Some very preliminary thoughts

on

local shear wave velocity inversion

using

rotational ground motion measurements

Andreas Fichtner

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Abstract

A simple plane wave approach suggests that the local shear wave velocity c_s is equal to the ratio of the ground acceleration $\dot{\mathbf{u}}$ and the rotation $\nabla \times \mathbf{u} =: \boldsymbol{\omega}$ in symbols: $|\dot{\mathbf{u}}|/|\boldsymbol{\omega}| = c_s$. Even though $\dot{\mathbf{u}}$ and $\boldsymbol{\omega}$ may be measured at one single point, the ratio of their respective absolute values can in reality not be the shear velocity at that point because this would require the wave field to change its properties very abruptly.

In what follows, I suggest a simple recipe that allows to determine the region in which $||\dot{\mathbf{u}}||_2/||\boldsymbol{\omega}||_2$ depends on the shear velocity structure. It can indeed be shown that distant shear velocity perturbations have little influence on the numerical value of $||\dot{\mathbf{u}}||_2/||\boldsymbol{\omega}||_2$. Still, the shear velocity measurement is not pointwise. This suggests that a regional shear velocity tomography should be possible even with only one available station, i.e., one ring laser.

We will proceed as follows: (1) The general strategy will be outlined in a brief introduction. (2) In order to understand (3) we will derive expressions for sensitivity densities corresponding to point measurements of $||\dot{\mathbf{u}}||_2$ and $||\boldsymbol{\omega}||_2$. (3) The results from (2) will be applied in a simple but yet very informative example. (4) An outlook to possible future applications will conclude this short collection of ideas.

1 Introduction

In this introduction we will outline the principal ideas and concepts, as well a the necessary mathematical symbolisms.

Let $\mathbf{u}(\mathbf{p};\mathbf{x}^r,t)$ denote a displacement field recorded over time t at the location $\mathbf{x}=\mathbf{x}^r$ and depending on some model parameters $\mathbf{p}=(\rho,\lambda,\mu,...)$. The assumption that $\mathbf{u}(\mathbf{p};\mathbf{x},t)$ is a plane shear wave, i.e.,

$$\mathbf{u}(\mathbf{p};\mathbf{x},t)=\mathbf{A}\,e^{-\mathbf{i}(\mathbf{k}\cdot\mathbf{x}-\omega t)}\,,\quad \mathbf{A}\perp\mathbf{k}\,, \tag{1}$$

immediately yields

$$\frac{|\dot{\mathbf{u}}(\mathbf{p};\mathbf{x}^r)|}{|\boldsymbol{\omega}(\mathbf{p};\mathbf{x}^r)|}=c_s=\sqrt{\frac{\mu}{\rho}}\,. \tag{2}$$

[A similar relation can be found by dividing acceleration amplitudes and rotation rate amplitudes. This may be more convenient in practice because rotation rates are the output of the ring laser. However, as we shall see later, acceleration measurements lead to expressions in the adjoint equations which involve the fourth (!!!) time derivative of the displacement field. This is evidently a very undesirable quantity, at least when you're dealing with discrete signals.] The idea that $|\dot{\mathbf{u}}(\mathbf{p};\mathbf{x}^r)|/|\boldsymbol{\omega}(\mathbf{p};\mathbf{x}^r)|$ indeed yields the local, i.e., pointwise, shear wave velocity is tempting but unlikely to be correct. The assumed plane waves only exist in unbounded and homogeneous media. So, even when we assumed local homogeneity, the measurement would still be done at the surface of the Earth where the boundedness of the medium is most evident.

It seems obvious and intuitively clear that the quantity $|\dot{\mathbf{u}}(\mathbf{p};\mathbf{x}^r)|/|\boldsymbol{\omega}(\mathbf{p};\mathbf{x}^r)|$ must be affected by the shear wave structure in a wider vicinity of the measurement point. But how much is it affected and in which regions exactly? To answer these questions, I propose to compute the sensitivity of the *apparent shear wave speed*

$$c_a(\mathbf{p};\mathbf{x}^r):=\frac{||\dot{\mathbf{u}}(\mathbf{p};\mathbf{x}^r)||_2}{||\boldsymbol{\omega}(\mathbf{p};\mathbf{x}^r)||_2} \tag{3}$$

with respect to the *true shear wave speed* c_s. If c_a only depended on the very local shear velocity structure then the derivative of c_a with respect to c_s should vanish almost everywhere. If not, then non-zero values of this derivative should extend well into the medium around $\mathbf{x}^r$.

