LMU Geophysics/Seismology

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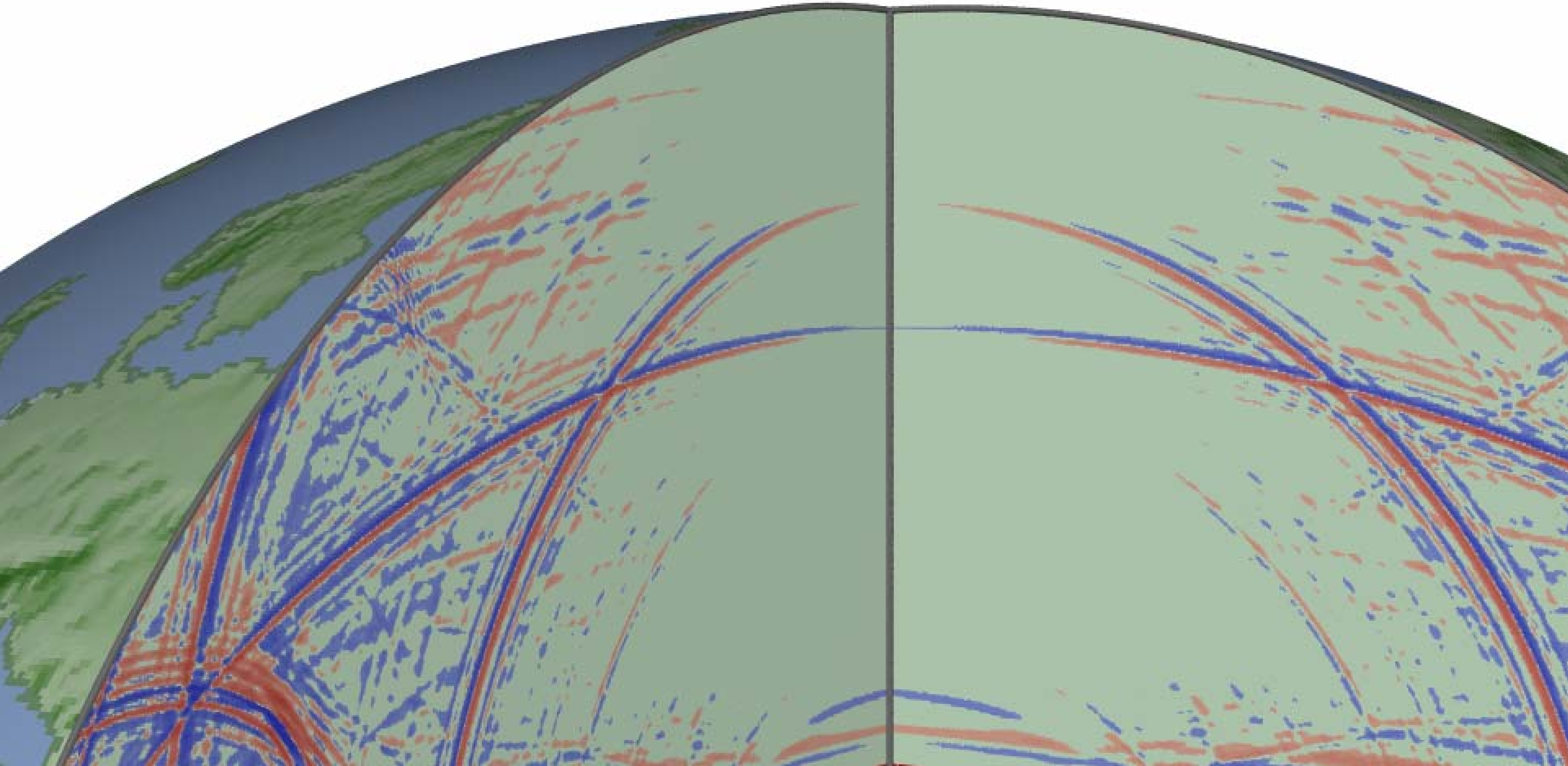
