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# Computational wave propagation: advances in global and regional seismology

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**Summary.** We provide a major advance in imaging the Earth's interior by simulating wave propagation in both 2D and 3D using several numerical methods. For the Earth's deep interior this is done on a global scale using axi-symmetric models and 3D spherical sections. We also apply numerical tools to calculate earthquake scenarios on a regional scale for prediction of ground motion (e.g. peak motion amplitude, shaking duration), taking into account amplification effects of low velocity zones in active faults and basin structures, topography effects, shear wave splitting effects due to anisotropy and attenuation due to visco-elasticity. These predictions are very useful for risk evaluation and civil engineering purposes (2). We further simulate earthquake sources as dynamic fault ruptures in the context of typical fault-zone velocity structures and material interfaces (3). We test the ability of a ring laser for seismological purposes, especially in the context of rotational motions and support the interpretation of data with numerical parameter studies (4).

## 1 Introduction

The use of wave propagation as a tool to image the properties of any kind of material (gas, liquid, or solid) plays a fundamental role in the of Earth sciences (e.g. seismology, volcanology, rock physics, geodynamics, atmospheric sciences) and industry (exploration, non-destructive testing, etc.). Wave propagation techniques and imaging tools are widely used to monitor critical structures (dams, water reservoirs, waste deposits), as well as in environmental investigations, and the research fields of seismic and volcanic hazards.

Research in elastic wave propagation using advanced computational methods has the potential for major breakthroughs and numerous applications in these associated fields. The rapid development in computational technology has reached the point that it is now possible for us to understand the complete three-dimensional behaviour of sources and wave propagation in real systems.

Only recently it has the ability to simulate the complete behaviour (in terms of observed frequencies) of realistic physical systems (e.g. the ringing of a sedimentary basin after a local earthquake or the vibrations of the planet Earth after a large earthquake).

In seismology there has been a gap between observations and theory for several decades: the quality and quantity of observations by far exceeds our capabilities in theory and processing. So far, only small fractions of the available data are being used for example in imaging the structure of the Earth's deep interior or understanding earthquake rupture properties. The modelling of the complete seismic wavefield is still in its infancy and it is presently difficult to synthesize seismic waves in complex three-dimensional natural media, i.e. media that are highly heterogeneous, anisotropic, anelastic, cracked or porous, even though basic solutions for elastic wave propagation problems exist. The increasing power of computers makes it now possible to address scientific problems which were previously out of reach. The need to go beyond relatively crude approximations in the simulation and imaging process (e.g. ray theory, linear approximations, 2D solutions) has been expressed for years. In this project we aim to go beyond the algorithm development stage and apply numerical simulations to realistic problems.

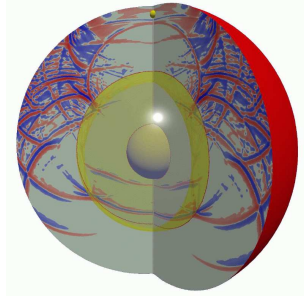
## 2 Numerical simulations of earthquakes and seismic wave propagation

### 2.1 Seismic wave propagation on a global scale

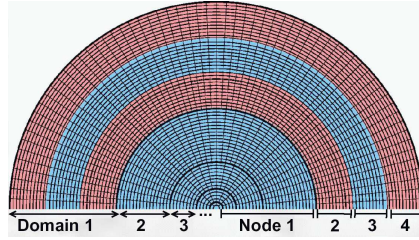
We employed a finite-difference method to simulate global seismic wave propagation in a axi-symmetric model. Axi-symmetric methods allow computation and storage of the model properties in 2D domains. This method allows much higher frequencies of global seismic wave propagation when compared to full 3D methods, which require the model properties and wave fields to be stored and computed for all dimensions. In order to obtain the corresponding 3D model space, one has to virtually rotate the 2D domains around the symmetry axis, as shown in Figure 1.

