**Computational seismology - *Finite Differences***

1) Show that

$$\frac{-1/12\, f(x+2\Delta x) + 4/3\, f(x+\Delta x) - 5/2\, f(x) + 4/3\, f(x-\Delta x) - 1/12\, f(x-2\Delta dx)}{dx^2}$$

is an approximation for the second derivative of f(x) with respect to x at position x. Hint: Use Taylor's theorem. What is the order of the error term?

2) Approximate the constant density acoustic wave equation in two dimensions using finite differences and formulate a pseudo-code

$$\partial_t^2 p = c^2 \left(\partial_x^2 + \partial_z^2\right) p + s$$

3) A seismometer consists of a spring with damping parameter $\varepsilon$, and eigenfrequency $\omega_0$. The seismometer is excited by the ground motion $\ddot{u}(t)$. The relative motion of the seismometer mass x(t) is governed by the following equation

$$\ddot{x} + 2\varepsilon\dot{x} + \omega_0^2 x = \ddot{u}$$

Replace the derivatives on the l.h.s. with finite differences. Solve for x(t+dt). Note: Center the differences at the same point in time.

4) a) You want to simulate 2-D acoustic wave propagation in a medium with size 1000km x 1000km. You want to model wave propagation up to a period of 10 seconds. The maximum velocity c is 8km/s, the minimum velocity is 4km/s. Your numerical algorithm requires 20 grid points per wavelength to be accurate for the propagation distances of interest. What space increment dx do you need for the simulation?
b) The stability criterion says that maximum velocity c, space increment dx and time increment dt are related by const=c dt/dx. You want a seismogram length of 500 seconds. How many time steps do you have to simulate, when const=0.5?

5) Certain isotopes (e.g., $^9$Be) are washed into the sea by rivers and then mixed by advection through ocean currents and diffusion. In addition, the isotopes are removed from the system through biomechanical processes. These processes can be described through the diffusion-advection-reaction equation (concentration C(x,t), diffusivity k (const), reactivity R(x), source p(x), velocity v(x)). Substitute in the 1-D equation below the partial differentials with finite differences and extrapolate to C(t+dt):

$$\partial_t C = k\partial_x^2 C + v_x \partial_x C - RC + p$$

How could a „ring-current“ be simulated with this 1-D equation?