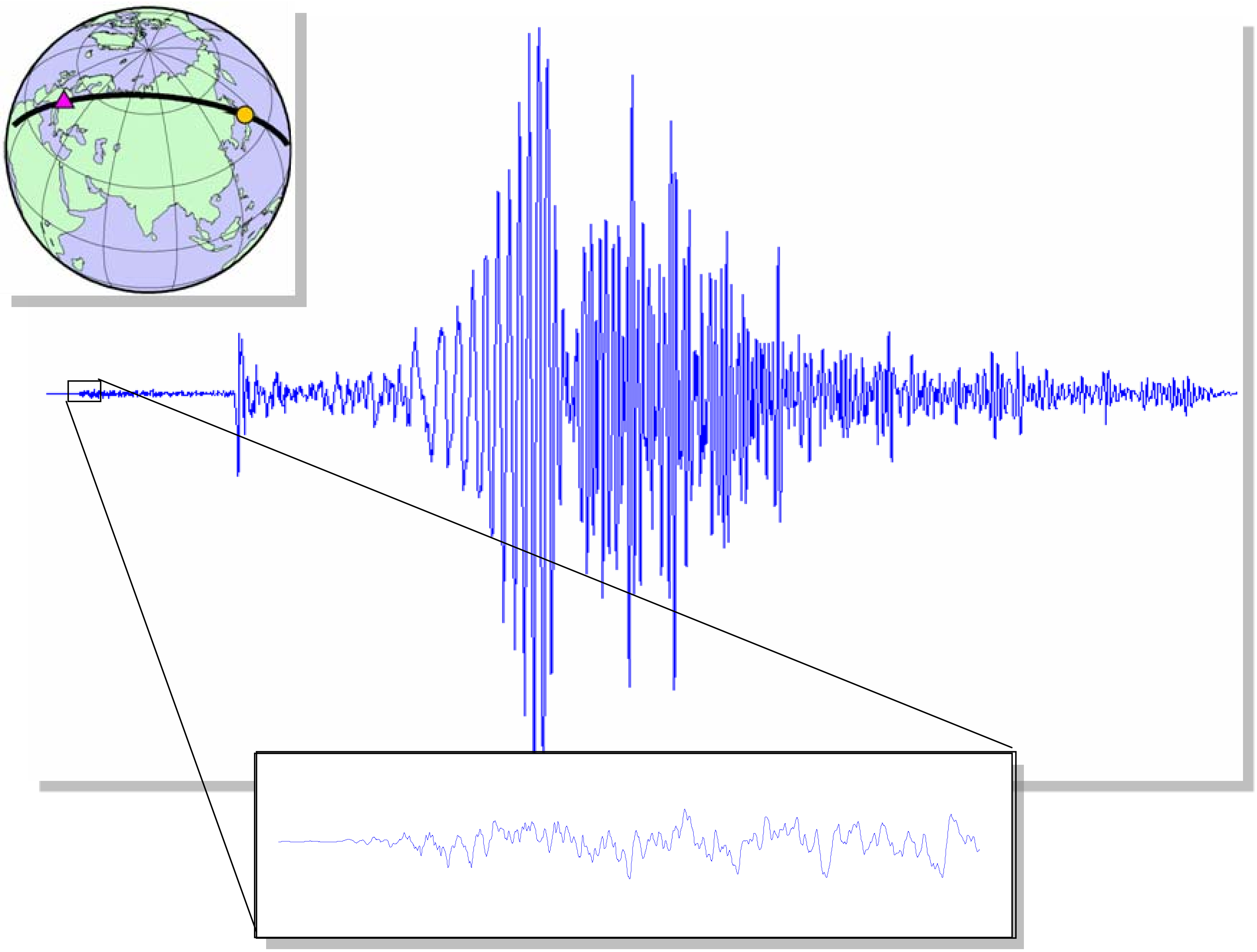
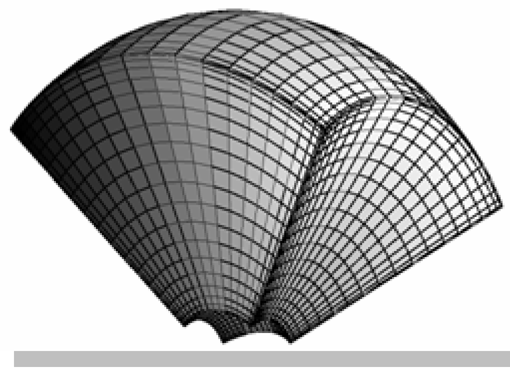
Käser, Marcus Mohr, Karin Sigloch, a.o.



