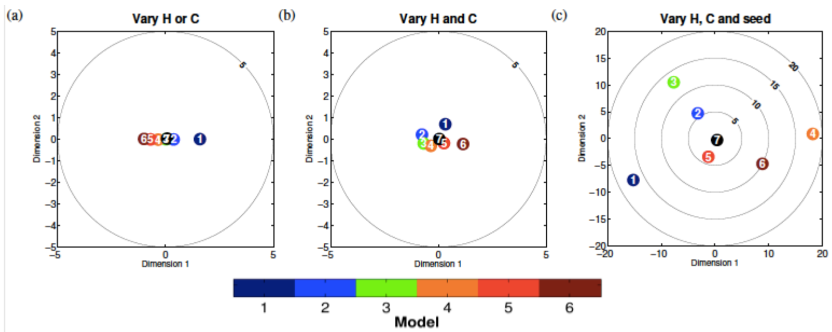
- **Method:**
  - Construct matrix ***D*** with elements that measure dissimilarity (SE, AE, or other)
  - Construct matrix ***B*** from ***D***, by double-centering ***D*** (for symmetry purposes)
  - Apply SVD to ***B***, such that $\mathbf{B} = \mathbf{V \Lambda V}^T$
  - Select *n*-points in *p*-dimensional space from $x_{ij} = V_{ij}\,\lambda_j^{1/2}$, $i = 1 .. n, j = 1 ... p$
  - Coordinates of *x* are constructed such that either a mean-model is the reference, located then in the center of the point cloud, or that any selected model (known solution) becomes the reference

New Developments

SIV Comparisons: More quantitative

- **Multi-dimensional scaling**
  - Applied to the previous test cases for synthetic slip-model realization with different random-field parameterizations





New Developments

SIV Comparisons: More quantitative

- **Multi-dimensional scaling**
  - Applied to the **SIV inv1** initial models

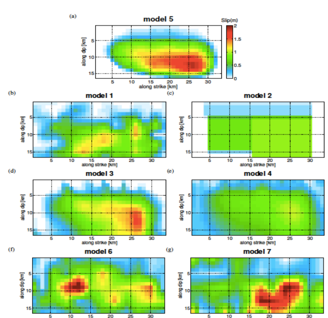
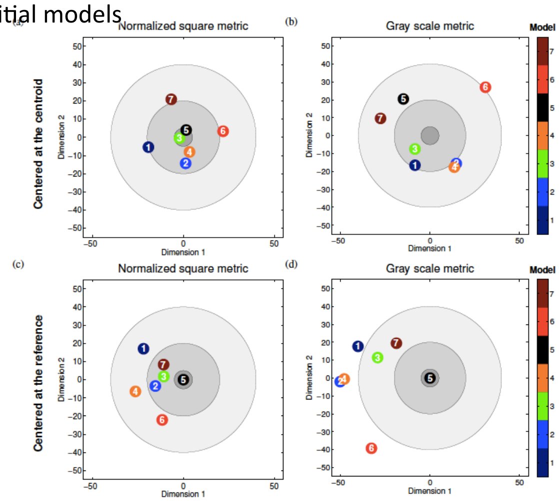


Table 1: Similarity scale.

Case	Excellent	Good	Fair	Poor
d_1*(%)	< 5	5 – 20	20 – 40	> 40
d_2†(%)	< 5	5 – 20	20 – 40	> 40