Denoting by D the functional derivative operator with respect to $\mathbf{p}$ and by $\mathbf{q}$ the differentiation direction in the model parameter space, we find

$$Dc_a(\mathbf{p};\mathbf{x}^r)(\mathbf{q})=\frac{D||\dot{\mathbf{u}}(\mathbf{p};\mathbf{x}^r)||_2(\mathbf{q})}{||\boldsymbol{\omega}(\mathbf{p};\mathbf{x}^r)||_2}-\frac{||\dot{\mathbf{u}}(\mathbf{p};\mathbf{x}^r)||_2\,D||\boldsymbol{\omega}(\mathbf{p};\mathbf{x}^r)||_2(\mathbf{q})}{||\boldsymbol{\omega}(\mathbf{p};\mathbf{x}^r)||_2^2}\,, \tag{4}$$

or the physically more reasonable expression

$$\frac{1}{c_a}Dc_a(\mathbf{p};\mathbf{x}^r)(\mathbf{q})=\frac{D||\dot{\mathbf{u}}(\mathbf{p};\mathbf{x}^r)||_2(\mathbf{q})}{||\dot{\mathbf{u}}(\mathbf{p};\mathbf{x}^r)||_2}-\frac{D||\boldsymbol{\omega}(\mathbf{p};\mathbf{x}^r)||_2(\mathbf{q})}{||\boldsymbol{\omega}(\mathbf{p};\mathbf{x}^r)||_2}\,. \tag{5}$$

Equation (5) suggests a simple recipe for the computation of $\frac{1}{c_a}Dc_a(\mathbf{p};\mathbf{x}^r)(\mathbf{q})$: First, compute the sensitivity of the velocity amplitude (more precisely: the L_2 norm of the velocity) with respect to c_s. Then compute the sensitivity of the rotation amplitude with respect to c_s. Finally, subtract one from the other. Done!

So, it seems that we have to take a closer look on sensitivities for different amplitude measurements:

2 Sensitivity kernels

The computation of the sensitivities and the physically more interpretable volumetric densities will be based on the adjoint method. Without going into too much detail, we shall accept at this point that the derivative with respect to the model parameters $\mathbf{p}$ of a given objective function $\mathfrak{E}(\mathbf{u}(\mathbf{p}))$ is

$$D\mathfrak{E}(\mathbf{u}(\mathbf{p}))(\mathbf{q})=\int_t\int_G \boldsymbol{\psi}(\mathbf{x},t)\cdot\partial_p\mathbf{L}(\mathbf{u},\mathbf{p})(\mathbf{q})\,dt\,dG\,. \tag{6}$$

The variable $\boldsymbol{\psi}$ denotes the adjoint field which is the solution of the adjoint problem. The partial derivative with respect to $\mathbf{p}$ of the wave operator is symbolised by $\partial_p\mathbf{L}$. Sensitivity densities can be obtained by omitting the integration over the volume G is which the wave field, i.e., the solution of the wave equation, is defined.

In general, the source term of the adjoint wave equation - and therefore the properties of the adjoint field $\boldsymbol{\psi}$ - are determined by the particular choice of the objective function $\mathfrak{E}$. Evidently, we are interested in the two cases $\mathfrak{E}(\mathbf{u}(\mathbf{p})) = ||\dot{\mathbf{u}}(\mathbf{p})||_2$ and $\mathfrak{E}(\mathbf{u}(\mathbf{p})) = ||\boldsymbol{\omega}(\mathbf{p})||_2$.

2.1 Velocity amplitude measurements

We first consider the case of $\mathfrak{E}(\mathbf{u}(\mathbf{p})) = ||\dot{\mathbf{u}}(\mathbf{p})||_2$, or more precisely

$$\mathfrak{E}(\mathbf{u}(\mathbf{p})) = ||\dot{\mathbf{u}}(\mathbf{p})||_2 = \left(\int\limits_{t_a}^{t_b} \dot{\mathbf{u}}^2(\mathbf{p}; \mathbf{x}^r, t)\, dt \right)^{1/2} . \tag{7}$$