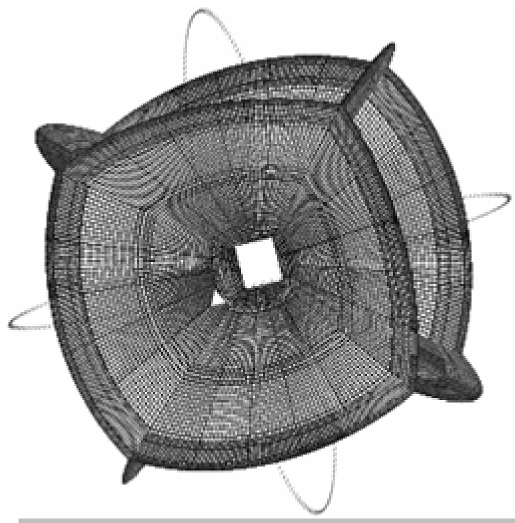


Spatial Scales and Memory

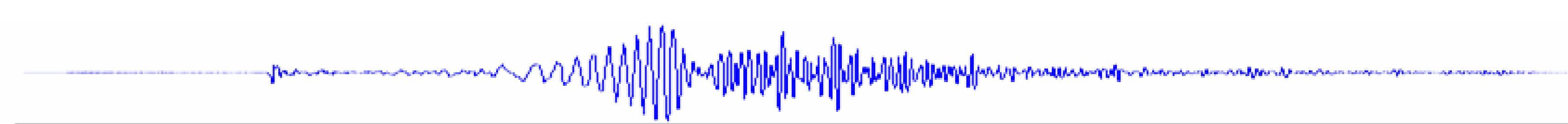
(back of the envelope)