*d_1, Normalized square metric
†d_2, Gray scale metric



New Developments

SIV Comparisons: More quantitative

- **Inv1:** Comparison of simple scalar measures for now 10 models

scalar metrics

multi-dimensional scaling

spatial prediction comparison test

Benchmark id: inv1

Scalar source-model metrics for the different solutions

SN	Solution	Mo (Nm)	Mw	Max. Slip (m)	dz,dx	Dimensions (km)		Eff. Dimensions (km)		Slip Centroid (km)			Waveform misfits x 100			
						Width	Length	Width	Length	Xc	Yc	Zc	1-Norm	2-Norm	RMSE	%VR
1	**SIVdata**	**1.060e+19**	**6.65**	**1.87**	**0.50, 0.50**	**18.48**	**36.50**	**11.18**	**23.34**	**1.37**	**1.97**	**11.66**	**0.0**	**0.0**	**0.0**	**100.0**
2	causse	9.740e+18	6.63	1.33	2.50, 2.50	19.98	35.00	12.84	23.89	0.30	-1.97	14.94	340.3	9.8	0.9	56.3
3	fsg	1.200e+19	6.69	1.86	0.50, 0.50	18.00	36.00	12.20	23.96	1.32	1.97	11.68	207.7	9.4	0.7	65.1
4	gallovic	8.700e+18	6.59	0.91	1.00, 1.00	20.00	35.00	13.10	23.34	0.64	-1.94	12.00	*	*	*	*
5	gallovic2	8.700e+18	6.59	1.55	1.00, 1.00	20.00	35.00	13.60	23.42	1.28	-2.14	13.16	12.3	0.1	0.3	94.4
6	gallovic3	8.700e+18	6.59	1.02	1.00, 1.00	20.00	35.00	14.65	25.76	0.94	-1.94	11.98	*	*	*	*
7	hobyt	1.100e+19	6.66	3.22	4.00, 3.00	20.00	39.00	15.21	28.76	0.64	2.00	15.25	223.5	3.9	0.5	76.3
8	hobyy1	1.300e+19	6.71	2.38	4.00, 3.00	20.00	39.00	15.31	30.55	0.09	1.87	14.52	204.4	3.0	0.5	81.6
9	hobyy3	1.300e+19	6.71	2.15	4.00, 3.00	20.00	39.00	15.59	30.02	-0.33	2.06	15.62	204.4	3.0	0.5	81.6
10	somala	1.060e+19	6.65	6.22	0.34, 0.25	17.42	35.25	12.88	23.40	1.75	-2.06	12.00	171.8	4.5	0.5	82.1
11	somala1	1.060e+19	6.65	2.11	0.35, 0.50	10.01	34.50	7.56	24.55	1.14	-1.93	11.27	97.3	0.3	0.2	95.2

- The table lists standard parameters of the source. Mo refers to seismic moment. Moment magnitude Mw is computed as Mw = 2/3*(log10(Mo) - 9.05). The effective source dimensions (or Eff. Dimensions) is computed from autocorrelation width of the slip distribution following Mai and Beroza (2000). The slip centroid (Xc, Yc, Zc) is estimated as slip-weighted averaged coordinates (x, y, and z)

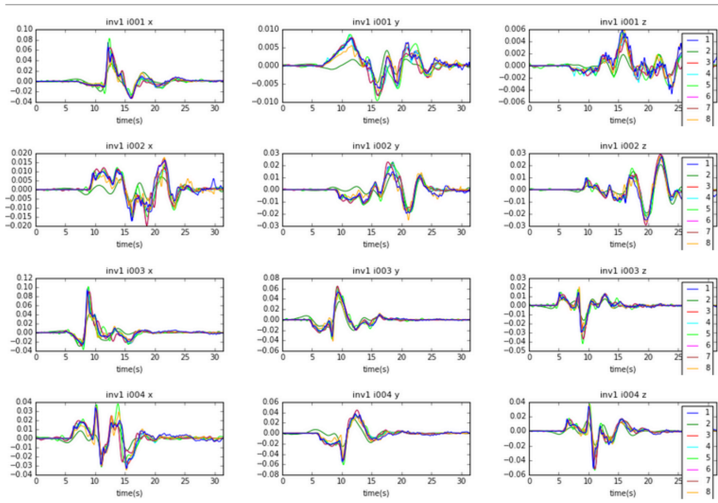
  The misfit metrics given below are computed for each waveform data and averaged for all the components.
  1-Norm (sum of average absoluture errors) = $\Sigma|y_i - f(x_i)|$
  2-Norm (sum of squares of the errors) = $\Sigma(y_i - f(x_i))^2$
  RMSE (root mean square errors) = $\sqrt{\Sigma(y_i - f(x_i))^2 / n}$
  Variance Reduction (VR) (scaled sum of the squares of the errors) = $1 - [\, \Sigma(y_i - f(x_i))^2 \,] / [\, \Sigma y^2]$

  Note : The waveform misfits are scaled up by multiplying with 100 to highlight differences in the small values. Additionally, the entries indicated with *, if the waveform data has not been provided.