Differentiation with respect to the model parameters $\mathbf{p}$ in the direction $\mathbf{q}$ gives

$$D||\dot{\mathbf{u}}(\mathbf{p})||_2(\mathbf{q}) = \frac{1}{||\dot{\mathbf{u}}(\mathbf{p})||_2} \int\limits_{t'=t_a}^{t_b} \dot{\mathbf{u}}(\mathbf{p}; \mathbf{x}^r, t') \cdot D\dot{\mathbf{u}}(\mathbf{p}; \mathbf{x}^r, t')(\mathbf{q})\, dt' . \tag{8}$$

One of the fundamental results of the adjoint method applied to the wave equation is (The references are well-known.)

$$D\dot{u}_k(\mathbf{p}; \mathbf{x}^r, t')(\mathbf{q}) = -\int\limits_{t=t_0}^{t_1} \int\limits_{G} \frac{\partial}{\partial t'} \mathbf{g}_k^\dagger(\mathbf{x}^r, t'; \mathbf{x}, t) \cdot \partial_p \mathbf{L}(\mathbf{u}, \mathbf{p})(\mathbf{q})\, dt\, dG , \tag{9}$$

where $\mathbf{g}_k^\dagger(\mathbf{x}^r, t'; \mathbf{x}, t)$ is the adjoint Green's function with its source acting in $\mathbf{e}_k$ direction at time t' and location $\mathbf{x}^r$, i.e., at the receiver. Introducing (9) into (8) yields

$$D||\dot{\mathbf{u}}(\mathbf{p})||_2(\mathbf{q}) = -\frac{1}{||\dot{\mathbf{u}}(\mathbf{p})||_2} \int\limits_{t'=t_a}^{t_b} \int\limits_{t=t_0}^{t_1} \int\limits_{G} \dot{u}_k(\mathbf{p}; \mathbf{x}^r, t') \partial_{t'} \mathbf{g}_k^\dagger(\mathbf{x}_r, t'; \mathbf{x}, t) \cdot \partial_p \mathbf{L}(\mathbf{u}, \mathbf{p})(\mathbf{q})\, dt'\, dt\, dG . \tag{10}$$

We now isolate the integration over t' and define an adjoint field $\boldsymbol{\psi}_v$ in order to bring (10) into the canonical form (6):

$$\boldsymbol{\psi}_v(\mathbf{x}, t) := \frac{1}{||\dot{\mathbf{u}}(\mathbf{p})||_2} \int\limits_{t'=t_a}^{t_b} \ddot{u}_k(\mathbf{p}; \mathbf{x}^r, t')\, \mathbf{g}_k^\dagger(\mathbf{x}^r, t'; \mathbf{x}, t)\, dt' . \tag{11}$$

This simplifies (10) to

$$D||\dot{\mathbf{u}}(\mathbf{p})||_2(\mathbf{q}) = \int\limits_{t=t_0}^{t_1} \int\limits_{G} \boldsymbol{\psi}_v(\mathbf{x}, t) \cdot \partial_p \mathbf{L}(\mathbf{u}, \mathbf{p})(\mathbf{q})\, dt\, dG . \tag{12}$$

Equation (11) suggests that the adjoint source corresponding to velocity amplitude measurements in the L_2 sense is simply a point source at the receiver $\mathbf{x}^r$ that radiates the recorded acceleration "backward in time" (I don't like that populistic expression. It does not make much sense but seems to have infiltrated the seismological vocabulary list.), i.e.,

$$\mathbf{f}_v^\dagger(\mathbf{x}, t) = \ddot{\mathbf{u}}(\mathbf{x}^r, t)\, \delta(\mathbf{x} - \mathbf{x}^r) . \tag{13}$$

2.2 Rotation measurements

In the case that our measurements are the rotation amplitudes

$$\mathfrak{E}(\mathbf{u}(\mathbf{p})) = ||\boldsymbol{\omega}(\mathbf{p})||_2 = \left(\int\limits_{t_a}^{t_b} [\boldsymbol{\omega}(\mathbf{p}; \mathbf{x}^r, t)]^2\, dt \right)^{1/2} , \tag{14}$$

we proceed just as before. Differentiating (14) with respect to $\mathbf{p}$ in the direction $\mathbf{q}$, gives

$$D||\boldsymbol{\omega}(\mathbf{p})||_2(\mathbf{q}) = \frac{1}{||\boldsymbol{\omega}(\mathbf{p})||_2} \int\limits_{t=t_a}^{t_b} \omega_i(\mathbf{p};\mathbf{x}^r,t')\, D\omega_i(\mathbf{p};\mathbf{x}^r,t')(\mathbf{q})\, dt' \,. \tag{15}$$