One disadvantage of traditional 2D Cartesian methods is that point sources and the correct 3D geometrical spreading can not be simulated. Axi-symmetric methods overcome this problem, since point sources are possible, as long as they have an axi-symmetric radiation pattern. Axi-symmetric models and wavefields are capable of answering interesting questions investigating global seismology. Of these, some include: diffractions at the core mantle boundary, teleseismic effects from subduction zones, and scattering of the seismic wavefield within the mantle by small scale heterogeneities.

*Numerical Methods.* To compute seismograms for such geometries we developed an algorithm for the PSV case where the model space was discretized



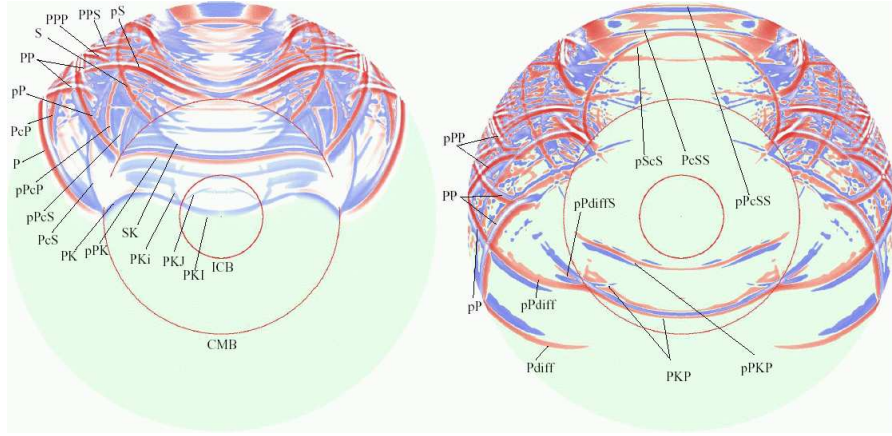
**Fig. 1.** Snapshot of the 3D wavefield inside the earth. The PSV 2D model slide has to be virtually rotated to achieve the full spherical 3D model space.



**Fig. 2.** Sketch of the discretization for the axi-symmetric PSV formulation. There are several grid-refinements necessary towards the center.

into finite-difference grids. In order to cover the whole Earth, we extended the acoustic multi-domain method used by Thomas et al. [10] to the PSV case. This method uses multiple grids (domains) defined in polar coordinates. To avoid numerical instabilities due to the decreasing grid spacing towards the center, a grid refinement is done several times at different depth levels. Figure 2 shows these domains where the first domain covers the depth range from the surface to  $1/2$  the Earth's radius, the second domain from  $1/2$  to  $3/4$  the Earth's radius and so forth. Also shown is the decomposition of the model space into four nodes, where the individual nodes are marked by alternating colors. After each time step the boundary regions of adjacent nodes exchange values in order to allow wave propagation across node boundaries. For larger numbers of nodes, the decomposition scheme is relatively easy if one increases the number of nodes by a factor of two: the former domains 2-4 are redistributed on six nodes. Node 1 is split up horizontally within the uppermost domain into two nodes.

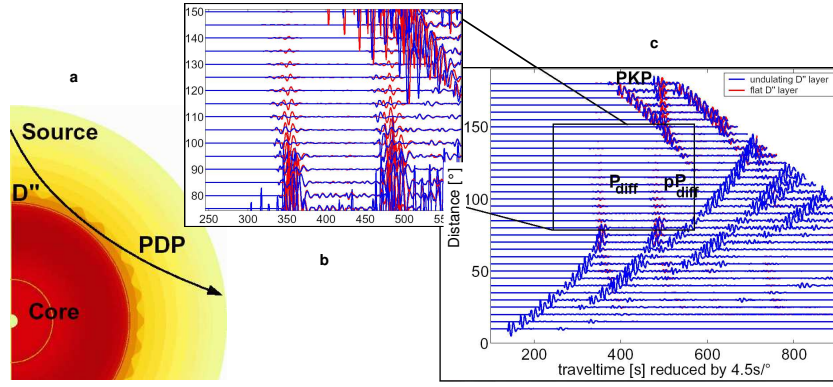
*PREM and a mantle with topography.* Two snapshots of PSV wave propagation for the PREM (spherical-symmetric reference model) are shown in Figure 3. To investigate a more complex model we implemented a D" layer 300km above the core mantle boundary with sinusoidal topography as shown in Figure 4a. This model was motivated by teleseismic observations of phases that



