The seismic inverse problem: a subjective primer[1]

Elastic wave propagation:

A sudden change in stress (e.g., through an explosion, or an earthquake) in the (visco-) elastic Earth, leads to attenuating elastic waves radiating from the source location, as stress or strain waves, measurable as local displacement vector **u**(x,y,z) around a reference point. This three-component vector **u** is commonly observed in local or global seismometer networks in frequency bands between 0.01 Hz to about 10 Hz. Examples are shown in Fig. 1 (black traces), in which the vertical component of ground motion u_z is shown for a time window containing the arrival of Rayleigh-type surface waves. The dominant period is about 50 seconds corresponding to wavelengths inside the Earth of 150-400km. The "*forward*" problem consists of calculating synthetic seismograms for a 3-D Earth model and all receiver locations (e.g., red traces in Fig 1 for a final tomographic model). We assume the earthquake source location and the source mechanism known. Thus, the remaining "*inverse*" problem is to find the 3D structure(s) that minimize the misfit between predicted synthetic and observed seismograms. The forward calculations are carried out using a parallel implementation of a spectral-element description of 3-D elastic wave propagation. One simulation for a spherical section as shown in Figure 1 takes about 30 mins on several processors on HLRB-II.

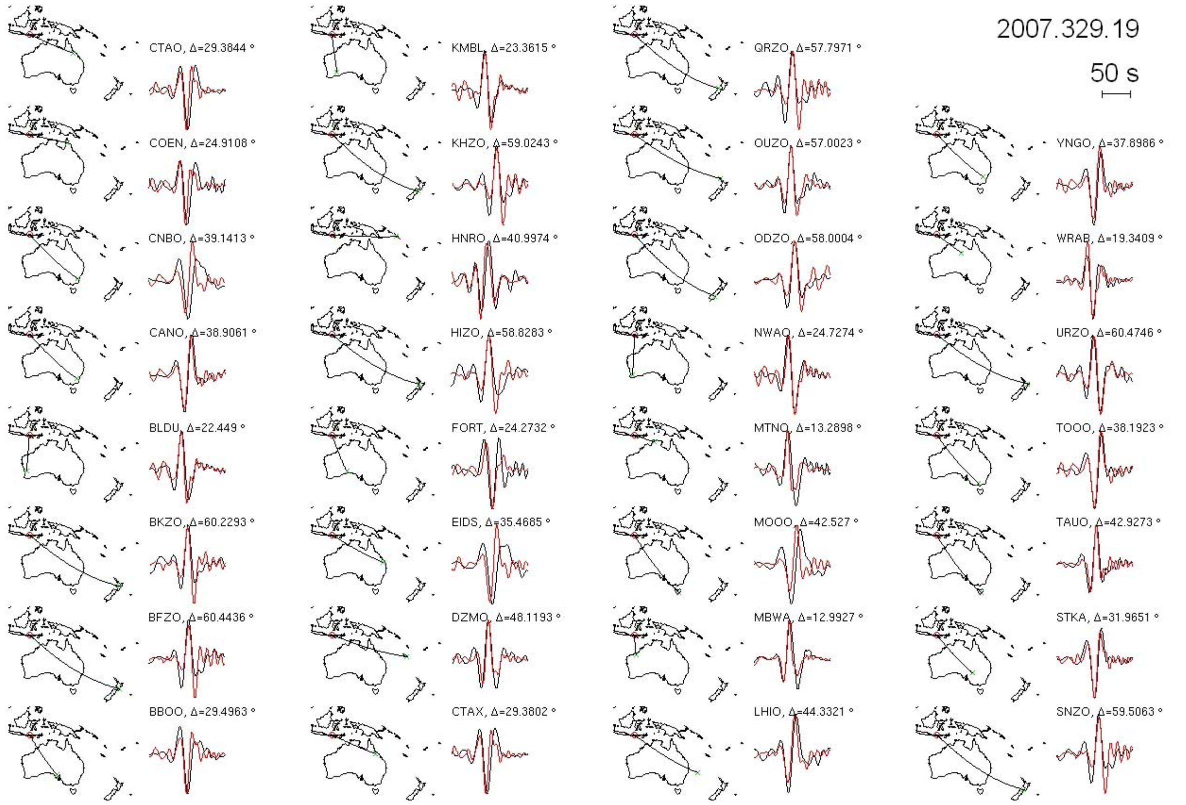

Fig. 1: Vertical-component surface waveforms for event 2007.329.19. Data are plotted in black and synthetics in red. Each subfigure corresponds to a different receiver location. A straight line is drawn connecting earthquake source and receiver location.

The inverse problem:

When a sufficiently well known initial model is available the inverse problem can be considered quasi-linear which makes the use of adjoint type methods - based on linear

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[1] Compiled by H. Igel with A. Fichtner and K. Sigloch, Geophysics Section, Department of Earth Sciences, LMU