We already had the differentiated version of

$$Du_k(\mathbf{p};\mathbf{x}^r,t')(\mathbf{q}) = -\int\limits_{t=t_0}^{t_1} \int\limits_G \mathbf{g}_k^\dagger(\mathbf{x}^r,t';\mathbf{x},t)\cdot \partial_p \mathbf{L}(\mathbf{u},\mathbf{p})(\mathbf{q})\, dt \,. \tag{16}$$

From this we obtain the rotation as follows:

$$\omega_i(\mathbf{p};\mathbf{x}^r,t') = \epsilon_{ijk} \frac{\partial}{\partial x_j^r} u_k(\mathbf{p};\mathbf{x}^r,t') \,, \tag{17}$$

and therefore

$$D\omega_i(\mathbf{p};\mathbf{x}^r,t')(\mathbf{q}) = -\epsilon_{ijk} \frac{\partial}{\partial x_j^r} \int\limits_{t=t_0}^{t_1} \int\limits_G \mathbf{g}_k^\dagger(\mathbf{x}^r,t';\mathbf{x},t)\cdot \partial_p \mathbf{L}(\mathbf{u},\mathbf{p})(\mathbf{q})\, dt\, dG \,. \tag{18}$$

Introducing the last expression into equation (15) yields

$$\begin{aligned} D\omega_i(\mathbf{p};\mathbf{x}^r,t')(\mathbf{q}) = -\int\limits_{t=t_0} \int\limits_{t'=t_a}^{t_b} \int\limits_G \omega_i(\mathbf{p};\mathbf{x}^r,t')\, \epsilon_{ijk} \frac{\partial}{\partial x_j^r} \mathbf{g}_k^\dagger(\mathbf{x}^r,t';\mathbf{x},t) \\ \cdot\, \partial_p \mathbf{L}(\mathbf{u},\mathbf{p})(\mathbf{q})\, dt\, dt'\, dG \,. \end{aligned} \tag{19}$$

Again, we isolate the integration over t' and define an adjoint field $\boldsymbol{\psi}_\omega$ that corresponds to rotation amplitude measurements in the L_2 sense:

$$\boldsymbol{\psi}_\omega(\mathbf{x},t) := -\frac{1}{||\boldsymbol{\omega}(\mathbf{p})||_2} \int\limits_{t'=t_a}^{t_b} \epsilon_{ijk}\omega_i(\mathbf{p};\mathbf{x}^r,t')\, \frac{\partial}{\partial x_j^r} \mathbf{g}_k^\dagger(\mathbf{x}^r,t';\mathbf{x},t)\, dt' \,. \tag{20}$$

The derivative (15) can then also be written in canonical form

$$D||\boldsymbol{\omega}(\mathbf{p})||_2(\mathbf{q}) = \int\limits_{t=t_0}^{t_1} \int\limits_G \boldsymbol{\psi}_\omega(\mathbf{x},t)\cdot \partial_p \mathbf{L}(\mathbf{u},\mathbf{p})(\mathbf{q})\, dt\, dG \,. \tag{21}$$

It follows that the adjoint source for $||\boldsymbol{\omega}(\mathbf{p})||_2$ is

$$\mathbf{f}_\omega^\dagger(\mathbf{x},t) = -\frac{\mathbf{e}_k}{||\boldsymbol{\omega}(\mathbf{p})||_2}\, \epsilon_{ijk}\, \omega_i(\mathbf{p};\mathbf{x}^r,t)\, \frac{\partial}{\partial x_j^r} \delta(\mathbf{x}-\mathbf{x}^r) \,. \tag{22}$$

A closer look at (22) reveals some interesting details: The adjoint source for rotation amplitude measurements is an **anti-symmetric moment tensor** source. Even though such a source would be nonsense in the real physical world (Would it?), it is still mathematically meaningful: An anti-symmetric moment-tensorial source radiates only S waves and no P waves. This makes sense, because otherwise the apparent S velocity c_a would depend on the P velocity structure. So, in a certain sense, one could even have expected that the adjoint source is an anti-symmetric moment tensor source, i.e., the simplest possible source that only radiates S waves. Explicit expressions for the cartesian components of $\mathbf{f}_\omega$ are