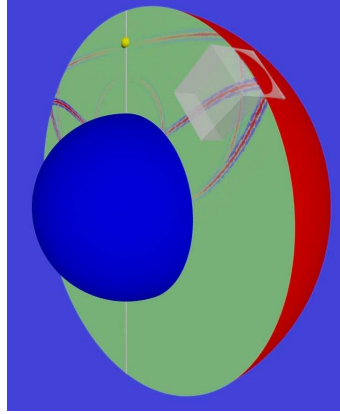
**Fig. 3.** Two snapshots of PSV wave propagation for the PREM model: The source depth is set to 600km in order to generate prominent depth phases. The dominant period is 25s and the color scaling is non-linear to show both the strong surface waves and the weak body waves. In the left snapshot the wave field after 550s is shown. Besides various other phases, an S wave in the inner core (PKJKP) is remarkable. The right figure shows the wavefield after 850s where PKP is about to leave the core with the bc and df branches.

turn in the lowermost mantle near the core mantle boundary showing a large amount of scattering, explainable by small scale topography in the D'' region. The amplitude of the modelled D'' variation is 50km and the velocity contrast is 5% with respect to PREM. The phase named PDP (see Figure 4a which dives into the D'' layer) is very sensitive to the D'' topography and therefore to the velocity perturbation. This can be seen in Figure 4b&c which shows a comparison of seismograms of the undulating D'' layer with an averaged D'' layer without topography. The zoomed seismogram section figure 4b with the time window around PDP shows prominent differences between the flat D'' layer model (red) and the undulating D'' layer (blue). Also the phase PKP shows large difference, since it travels through the D'' layer twice.

Another major technical task was the development of a code combining the axi-symmetric method with 3D spherical sections. This hybrid method uses the axi-symmetric finite-difference method to calculate the propagating wavefield from the source to teleseismic distances in a radially symmetric earth model. The wavefield is handed into a fully 3D finite-difference calculation once it reaches the interesting study area, e.g. a geological structure such as a mantle plume or a subduction zone. Thus all the 3D-wavefield effects like multiple reflections, refractions, and scattering can be investigated at a smaller computational cost than fully 3D models, allowing the effects of teleseismic waves to be studied in full 3D. This technique is illustrated in figure 5. Early results from this method show that the conversion of SH to P-SV waves caused



**Fig. 4.** (a) Sketch of a model that is motivated by teleseismic observations and implemented with a D'' layer extending to 300km above the CMB with sinusoidal topography. (b/c) The zoomed seismogram section (b) with the time window around PDP shows prominent differences between the flat D'' layer model (red) and the undulating D'' layer (blue). Also the PKP phase shows large difference since it travels two times through the D'' layer.



**Fig. 5.** The axisymmetric global wavefield and the 2D computational with the position of the source and the symmetry axis is shown together with the 3D section. Waves in the core are not modelled because shear waves do not penetrate the liquid core.

by a plume have a systematic but very weak pattern, and thus the detectability of mantle plumes using teleseismic data remains difficult.

## 2.2 Earthquake scenarios

Our 3D finite-difference was also employed to simulate seismic wave propagation on a regional scale to study the influence of a sedimentary basin on

the resulting ground motion in three different areas: the Cologne Basin, the Beijing Metropolitan area and Los Angeles Basin. With the computational power of the Hitachi SR8000 supercomputer it is possible for us to simulate these regional areas with a dominant frequency of higher than 1 Hz. At these frequencies it is possible to compare the results with recorded data of historic earthquakes if available.

The absolute values of ground velocities at the surface as obtained from the FD-calculations are used to calculate the peak ground velocity at each grid point at the surface. From these values the Modified Mercalli Intensity can be calculated.