perturbations around a reference model – feasible. In the example case shown here this implies that for each of the available 60 earthquakes one forward simulation is necessary (an illustration of the ray coverage is shown in Fig. 2). The most important part is the calculation of the model update (i.e., the gradient of the misfit function) that will lead to the final model through an iterative process. It turns out that to get the model update one needs to inject the “adjoint sources” – related to the misfit between synthetic and observed seismograms – at the receiver locations. The model update is obtained by cross-correlating the forward field and the time-reversed field with sources at the receiver locations. Finally, the determination of the step-length for the model update needs to be carried out. Generally, for each earthquake three simulations per iteration are necessary.

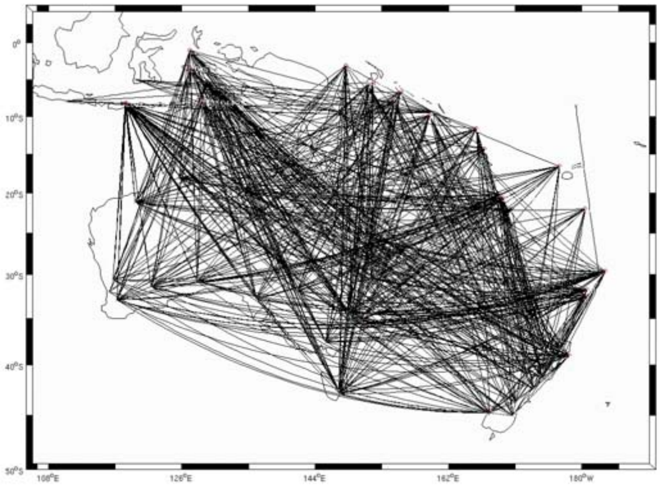

Fig. 2. Ray coverage for several dozen events.

The solution to the inverse problem: tomographic images

With this linearized approach one minimizes the misfit between observed and synthetic data in a deterministic way in order to obtain a best-fitting model. Such a model is shown in Fig. 3. Physically this is a 3-D function of seismic velocities. Because of the specific way the model update is calculated through cross-correlating wavefields, no particular parameterization of the model using some basis functions is necessary (but might be desirable in the future). That implies that basically each of the grid points is a degree of freedom (of course not in reality because of the long wavelength character of the wavefields). It is important to note that it is extremely difficult to calculate the resolution or uncertainties of the resulting “final” model and this should be one of the key issues to be addressed in our project. Note that the graphs presented here are the first ever waveform inversions of seismic data on a continental scale (thanks to Andreas) based on complete 3-D calculations. Full waveform inversion for seismological data is only now getting attention because of the tremendous computational resources necessary to solve realistic problems.