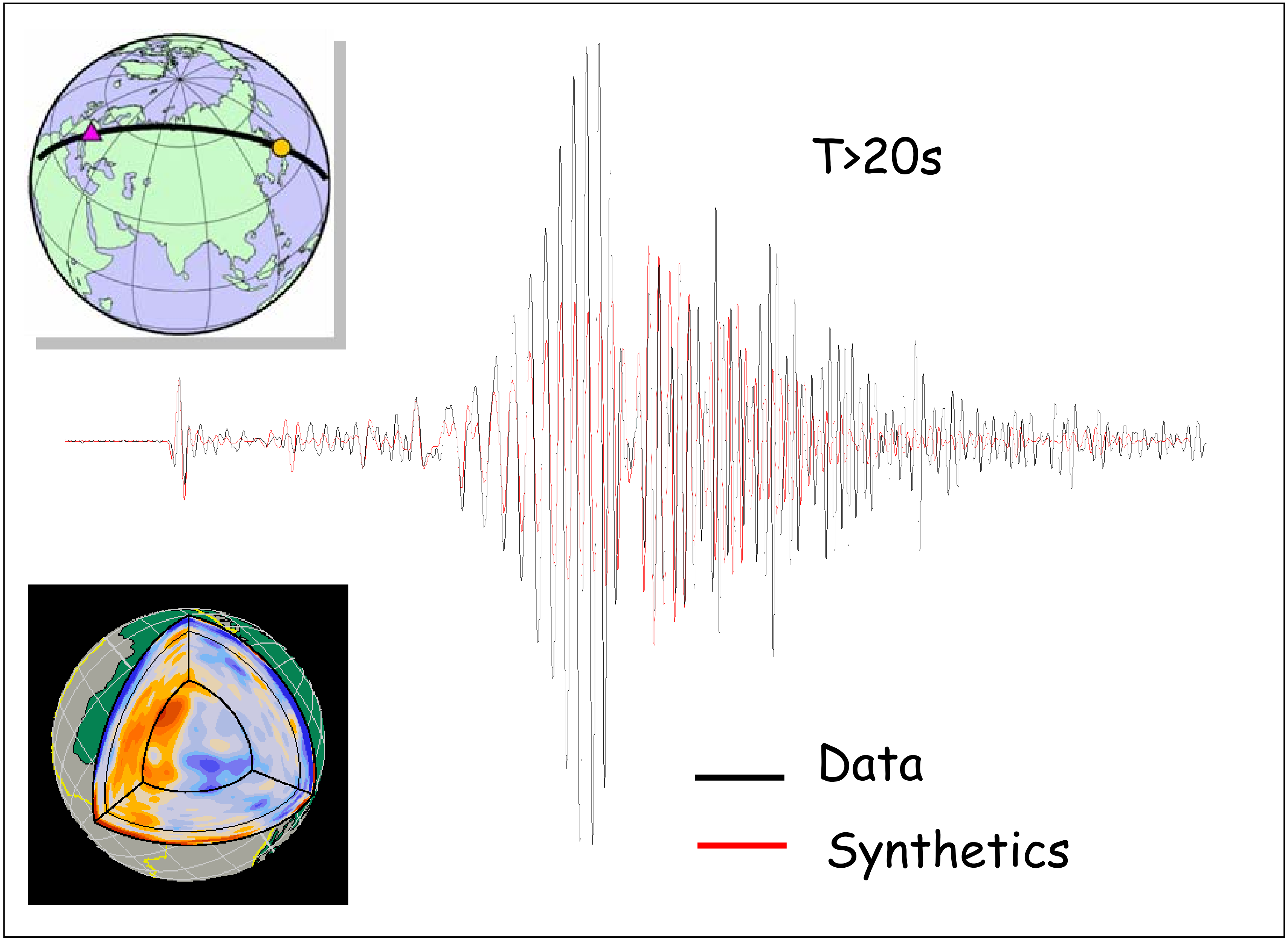


Highest frequency:	**0.1 Hz**
Shortest wavelength:	20 km (crust)
Shortest wavelength:	50 km (mantle)
Grid points per wavelength:	5
Grid spacing:	2000 m (crust)
Grid spacing:	5000 m (mantle)



Required grid points: $O(10^9)$
Required memory: O(100 GBytes)




T>20s
Data
Synthetics

The FORWARD Problem

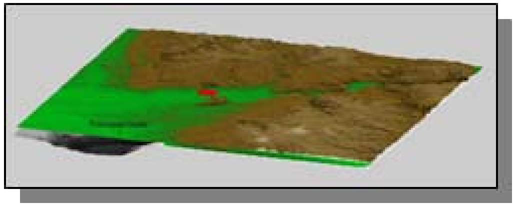
Elastic wave equations

$$\rho \partial_t^2 u_i = \partial_j (\sigma_{ij} + M_{ij}) + f_i$$

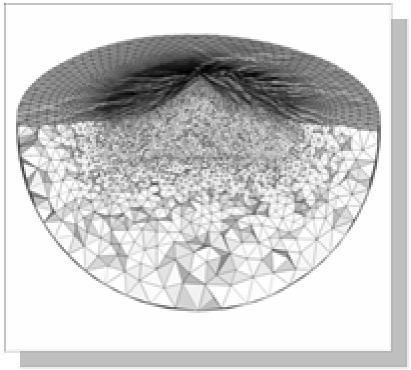
$$\sigma_{ij} = c_{ijkl} \varepsilon_{kl}$$

$$\varepsilon_{kl} = 1/2(\partial_k u_l + \partial_l u_k)$$

3D Model

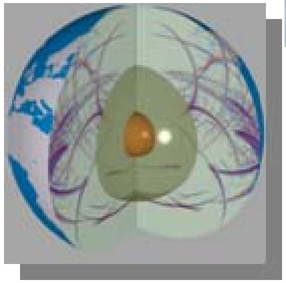


Grid

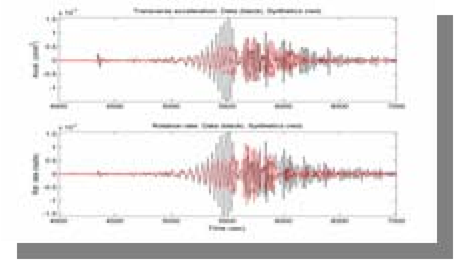


Parallelisation

Simulation



Synthetic seismograms



Elastic Wave Equations

theory of linear elasticity
(stress-strain, Hooke´s law)
+
Newton´s law
(acceleration through forces caused by stress)
=
velocity-stress formulation
(linear hyperbolic system)