The shaking duration can be defined as the time difference between the first and the last exceed of a threshold value. Diverse studies have shown that the three-dimensional structure of sediment basins have a large effect on the shaking duration.

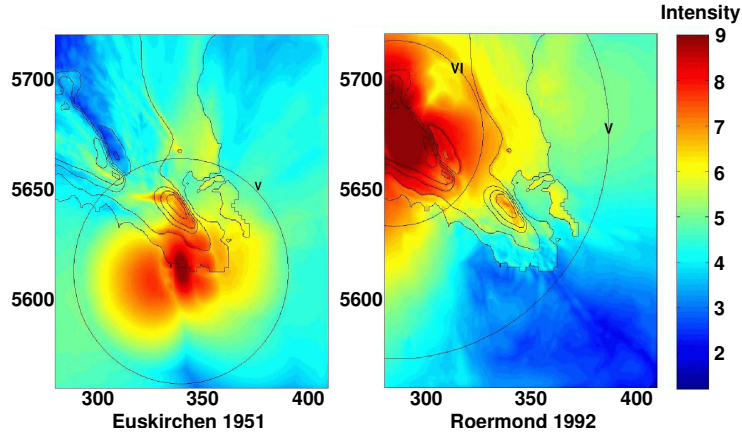
*Cologne Basin.* The Cologne Basin is one of the regions in central Europe with an elevated seismic risk. It has been well characterized tectonically and geologically, making it ideal for testing our scenario simulation techniques. A sensitive point in the scenario simulations is the accuracy of the model of seismic velocities in the study area. Ewald [5] has demonstrated the achievable simulation accuracy using a simplified 2-layer model consisting of semi-consolidated sediments and a host rock for three historic earthquakes in this region. The computational effort for a typical single scenario simulation is shown in table 1.

**Table 1.** Example of the computational parameters for a single scenario calculation of the Cologne Basin

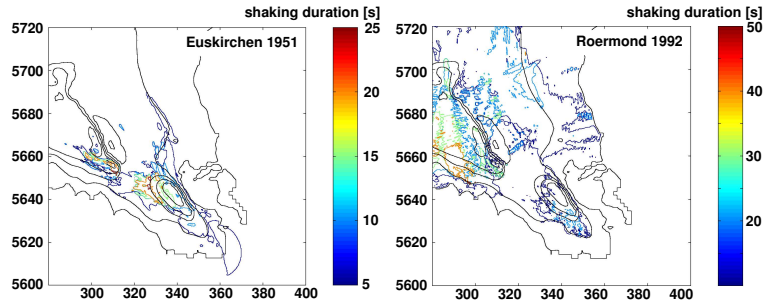
spatial discretization	0.2 km
temporal discretization	0.0198 km
Lowest S-wave velocity	1400 m/s
Grid Size (computational model)	800 x 900 x 200 pts
Number of time steps	3034
Simulation time	60 s
Memory usage	24 GB
Computation time	12 h

For relatively small earthquakes, similar to those in the Cologne Basin the point source solution of the source mechanism (as it is calculated from the recorded events) is a useful approximation for simulating the source mechanism. The source dimensions for these earthquakes is on the order of tens of hundreds of meters. For the case that further information about the slip time and space history is available, enhancements of the simulated results using simple extended source mechanisms are likely. Large earthquakes require a more sophisticated decomposition of the source mechanism. Fortunately big-

ger earthquakes also provide more information about the source. Information that is also mapped as damage in the rock near the surface.