Probabilistic description of the inverse problem and geostatistics:

Because we are basically always dealing with indirect parameter estimation (in most of the cases we never can verify what we predict as structure in the Earth’s deep interior) inverse problems have received a lot of attention in geophysics since decades ago. Tarantola (see papers below) formulated the inverse problem in probabilistic (Bayesian) terms and has ever since pushed for the use of Monte Carlo type methods for the solution of inverse problems. The – in my view – beautiful theory with the key statement that THE solution to the inverse problem is the a posteriori probability density function defined on the model space, is not easily implemented in practice, simply because any point in model space requires the solution of the forward problem. The probability of the specific model is then a function of the misfit between synthetic and observed seismograms. It is important to note that we are now in the situation that the simulation of O(10000) 3-D simulations is in sight. So one of the key questions to address is whether we can complement the adjoint inverse approach with Monte Carlo type techniques particularly in connection with resolution and uncertainty analysis.

The field of “geostatistics” has in this context played a role in the description of Earth models using random functions. Example: Given some prior distribution of any physical parameter in the Earth (e.g., seismic velocities, layer thicknesses, spatial wavelengths, location of major discontinuities) one can generate samples of Earth models that represent the prior information. An alternative “ansatz” to the inverse problem is to sample this prior distribution and to test each model against the observation and thereby building the a posteriori distribution. It would be a challenge to define ways to describe the geological/geophysical prior information on a region such as Australia or other continents a in sensible way and generate samples of such a prior distribution. Such an approach could be used to (1) to solve the inverse problem completely using Monte Carlo techniques, (2) to investigate resolution/uncertainty around a “final” model found by adjoint techniques; or (3) find a family of acceptable models descriptive of the prior that are used as starting models for local search techniques (adjoint).