New Developments

SIV Comparisons: More quantitative

- **Inv1:** Recall -- waveform fits are all very good



New Developments

SIV Comparisons: More quantitative

- **Inv1: Multi-dimensional scaling between 10 models**

scalar metrics

multi-dimensional scaling

spatial prediction comparison test

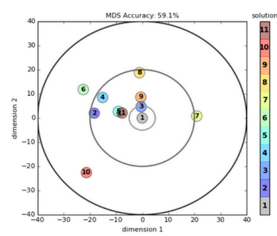
Benchmark id: inv1

Multi-Dimensional Scaling (MDS) analysis

Select Solutions for the comparisions (atleast 3, default: none for 'all')

SIVdata (1): causse (2): fsg (3): gallovic (4): gallovic2 (5): gallovic3 (6): hobyt (7): hobyy1 (8): hobyy3 (9): somala (10): somala1 (11):

Reference solution: SIVdata (default: none, also if this is not one of selected Solutions) reload



Ranking with respect to the reference

Excellent	Good	Fair	Poor
SIVdata (1) fsg (3)	causse (2) gallovic (4) gallovic2 (5) hobyy1 (8) hobyy3 (9) somala1 (11)	gallovic3 (6) hobyt (7) somala (10)	

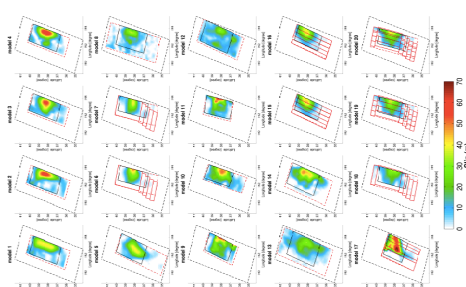
Dissimilarity matrix

Solution	SIVdata	causse	fsg	gallovic	gallovic2	gallovic3	hobyt	hobyy1	hobyy3	somala	somala1
SIVdata	0.00	27.53	2.87	19.72	7.56	23.78	22.34	21.85	21.10	29.65	11.59
causse	27.53	0.00	27.55	23.97	19.07	19.26	35.88	39.64	28.30	32.31	24.53
fsg	2.87	27.55	0.00	19.68	5.73	20.18	22.67	17.49	16.18	32.42	8.81
gallovic	19.72	23.97	19.68	0.00	11.29	9.93	41.64	25.50	32.28	35.29	18.72
gallovic2	7.56	19.07	5.73	11.29	0.00	11.07	29.17	21.23	18.44	25.50	9.86
gallovic3	23.78	19.26	20.18	9.93	11.07	0.00	45.36	20.78	27.40	31.32	16.45
hobyt	22.34	35.88	22.67	41.64	29.17	45.36	0.00	36.69	31.84	51.51	28.08
hobyy1	21.85	39.64	17.49	25.50	21.23	20.78	36.69	0.00	33.59	46.77	19.38
hobyy3	21.10	28.30	16.18	32.28	18.44	27.40	31.84	33.59	0.00	43.57	20.87
somala	29.65	32.31	32.42	35.29	25.50	31.32	51.51	46.77	43.57	0.00	24.95
somala1	11.59	24.53	8.81	18.72	9.86	16.45	28.08	19.38	20.87	24.95	0.00

New Developments

A real case application: Tohoku slip models

- **Multi-dimensional scaling**
  - Applied to 20 models of the Tohoku earthquake



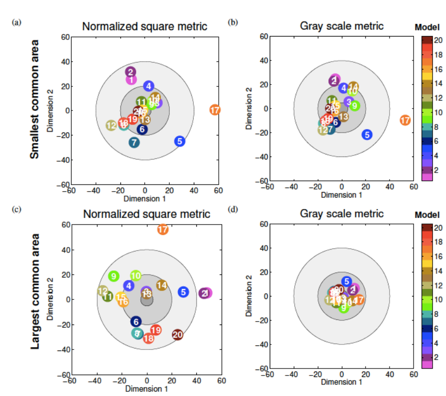


Table 3: Tohoku slip model similarity compared to mean model (smallest common area).