$$f_\omega^1(\mathbf{x},t) = \frac{1}{||\boldsymbol{\omega}(\mathbf{p})||_2} \left[\omega_2(\mathbf{p};\mathbf{x}^r,t)\, \frac{\partial}{\partial x_3} - \omega_3(\mathbf{p};\mathbf{x}^r,t)\, \frac{\partial}{\partial x_2} \right] \delta(\mathbf{x}-\mathbf{x}^r) \,, \tag{23a}$$

$$f_\omega^2(\mathbf{x},t) = \frac{1}{||\boldsymbol{\omega}(\mathbf{p})||_2} \left[\omega_3(\mathbf{p};\mathbf{x}^r,t)\, \frac{\partial}{\partial x_1} - \omega_1(\mathbf{p};\mathbf{x}^r,t)\, \frac{\partial}{\partial x_3} \right] \delta(\mathbf{x}-\mathbf{x}^r) \,, \tag{23b}$$

$$f_\omega^3(\mathbf{x},t) = \frac{1}{||\boldsymbol{\omega}(\mathbf{p})||_2} \left[\omega_1(\mathbf{p};\mathbf{x}^r,t)\, \frac{\partial}{\partial x_2} - \omega_2(\mathbf{p};\mathbf{x}^r,t)\, \frac{\partial}{\partial x_1} \right] \delta(\mathbf{x}-\mathbf{x}^r) \,. \tag{23c}$$

So far the theory. It seems that we now need a simple example in order to see what equations (1) to (23c) actually mean:

3 An example

The example is deliberately simple and non-realistic, in order to illustrate as many interesting aspects as possible in an understandable manner.

As medium we use PREM in a box of 500 km extension in each direction. A source in the form of a single force pointing in y direction is located at a depth of 250 km. It radiates a Ricker wavelet (first derivative of a Gaussian) with a dominant period of 20 s, i.e., with a quite low frequency. The receiver is positioned at an epicentral distance of 300 km so that S wave motion is recorded basically only in the y direction. Significant rotational motion is recorded only in vertical direction. The translational and rotational seismograms are shown in figures (1) and (2), respectively.

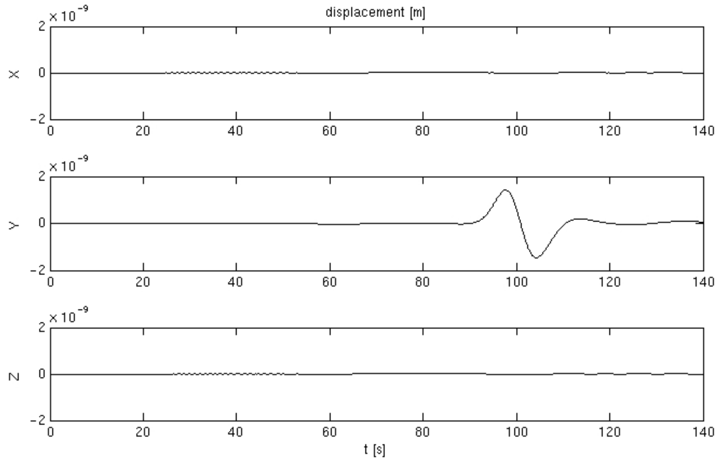

Figure 1: *Displacement seismograms recorded at the surface at an epicentral distance of* 300 *km. Most of the translational motion is in y direction - by construction.*

We now follow our recipe and construct adjoint sources. For the relative derivatives that we are interested in (see equation (5)) our two adjoint sources are

$$\mathbf{f}_v^\dagger(\mathbf{x},t) = \frac{\mathbf{e}_y}{||\dot{\mathbf{u}}(\mathbf{p})||_2^2}\,\ddot{u}_y(\mathbf{p};\mathbf{x}^r,t)\,\delta(\mathbf{x}-\mathbf{x}^r)\,, \tag{24a}$$

$$\begin{aligned}\mathbf{f}_\omega^\dagger(\mathbf{x},t) &= -\frac{\mathbf{e}_x}{||\boldsymbol{\omega}(\mathbf{p})||_2^2}\,\omega_z(\mathbf{p};\mathbf{x}^r,t)\,\frac{\partial}{\partial y}\delta(\mathbf{x}-\mathbf{x}^r) \\ &\quad + \frac{\mathbf{e}_y}{||\boldsymbol{\omega}(\mathbf{p})||_2^2}\,\omega_z(\mathbf{p};\mathbf{x}^r,t)\,\frac{\partial}{\partial x}\delta(\mathbf{x}-\mathbf{x}^r)\,.\end{aligned} \tag{24b}$$