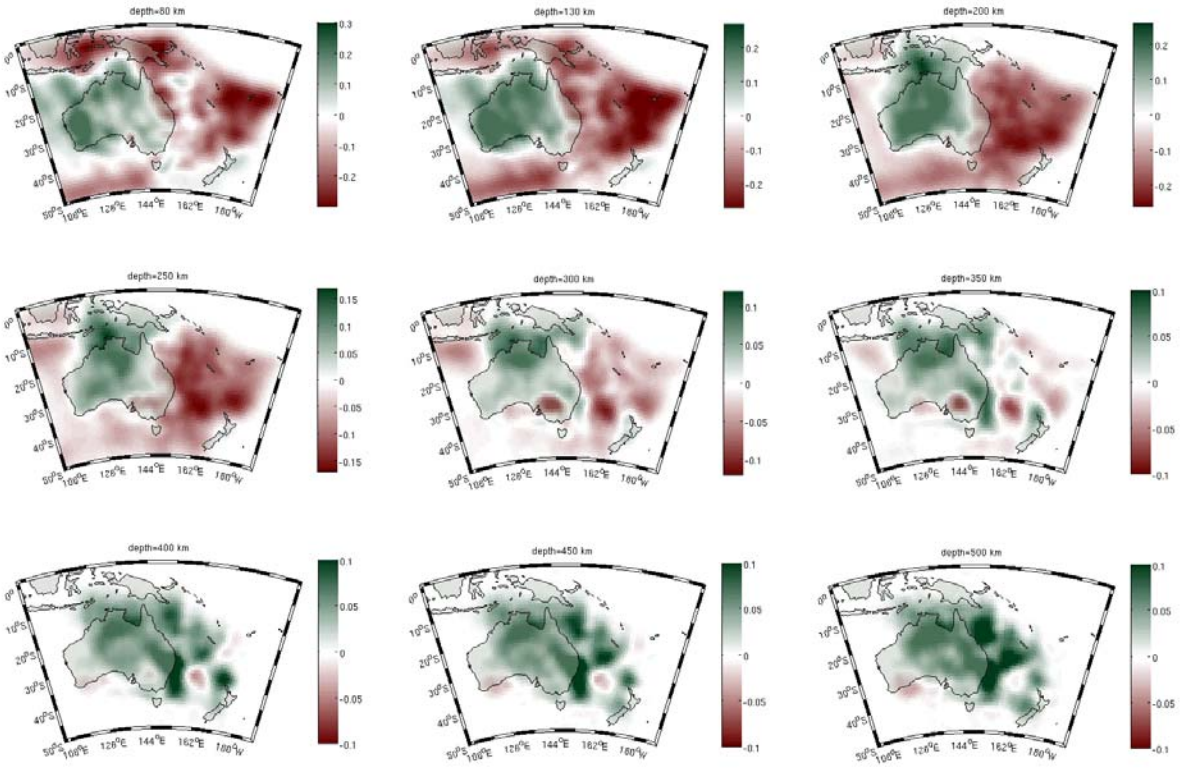

Fig. 3. Absolute perturbation of the S wave speed in km/s at depths of (from upper left to lower right) 80 km, 130 km, 200 km, 250 km, 300 km, 350 km, 400 km, 450 km and 500 km. The background model is PREM.

Challenges and major questions:

Here are some general issues that could be addressed in the MACES project but of course this will be finalised in the discussion:

- What are optimal parameterizations of the Earth model minimizing the degrees of freedom in the model space?
- How could one describe in a quantitative way a priori information for continental-scale Earth model (using probabilities) taking into account geological/geophysical information?
- How can we quantify and visualize uncertainties and resolution of tomographic Earth models?
- Is it possible (and does it make sense) to combine Monte Carlo and adjoint techniques to solve the seismic inverse problem?

We propose to continue to work on the seismic inverse problem on a continental scale because (1) all computational tools for forward and inverse problem are set up for this problem; (2) a specific regional data set is available and results from a preliminary inversion using adjoint methods are available; (3) it is an emerging field in geophysics.