Case	Excellent	Good	Fair	Poor
d_1*	15,16	3,4,6,8,9,10,11,13,14,18,19,20	1,2,5,7,12	17
d_2†	15,16	3,4,6,7,8,9,10,11,13,14,18,19,20	1,2,5,12	17

New Developments

A real case application: Tohoku slip models

- **Multi-dimensional scaling**
  - Applied to 20 models of the Tohoku

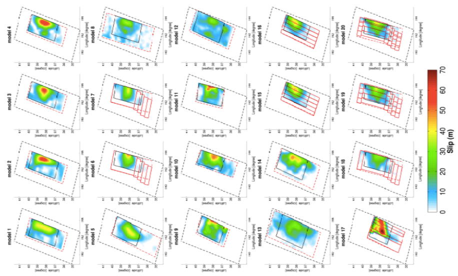
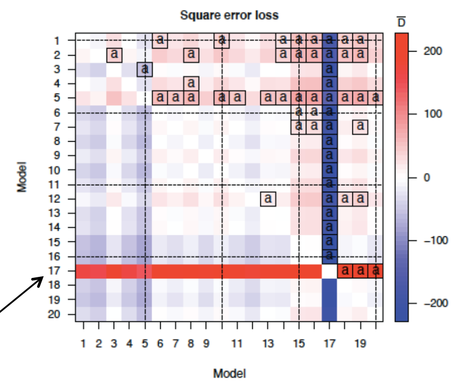


Table 3: Tohoku slip model similarity compared to mean model (smallest common area).

Case	Excellent	Good	Fair	Poor
d_1*	15,16	3,4,6,8,9,10,11,13,14,18,19,20	1,2,5,7,12	17
d_2†	15,16	3,4,6,7,8,9,10,11,13,14,18,19,20	1,2,5,12	17

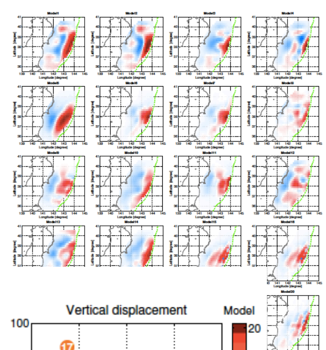
*d_1, Normalized square metric
†d_2, Gray scale metric

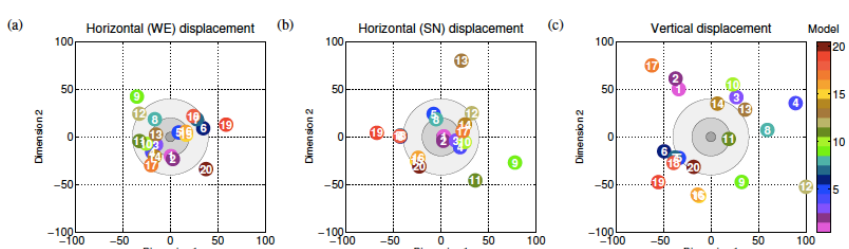


New Developments

A real case application: Tohoku slip models

- **Multi-dimensional scaling**
  - Applied to 20 models of the Tohoku earthquake and <u>their predicted ocean-floor displacements</u>





New Developments

Some General Conclusions

- **Source Inversion Validation**
  - Through a series of benchmarks we aim at being able to discriminate "strong" source-inversion methods from "weak" ones, and to identify where deficiencies could be
  - The project & efforts are ongoing, but already have been used to develop and test new methods, or to 'calibrate' existing ones
- **Quantitative Source Model Comparison**
  - The Spatial Prediction Comparison Test (SPCT) seems to be a useful tool to quantify how well a given 2D field (slip model) "fits" a reference solution
  - Using a multidimensional scaling approach allows to further quantify in which sense the models are different (amplitude; patch location ..), and to propose some form of ranking for the models