The functions $\ddot{u}_y/||\dot{\mathbf{u}}||_2^2$ and $\omega_z/||\boldsymbol{\omega}||_2^2$ are displayed in figures (3) and (4), respectively. Note that they already bear some physical meaning: The amplitude of the adjoint source time function corresponding to the rotation measurements is roughly five orders of magnitude larger than the amplitude of the source time function corresponding to the velocity measurements. However, the adjoint source for the rotation measurement is a moment tensor source and not a vectorial source. Hence, the amplitude of the adjoint field $\boldsymbol{\psi}_\omega$ will be proportional to $\dot{\omega}_z/c_s^3$. This will then be approximately the same order of magnitude that we can expect for the amplitude of adjoint field $\boldsymbol{\psi}_v$, namely $\ddot{u}_y/c_s^2$. One may therefore predict that $||\dot{\mathbf{u}}(\mathbf{p})||_2^{-1}\,D||\dot{\mathbf{u}}(\mathbf{p})||_2(\mathbf{q})$ and $||\boldsymbol{\omega}(\mathbf{p})||_2^{-1}\,D||\boldsymbol{\omega}(\mathbf{p})||_2(\mathbf{q})$ will reach the same maximum values. In more simple words: **Velocity amplitudes are about as sensitive to shear velocity structure than rotation amplitudes.**

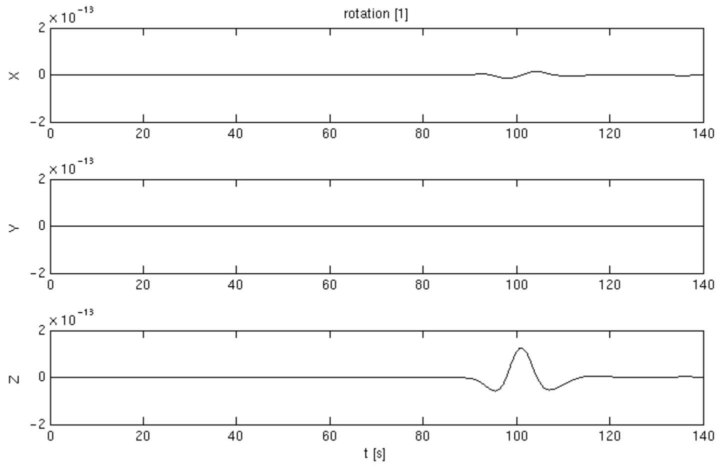

Figure 2: *Rotation seismograms recorded at the surface at an epicentral distance of* 300 *km. Most of the rotational motion is in vertical direction.*

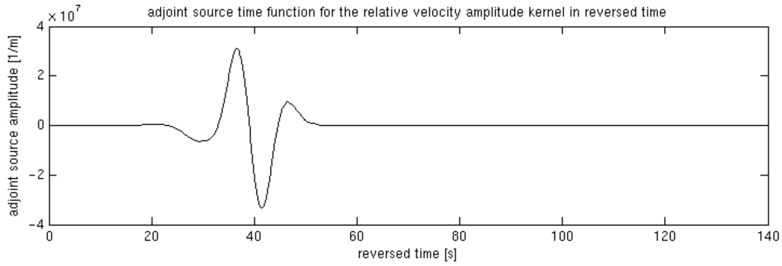

Figure 3: *Source-time function in reversed time for the velocity amplitude measurement. The function is proportional to* $\ddot{u}_y$ *and inversely proportional to* $||\dot{\mathbf{u}}||_2^2$.

Our qualified guess is confirmed by figures (5) and (6) which show the volumetric densities of the relative derivatives $||\dot{\mathbf{u}}(\mathbf{p})||_2^{-1}\, D||\dot{\mathbf{u}}(\mathbf{p})||_2(\mathbf{q})$ and $||\boldsymbol{\omega}(\mathbf{p})||_2^{-1}\, D||\boldsymbol{\omega}(\mathbf{p})||_2(\mathbf{q})$, respectively. The first impression is indeed that both kernels (= volumetric sensitivity densities) look very similar. This is not surprising because the radiation patterns of the corresponding adjoint sources are very similar also - at least far from the receiver. In the vicinity of the receiver, which is the adjoint source location, the two adjoint fields differ in slightly in the radiation patterns and in the contributions of various near and intermediate field terms. In the far field, however, they become more and more indistinguishable. One can easily show that using the Green's functions for a homogeneous medium. What seems to be at this point a rather unimportant statement turns out to physically quite profound! **That the adjoint fields corresponding to two different physical quantities become indistinguishable in the far field implies that the ratio of the two quantities in independent of the structure near the source!!!**