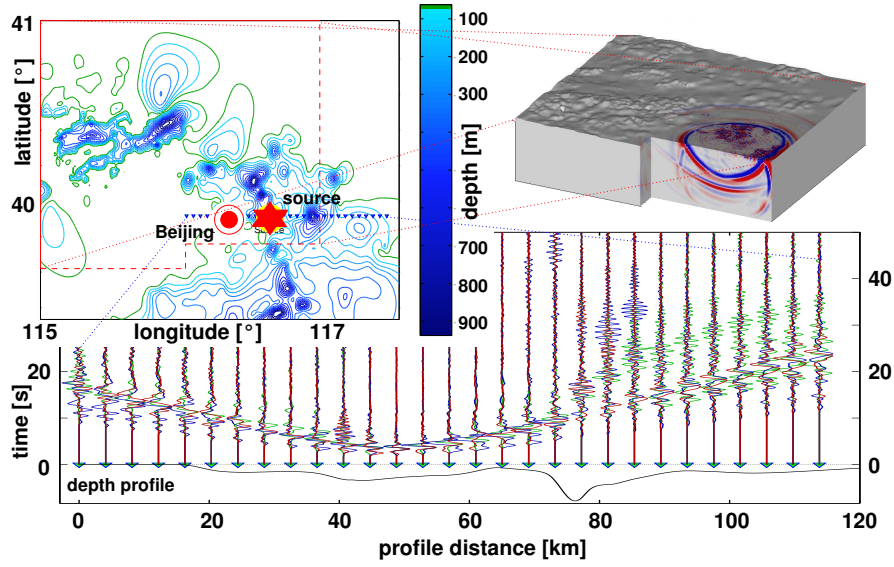


**Fig. 6.** Maps showing the Modified Mercalli Intensity for the Euskirchen 1951 and the Roermond 1992 earthquake in the Cologne Basin as obtained from the finite-difference calculations



**Fig. 7.** Maps showing the shaking duration for the Euskirchen 1951 and the Roermond 1992 earthquake in the Cologne Basin as obtained from the finite-difference calculations

*Beijing metropolitan area.* The Beijing Metropolitan area is situated in a seismically active region. Several historical earthquakes occurred in this region, such as the 1665 Tongxian earthquake, the 1679 Sanhe-Pinggu earthquake and the 1720 Shacheng earthquake. Using a model of seismic velocities based upon seismic reflection surveys of the basin, earthquake scenario simulations were carried out similar to those in the Cologne Basin.



**Fig. 8.** Example of an earthquake scenario in the Beijing metropolitan area. The earthquake is initiated by a point source approximation of the the 1665 Tongxian historic earthquake. The Figure illustrates the basin topography as an isoline plot, the wavefield as emanated from the source and the ground motion as recorded from virtual seismometers at the surface along an east-west-profile. Clearly one can see the amplification and ringing due to the basin structure.

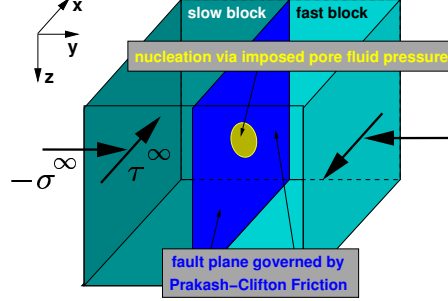
### 2.3 Simulation of earthquake source dynamics

For the simulation of large earthquake scenarios and prediction of strong ground motion, a detailed knowledge of the space and time history of earthquake slip is necessary. This is very intuitive since large earthquakes can rupture the Earth's crust for several tens or hundreds of kilometers, like the 7.9 magnitude 2002 Alaska earthquake which had a rupture propagation distance of about 260 km. Earthquakes of this size basically rupture the entire crust. Once the source history is known, one can use these kinematics for scenario simulations as described in section 2.2. Due to limitations in experimental and theoretical solutions the determination of those earthquake source parameters is very difficult. Numerical simulation of dynamic faulting and seismic wave propagation therefore gives a valuable tool for achieving advanced solutions of earthquake source dynamics through broad parameter studies. We have implemented frictional boundaries into the 2D and 3D finite-difference codes, to study source dynamics and faulting and tested them for their accuracy [4]. These potential faults are governed by friction laws that are based on laboratory measurements. Depending on the goals of the study a simple or a more sophisticated friction law may be the more appropriate choice. However, the



frictional conditions in the deeper crust remain to be probed, and the different behaviour of types of friction must be studied to come to stable conclusions.

