APPENDIX A: CALCULATIONS

1. High Quality Data

In the stretching technique we are looking to maximize the cross correlation coefficient (equation (4) in the paper):

\[
CC_k(\varepsilon) = \frac{\int_{t_1}^{t_2} h_k(t(1 - \varepsilon)) h_0(t) dt}{\sqrt{\int_{t_1}^{t_2} h_k^2(t(1 - \varepsilon)) dt \int_{t_1}^{t_2} h_0^2(t) dt}}
\] (A1)

In section IIIA we estimate \(CC_k\) for high quality data, without electronic or other noise. Our signals before and after a small temperature change then become:

\[
h_0(t) = G_0(R, R, t) \otimes e(t)
\] (A2)

and

\[
h_k(t) = G_k(R, R, t) \otimes e(t) = [G_0(R, R, t(1 + \varepsilon_k)) + f(t)] \otimes e(t)
\] (A3)

where \(\varepsilon_k\) is the amount by which the record is stretched, and \(f(t)\) represents the small fluctuations due to tiny physical changes in the medium as it expands slightly. Both \(h_0\) and \(h_k\) are assumed to be stationary. Applying (A2) and (A3) to (A1), we get:

\[
CC_k(\varepsilon) = \frac{\int_{t_1}^{t_2} [G_0(R, R, t(1 + \varepsilon_k)) + f(t)] \otimes e(t) [G_0(R, R, t)] \otimes e(t) dt}{\sqrt{\int_{t_1}^{t_2} [G_0(R, R, t(1 + \varepsilon_k)) + f(t)]^2 dt \int_{t_1}^{t_2} [G_0(R, R, t)]^2 dt}}
\] (A4)

We consider the simple case where \(t_1 = 0\) and \(t_2 = T\). We know that:

\[
\rho(t) = \frac{e(t) \otimes e(t)}{\int e^2(t) dt}
\] (A5)

and simplify the expression to:

\[
CC_k = \frac{\int_0^T [G_0^2 + G_0f] \otimes \rho(t) dt}{\sqrt{\int_0^T [G_0^2] \otimes \rho(t) dt \int_0^T [G_0^2 + f^2 + 2G_0f] \otimes \rho(t) dt}}
\] (A6)

Before calculating the mean value of \(CC_k\), we assume that the Green functions at different times \(G_0(t)\) and \(G_0(t')\) are random, \(\delta\)-correlated signals, with zero mean. This means that \(\langle G(t)G(t')\rangle \approx \delta(t - t')\) and \(\langle G_0 \rangle = 0\). Furthermore, we suppose that the mean intensity of the Green function will remain unchanged before and after a temperature change: \(\langle G_0^2 \rangle = \langle G_k^2 \rangle = \langle G^2 \rangle\). The mean of any crossterms with the fluctuations \(\langle G_0f \rangle\) are set to zero. We use that \(\langle \int_0^T G_0^2 dt \rangle = T\langle G_0^2 \rangle\). With all this we can estimate the mean of \(CC_k\):
\[ A = \langle CC_k \rangle = \frac{T(G_0^2)}{T\sqrt{(G_0^2)/(G_0^2 + f^2))}} = \frac{\sqrt{G_0^2}}{\sqrt{(G_0^2 + f^2)}} \] (A7)

which is the constant \( A \) in equation (7) in the paper.

In order to find the amplitude of the fluctuations around the mean value we need to calculate the standard deviation of \( CC_k \). We can first estimate its variance: \( \text{var}(CC_k) = \langle CC_k^2 \rangle - \langle CC_k \rangle^2 \). Breaking it up into smaller pieces, we start by calculating the mean of \( CC_k^2 \):

\[ CC_k^2 = \frac{\int_0^T [G_0^2 + G_0 f] \otimes \rho(t) \ dt \int_0^T [G_0^2 + G_0 f] \otimes \rho(t') \ dt'}{\int_0^T [G_0^2] \otimes \rho(t) \ dt \int_0^T [G_0^2 + f^2 + 2G_0 f] \otimes \rho(t) \ dt} \] (A8)

\[ = \frac{\int_0^T \int_0^T [G_0^2 + G_0 f] [G_0^2 + G_0 f] \otimes \rho(t) \otimes \rho(t') \ dt \ dt'}{\int_0^T G_0^2 \otimes \rho(t) \ dt \int_0^T [G_0^2 + f^2 + 2G_0 f] \otimes \rho(t) \ dt} \] (A9)

Again, crossterms with \( \langle G_0 f \rangle \) are zero. The same assumptions as before equation (A7) hold, and we use that:

\[ \int \rho(t)^2 \ dt \approx \frac{\Delta \omega}{2\pi} \] (A10)