To see whether this is indeed true for our example, we simply subtract $||\boldsymbol{\omega}(\mathbf{p})||_2^{-1}\, D||\boldsymbol{\omega}(\mathbf{p})||_2(\mathbf{q})$ from $||\dot{\mathbf{u}}(\mathbf{p})||_2^{-1}\, D||\dot{\mathbf{u}}(\mathbf{p})||_2(\mathbf{q})$ to get $c_a^{-1}\, Dc_a(\mathbf{p})(\mathbf{q})$. The result can be seen in figure (7). ((fig 5) $-$ (fig 6) = (fig 7) = sensitivity of the apparent shear velocity with respect to the true shear velocity.) **As expected, the apparent shear velocity $c_a = ||\dot{\mathbf{u}}||_2/||\boldsymbol{\omega}||_2$ measured at $\mathbf{x}^r$ is sensitive mostly to true shear velocity structure near the receiver.** The contributions near the source are due to (1) numerical errors related to the improper implementation of a point source and (2) to the small distance between source and receiver compared to the dominant wavelength. In other words, the source is still within a distance from the receiver

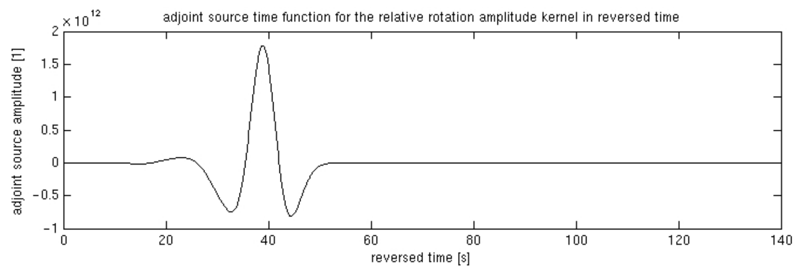

Figure 4: *Source-time function in reversed time for the rotation amplitude measurement. The function is proportional to ω_z and inversely proportional to $||\dot{\boldsymbol{\omega}}||_2^2$.*

where the radiation patterns of the adjoint fields $\boldsymbol{\psi}_v$ and $\boldsymbol{\psi}_\omega$ are distinguishable. **For teleseismic events, there will be no contribution at all near the source.**

It seems at this point evident that the size of the region in which c_a is significantly sensitive to c_s depends on the frequency of the incoming waves and on the width of the window that one considers. In general, higher frequencies and smaller windows will reduce the size of the region of significant sensitivity. The opposite effect will result from lower frequencies and larger windows.

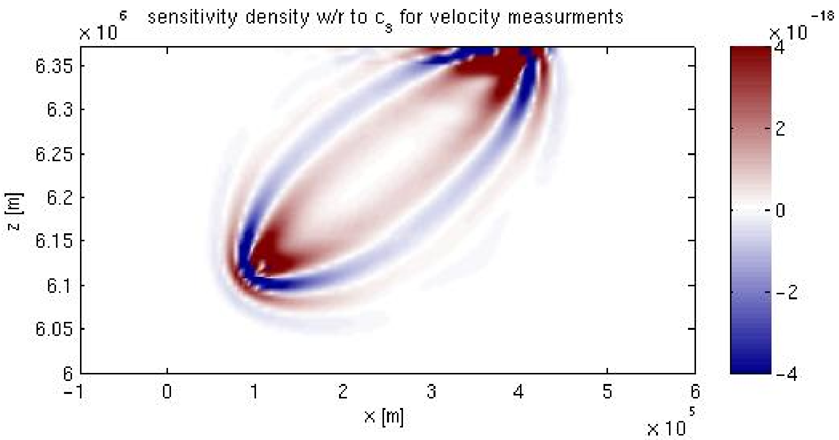

Figure 5: *Volumetric sensitivity density of $||\dot{\mathbf{u}}||_2$ divided by $||\dot{\mathbf{u}}||_2$ (relative sensitivity) with respect to the shear velocity c_s. Slice in the y plane through the source and receiver locations. The unit is $s \cdot m^{-1}$.*