In a heterogeneous medium:
space-dependent material coefficients

Lamé constants: $\lambda = \lambda(x,y,z)$,
$\mu = \mu(x,y,z)$,

density $\rho = \rho(x,y,z)$

$$
\begin{aligned}
\frac{\partial}{\partial t}\sigma_{xx} - (\lambda + 2\mu)\frac{\partial}{\partial x}u - \lambda\frac{\partial}{\partial y}v - \lambda\frac{\partial}{\partial z}w &= 0\,, \\
\frac{\partial}{\partial t}\sigma_{yy} - \lambda\frac{\partial}{\partial y}v - (\lambda + 2\mu)\frac{\partial}{\partial y}v - \lambda\frac{\partial}{\partial z}w &= 0\,, \\
\frac{\partial}{\partial t}\sigma_{zz} - \lambda\frac{\partial}{\partial x}u - \lambda\frac{\partial}{\partial y}v - (\lambda + 2\mu)\frac{\partial}{\partial z}w &= 0\,, \\
\frac{\partial}{\partial t}\sigma_{xy} - \mu(\frac{\partial}{\partial x}v + \frac{\partial}{\partial y}u) &= 0\,, \\
\frac{\partial}{\partial t}\sigma_{xz} - \mu(\frac{\partial}{\partial z}u + \frac{\partial}{\partial x}w) &= 0\,, \\
\frac{\partial}{\partial t}\sigma_{yz} - \mu(\frac{\partial}{\partial z}v + \frac{\partial}{\partial y}w) &= 0\,, \\
\rho\frac{\partial}{\partial t}u - \frac{\partial}{\partial x}\sigma_{xx} - \frac{\partial}{\partial y}\sigma_{xy} - \frac{\partial}{\partial z}\sigma_{xz} &= 0\,, \\
\rho\frac{\partial}{\partial t}v - \frac{\partial}{\partial x}\sigma_{xy} - \frac{\partial}{\partial y}\sigma_{yy} - \frac{\partial}{\partial z}\sigma_{yz} &= 0\,, \\
\rho\frac{\partial}{\partial t}w - \frac{\partial}{\partial x}\sigma_{xz} - \frac{\partial}{\partial y}\sigma_{yz} - \frac{\partial}{\partial z}\sigma_{zz} &= 0\,.
\end{aligned}
$$

Structure of the Hyperbolic System

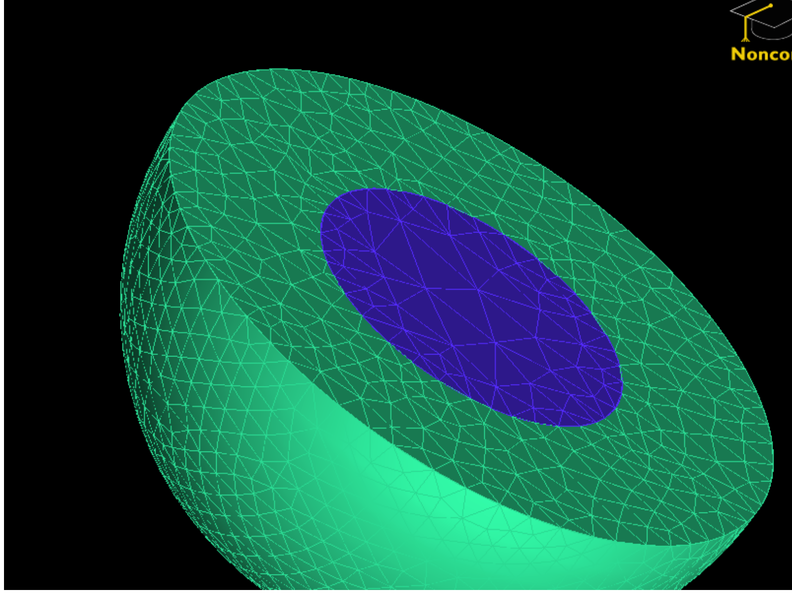
Compact vector-matrix notation gives

$$\frac{\partial Q_p}{\partial t} + A_{pq}\frac{\partial Q_q}{\partial x} + B_{pq}\frac{\partial Q_q}{\partial y} + C_{pq}\frac{\partial Q_q}{\partial z} = 0,$$