The mean value of \( CC_k^2 \) then becomes:

\[ \langle CC_k^2 \rangle = \frac{2\pi}{\Delta \omega} \frac{T((G_0^2)^2 + (G_0^2)(f^2))}{T^2((G_0^2)^2 + (f^2))} \] (A11)

Now the standard deviation is \( \sqrt{\text{var}(CC_k)} \), or, using \( \langle CC_k \rangle^2 \) from equation (A7), \( \sqrt{\langle B^2 \rangle} = \sqrt{\langle CC_k^2 \rangle - \langle CC_k \rangle^2} \):

\[ \sqrt{\langle B^2 \rangle} = \sqrt{\frac{2\pi}{\Delta \omega T}} \frac{\sqrt{(f^2)}}{\sqrt{(G_0^2) + (f^2)}} \] (A12)

which is equation (8) in the paper.

### 2. Low Quality Data

In section IIIB we consider a signal with some noise added, electronic or otherwise:

\[ S_0 = h_0 + n_0 \] (A13)

\[ S_k = h_k + n_k \] (A14)

The mean value of \( CC_k \) will be a bit different for this case:

\[ CC_k(\varepsilon) = \frac{\int_0^T [h_0 + n_0][h_k + n_k] \otimes e(t) \otimes e(t) \ dt}{\sqrt{\int_0^T [(h_0 + n_0) \otimes e(t)]^2 \ dt} \sqrt{\int_0^T [(h_k + n_k) \otimes e(t)]^2 \ dt}} \] (A15)

\[ = \frac{\int_0^T [h_0h_k + h_0n_k + h_kn_0 + n_0n_k] \otimes \rho(t) \ dt}{\sqrt{\int_0^T [h_0^2 + n_0^2 + 2h_0n_0] \otimes \rho(t) \ dt} \sqrt{\int_0^T [h_k^2 + n_k^2 + 2h_kn_k] \otimes \rho(t) \ dt}} \] (A16)
We assume that the mean of the crossterms involving noise (e.g. $\langle h_i n_j \rangle$ and $\langle n_i n_j \rangle$) are zero. We also assume that the mean of the main signal $h$ will stay the same after a temperature change: $\langle h_0^2 \rangle = \langle h_k^2 \rangle = \langle h^2 \rangle$. With this, the mean of $CC_k$ is:

$$A = \langle CC_k \rangle = \frac{\langle h^2 \rangle}{\langle h^2 \rangle + \langle n^2 \rangle},$$

which is equation (10) in the paper.

As before, the variance of $CC_k$ is given by $\text{var}(CC_k) = \langle CC_k^2 \rangle - \langle CC_k \rangle^2$:

$$CC_k^2 = \int_0^T \int_0^T \frac{[(h_0 + n_0)^2(h_k + n_k)^2] \otimes \rho(t) \otimes \rho(t') dt' dt}{T^2(\langle h^2 \rangle + \langle n^2 \rangle)^2}$$

$$= \int_0^T \int_0^T \frac{[(h_0^2 + n_0^2 + h_0 n_0)(h_k^2 + n_k^2 + h_k n_k)] \otimes \rho(t) \otimes \rho(t') dt' dt}{T^2(\langle h^2 \rangle + \langle n^2 \rangle)^2}$$

Again, crossterms with noise are set to zero. Using equation (A10), the mean of $CC_k^2$ is now:

$$\langle CC_k^2 \rangle = \frac{2\pi T \left[ \langle h^2 \rangle^2 + \langle n^2 \rangle^2 + 2\langle h^2 \rangle \langle n^2 \rangle \right]}{\Delta \omega T^2(\langle h^2 \rangle + \langle n^2 \rangle)^2}$$

and the variance of $CC_k$, using equation (A17):

$$\text{var}(CC_k) = \langle CC_k^2 \rangle - \langle CC_k \rangle^2 = \frac{2\pi}{\Delta \omega T} \frac{\langle n^2 \rangle^2 + 2\langle h^2 \rangle \langle n^2 \rangle}{\langle h^2 \rangle + \langle n^2 \rangle^2}$$

Now the standard deviation is just the square root of equation (A21):

$$\sqrt{\langle B^2 \rangle} = \sqrt{\frac{2\pi}{\Delta \omega T} \frac{\langle n^2 \rangle^2 + 2\langle h^2 \rangle \langle n^2 \rangle}{\langle h^2 \rangle + \langle n^2 \rangle}},$$

which is equation (11) in the paper.