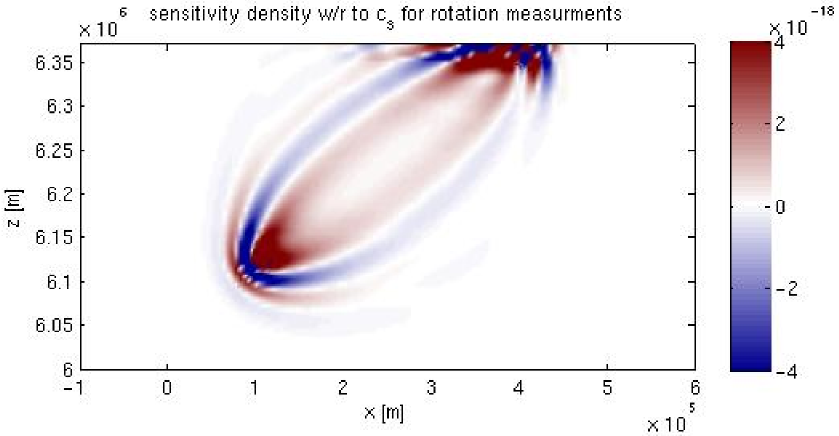

Figure 6: *Volumetric sensitivity density of* $||\dot{\boldsymbol{\omega}}||_2$ *divided by* $||\dot{\boldsymbol{\omega}}||_2$ *(relative sensitivity) with respect to the shear velocity* c_s*. Slice in the y plane through the source and receiver locations. The unit is* $s \cdot m^{-1}$.

4 Outlook

So, inversion?!? At least the theoretical foundation of a local shear velocity inversion based on the measurement of

$$c_a(\mathbf{p}) = \frac{||\dot{\mathbf{u}}(\mathbf{p})||_2}{||\boldsymbol{\omega}(\mathbf{p})||_2} \tag{25}$$

seems to be solid. This would be an attractive technique mainly because (in the case of distant events) the measurement and therefore the sensitivities are independent of many weakly constrained parameters such as source location, source time, source time function, moment tensor, structure in the deep Earth, structure near the source, ... Another advantage seems to be that the modelling can be done locally because there is no sensitivity at great distances from the receiver. The inversion would be simple in the sense that only one parameter is involved, namely the shear wave speed.

The fact that the depth extension of the sensitivity density is frequency-dependent (and by means of the above theory easily quantifiable) suggests that one could successively invert for shallower (or deeper) structures by increasing (or decreasing) the frequency. This, however, is nothing new in seismic tomography.

A significant problem is - as always - the forward modelling. A multi-scale approach seems to be the most reasonable solution. Global modelling could be used to generate a teleseismic wave in a local model which is then used to solve the inverse problem.

What remains to be done? Certainly, one should go through more examples in order to get some intuition for what the sensitivity kernels really mean. It would also be interesting to see whether anisotropic structure can affect the measurements of the apparent shear velocity c_a.

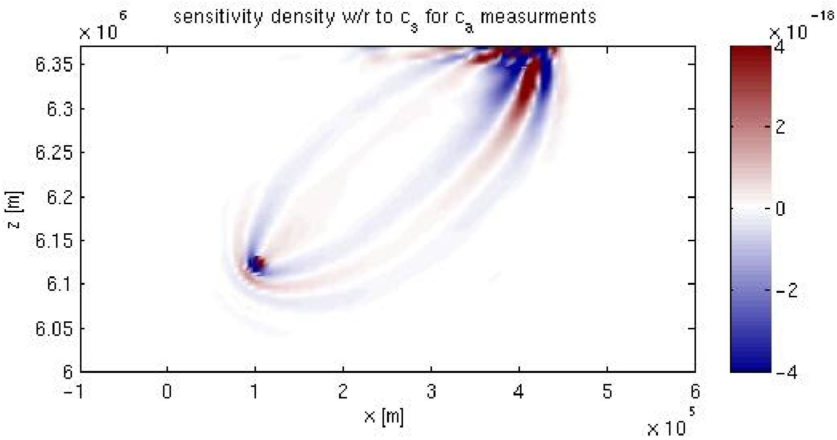

Figure 7: *Volumetric sensitivity density of the apparent shear velocity c_a divided by itself (relative sensitivity) with respect to the true shear velocity c_s. Slice in the y plane through the source and receiver locations. The unit is $s \cdot m^{-1}$.*