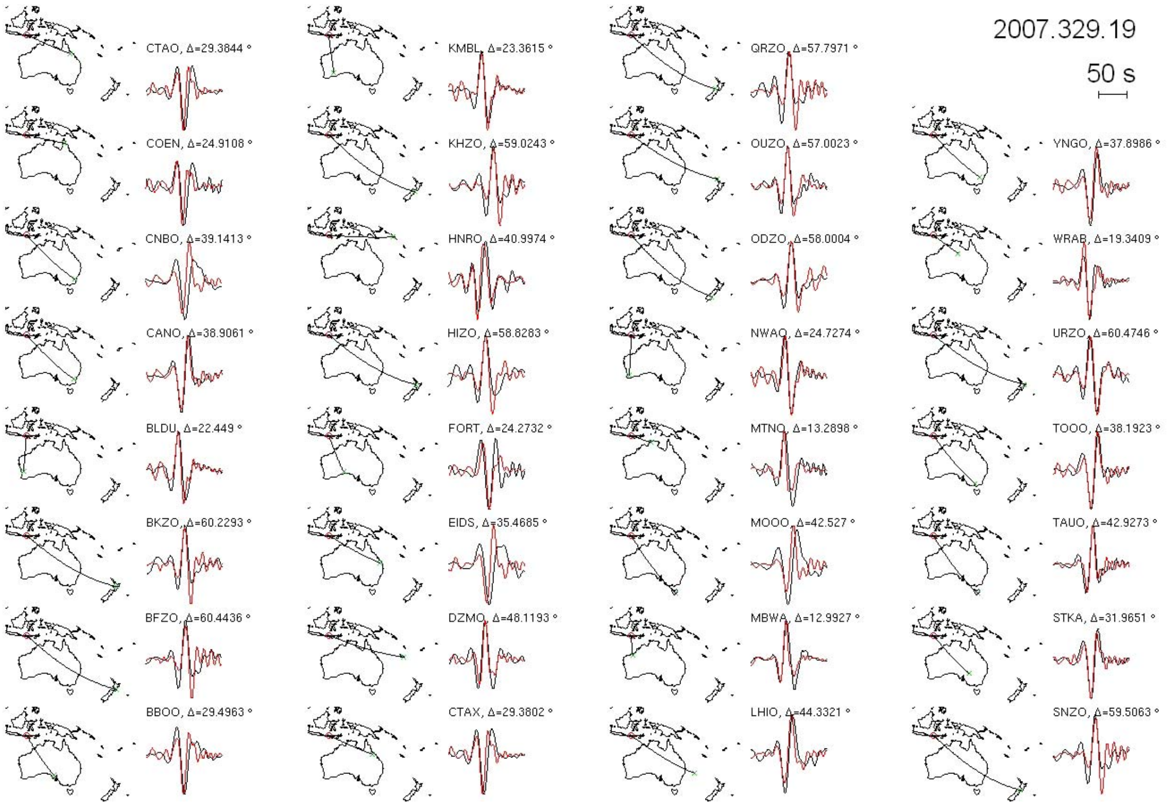

Fig. 3: Vertical-component surface waveforms for event 2007.329.19. Data are plotted in black and synthetics in red.

Commented Literature:

Click on the “pdf” to access the publications.

The seismic inverse problem in the adjoint formulation:

Inversion of Seismic Reflection Data in the Acoustic Approximation. Albert Tarantola, Geophysics, Vol. 49, No. 8, p 1259-1266, 1984. (pdf). *One of the first papers on the application of adjoint methods to seismic wave propagation.*

Fichtner, A., H.-P. Bunge, and H. Igel (2006), The adjoint method in seismology: I - Theory, Physics of The Earth and Planetary Interiors, 157(1-2), 86-104, doi:10.1016/j.pepi.2006.03.016. (pdf). *A mathematically more general formulation of the seismic waveform inversion problem.*

Fichtner, A., P. Bunge, and H. Igel (2006), The adjoint method in seismology: II - Applications: traveltimes and sensitivity functionals, Phys. Earth Planet. Int., 157(1-2), 105-123, doi:10.1016/j.pepi.2006.03.018. (pdf) *Paper with illustration of sensitivity kernels for seismic travel times*

Fichtner, Andreas, Brian L. N. Kennett, Heiner Igel, Hans-Peter Bunge, DOI:Theoretical background for continental- and global-scale full-waveform inversion in the time–frequency domain (p 665-685) 10.1111/j.1365-246X.2008.03923.x (pdf) *Some thoughts and applications of a new misfit criterion based on a time-frequency representation of waveform misfits, separation of travel-time effects and amplitude effects.*

Probabilistic description of the inverse problem, uncertainties, visualization:

Popper, Bayes and the inverse problem, by Albert Tarantola, Nature Physics, Vol. 2, August 2006, p 492-494, 2006. (pdf) *A qualitative description of the probabilistic approach to geophysical inverse problems stressing the necessity to sample a probability distribution descriptive of prior knowledge.*

How do we understand and visualize uncertainty ? Sambridge, M. Beghein, C. Simons, F. and Snieder, R., The Leading Edge, 25,542-546, 2006. (pdf) *Description of the problem faced when inverting seismograms and attempting to visualize resolution, and uncertainties in multidimensional model space.*

Monte Carlo Sampling of Solutions to Inverse Problems, by Klaus Mosegaard and Albert Tarantola. Journal of Geophysical Research , Vol. 10, No B7, p 12,431-12,447, 1995. (pdf) *Basic ideas on how to efficiently sample prior and posterior distributions of probability density functions describing solutions to geophysical inverse problems.*

Probabilistic Approach to Inverse Problems, by Klaus Mosegaard and Albert Tarantola, International Handbook of Earthquake & Engineering Seismology, Part A., p 237-265, Academic Press, 2002. (pdf) *More extensive than the previous publication, very instructive examples, and introduction of the concept of volumetric probabilities.*

(Geo-) Scientfic relevance of seismic inversion:

Sigloch, Karin, Nadine McQuarrie, and Guust Nolet (2008), Two-stage subduction history under North America inferred from multiple-frequency tomography, Nature Geoscience, doi:10.1038/ngeo231. (pdf) *An example of the relevance of tomography for questions on the dynamics of Earth's interior.*

More papers on parametrization issues, probabilistic inversion etc. can be found on the pages by Malcolm Sambridge and Albert Tarantola:

http://rses.anu.edu.au/~malcolm/index.php?p=pubs

http://www.ipgp.jussieu.fr/~tarantola/