Recent works indicated that rupture along a material interface has remarkably dynamic properties, which may be relevant to a number of geophysical problems. Those works also show that material interfaces are mechanically favoured locations for rupture propagation. This topic has gained interest in the seismological community as material interfaces are likely to exist on seismically active regions with a long slip history, such as the San Andreas Fault in California. In general, a rupture along a material interface governed by Coulomb friction does not have a continuum limit (e.g., Adams[1], Ranjith & Rice [9]). In order to achieve convergence of the numerical methods (boundary integral methods and finite-difference methods) a very fine numerical mesh and a regularization procedure (like the Prakash-Clifton friction or to some extent the Coulomb friction in a visco-elastic medium) is needed. The grid spacing here is in the order of decimeters - compared to 200m grid spacing for the scenario-simulations in the Cologne Basin. This means that a fully dynamic simulation at the scale of a large regional earthquake scenario is at the edge of the current computational limits. However, the numerical simulations concentrating on earthquake source dynamics help to explore and understand the non-linear phenomena of earthquake ruptures. Results will help resolving source parameters of observational earthquake data and can also be used for kinematic sources in earthquake scenario calculations.

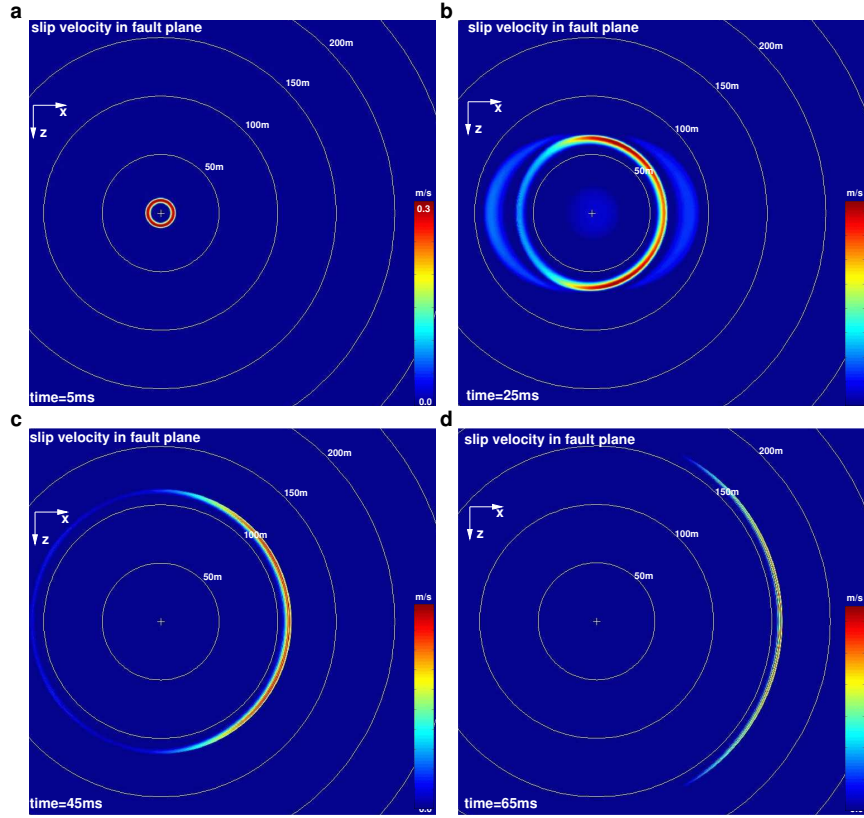


**Fig. 9.** Sketch of the 3D rupture propagation problem on a material interface using the Prakash-Clifton friction law. Results of such a model are shown in Figure 10. The required resolution using realistic values ( $\sigma = 100\text{MPa}$ ,  $v_p = 5200\text{m/s}$ ) is in the order of decimeters. For propagation distances of interest (several hundred meters) this leads to huge memory requirements.

