Linear(-ized) Inverse Problems

Geophysical examples

Inverse – or data-fitting – problems are usually written as

$$\mathbf{d} = \mathbf{g}(\mathbf{m})$$

or - in linearized form –

$$\Delta\mathbf{d} = \mathbf{G}\Delta\mathbf{m}$$

d is the data vector, G are the "kernels" and m is the mode vector. Give five different examples of inverse problems in Earth sciences and describe d, m , their dimensions, the degree of non-linearity – if possible – describe the forward problem.

Line-fitting: Refraction (overdetermined case)

The following **noisy refraction** data (do you remember the mathematical function?) were observed in s as a function of distance. You want to determine the slope and the intercept time using linear inverse theory ("regression", "line-fitting"). Calculate the G matrix and formulate the inverse problem for the overdetermined case.

d = [1.4102 2.7626 2.8500 3.3633 4.0844]

The data were observed at the following locations (in km)

x = [2 4 6 8 10]

Find a solution using intrinsic functions for Matrix inversion (like ***inv*** for example with Matlab) or Python. Estimate the data error by calculating the standard deviation of the data misfits.

Reflection seismics (exact solution)

Formulate the problem of reflection travel times from one layer as an inverse problem. The observations are arrival times t(x) and the model is the velocity v and the depth h of the layer. Assume as many data as unknowns and formulate the inverse problem in linearized form. The true model is v=5km/s and h=10km. Observations are taken at x_1=10km, x_2=20km. Use as starting model v=4km/s and h=5km. Is the resulting G matrix invertible or do you have to use the least-square solution? Calculate the necessary vectors and matrices and perform the first iteration of the inverse problem.

Direct solution for the linearized problem

$$\Delta\mathbf{m} = G^{-1}\Delta\mathbf{d}$$

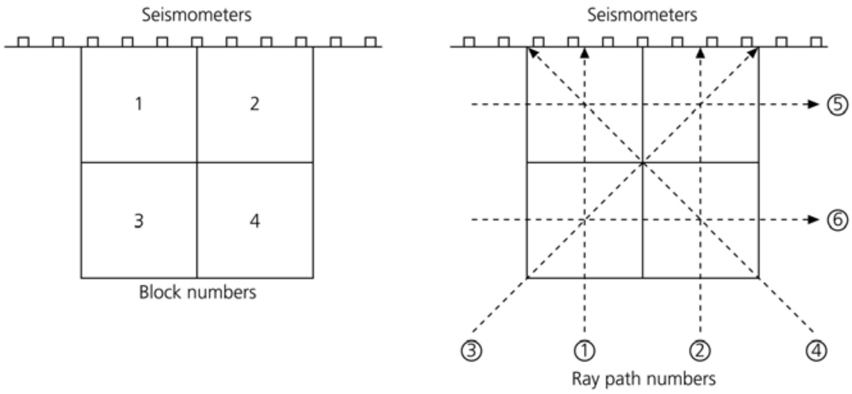
Least-square solution

$$\Delta\mathbf{m} = (G^T G)^{-1} G^T \Delta\mathbf{d}$$

Seismic tomography (overdetermined case)

Formulate the problem of the simple tomographic experiment to recover velocity perturbations from travel time perturbations. Calculate matrix G for the geometry given in the graph. Assume a side length l of the blocks. Check (e.g., using Matlab) whether the system can be inverted using the least-squares Ansatz. What is different in this toy example compared to a realistic tomographic experiment?

Figure 7.3-2: Ray path and block geometry for an idealized tomographic experiment.



If you have formulated the problem mathematically use the programs in the repository (tomography_perfectresolution.m) to analyze a possible implementation of this using Matlab.

Analyze the effect of underdetermination using the program tomography_svd.m. Save the program in a new file, extend the problem to a 3x3 cell situation and solve the inverse problem for an appropriate model perturbation using svd. Analyze the resulting model resolution matrix.

Earthquake Hypocentre Location

Write down the equations for the hypocentre location problem in 3-D. Linearize the problem and write down the problem in terms of

$$\Delta\mathbf{m} = (G^T G)^{-1} G^T \Delta\mathbf{d}$$

Extend the problem discussed in the lecture to include seismic velocity as an unknown. Develop the G matrix for this problem.

Apply the programs in subfolder (Basic, Basic+v, Basic+noise) to investigate the implementation of the problem.

Set up the hypocentre problem using the travel time data extracted from the weekend problem.