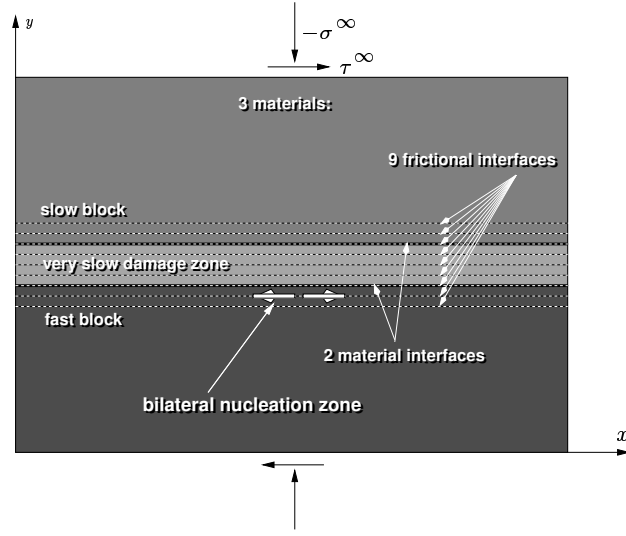
## 2.4 Numerical simulation of rotational motions

In the past, the theory of translational motions had been paid much more attention than the theory of rotational motions. There are basically no systematic theoretical studies of rotational effects in realistic media, partly because the effects were thought to be small, and also because no instruments existed that directly measure absolute or incremental rotation.

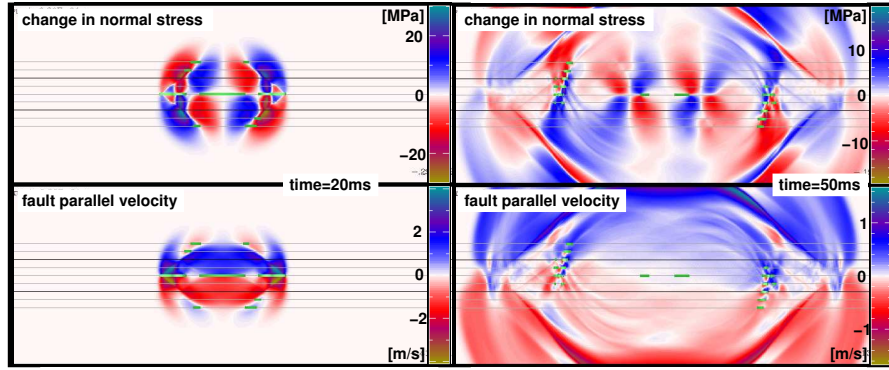
From the numerical point of view the technique of simulating the full seismic wavefield in three dimensions has only recently become possible due



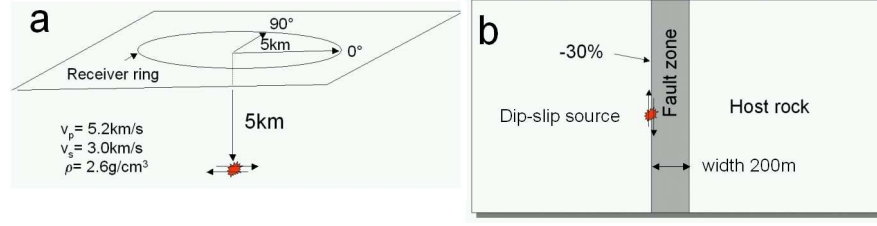
**Fig. 10.** (a)-(d): Evolution of slip velocity on a material interface governed by the Prakash-Clifton friction law under homogeneous stress loading. Although the rupture is nucleated symmetrically (a) simulating a loss in pore pressure, the rupture does not propagate symmetrically about the in-plane direction (b) and dies out quickly for the direction of slip in the faster medium (c). Depending on the stress loading and friction parameters, the remaining unilateral rupture (d) can propagate self-sustained and self-sharpening for a long propagation distance (large earthquake) or can die out as well (small earthquake). The unilateral rupture propagation is very characteristic for material interfaces and also observed on different earthquake faults, as in the San Andreas fault. In the shown 3D-simulation slip is restricted to the x-direction.



**Fig. 11.** A model for studying migration of in-plane dynamic rupture among nine frictional interfaces within three different elastic solids (a fast block, a slow block and a low velocity layer).

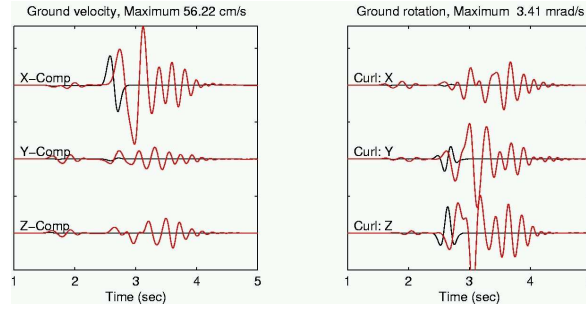


**Fig. 12.** Here we examine tendencies of in-plane rupture to migrate to material interfaces. Snapshots of the change in normal stress and the fault-parallel velocity are shown for evolution of 2D in-plane fault ruptures under Prakash-Clifton friction on nine parallel fault planes in a 3-media composition with two material interfaces. We use an efficient second-order finite-difference scheme as described by Andrews [2] and tested for its accuracy by Andrews and Ben-Zion [3]



**Fig. 13.** a: Model setup for wave calculations and receiver locations at the surface. b: Model setup for a fault-zone. Here the source is located at the edge of the fault

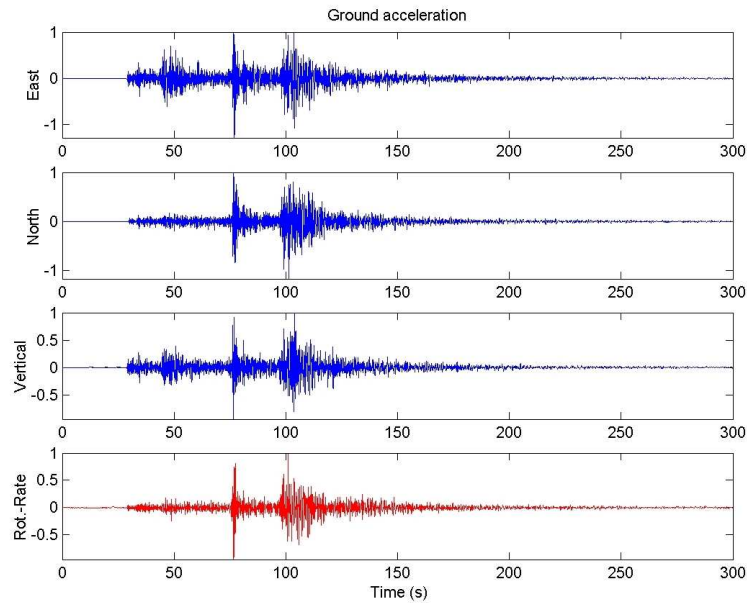
to advanced computational power. While the extraction of rotational motion from numerical calculations is straight forward, the field measurement of rotational motion and subsequent interpretation remains difficult. It is expected that rotational motions will provide interesting information on the physics of earthquake sources and might help to fill the gaps in our knowledge of the seismic source. Rotational motion will hopefully provide more characteristic information about the earthquake source dynamics and kinematics. We are currently investigate the ability of a ring-laser-gyroscope (installed at Wettzell) for these seismological purposes. We support the results with theoretical and numerical studies.



**Fig. 14.** 6-component seismograms (left velocity components, right components of rotation rate) for a homogeneous model (black traces) and a fault-zone typical velocity structure (red traces). Note the amplification of ground motion in both translational and rotational measurements

Outlook text goes here.

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**Fig. 15.** Records of the M5.5 earthquake in the Vosges on 22nd February, 2003. The top three traces are the components of ground acceleration at Wettzell. The bottom trace is the vertical component of the rotation rate recorded with the ringlaser. Traces are normalized.

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