with the vector of unknowns and Jacobian matrices

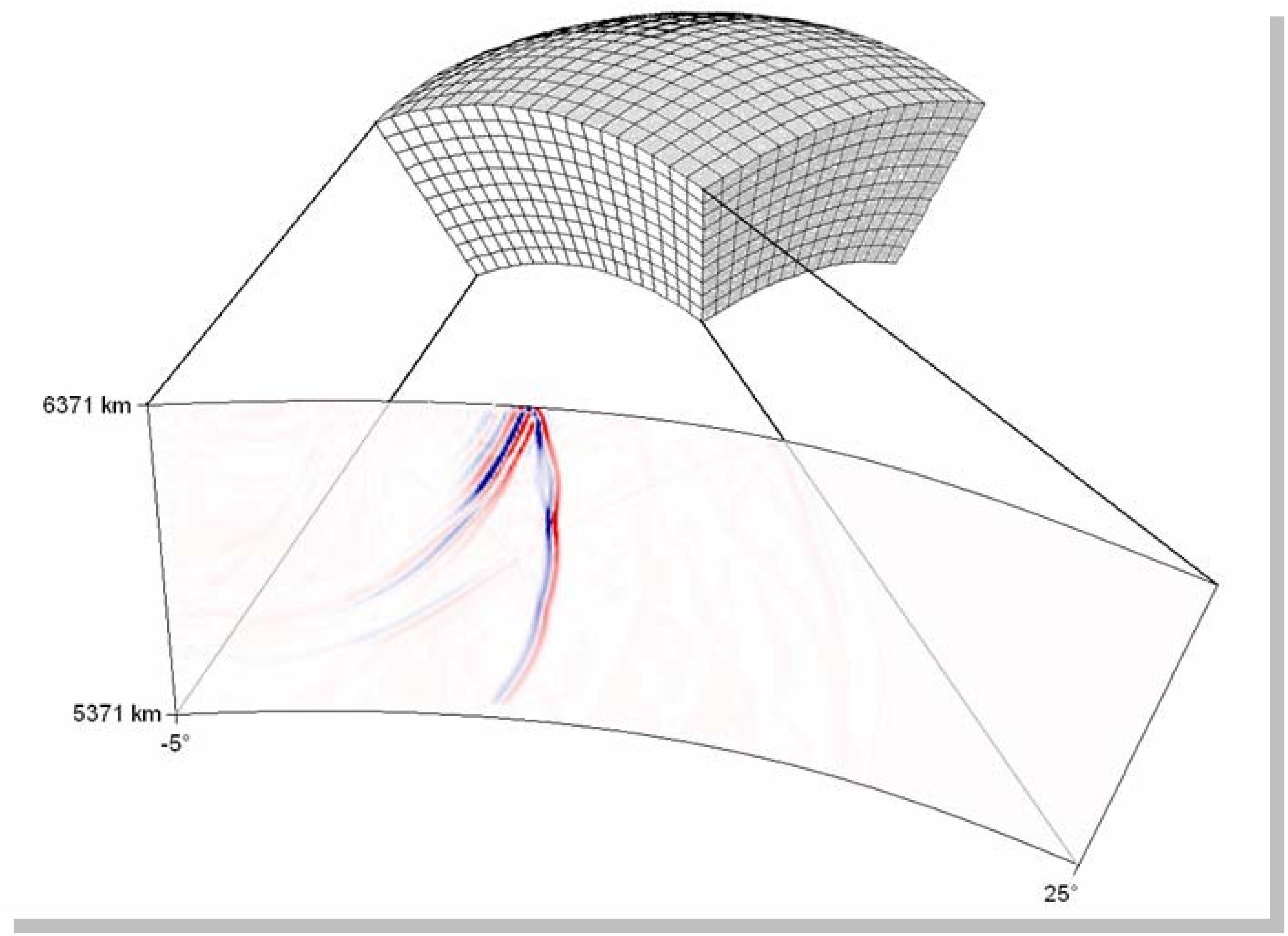
$$Q = \begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{xz} \\ \sigma_{yz} \\ u \\ v \\ w \end{pmatrix}, \qquad C_{pq} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\lambda \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\lambda \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -(\lambda + 2\mu) \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\mu & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{\rho} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{\rho} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{\rho} & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Numerical methods



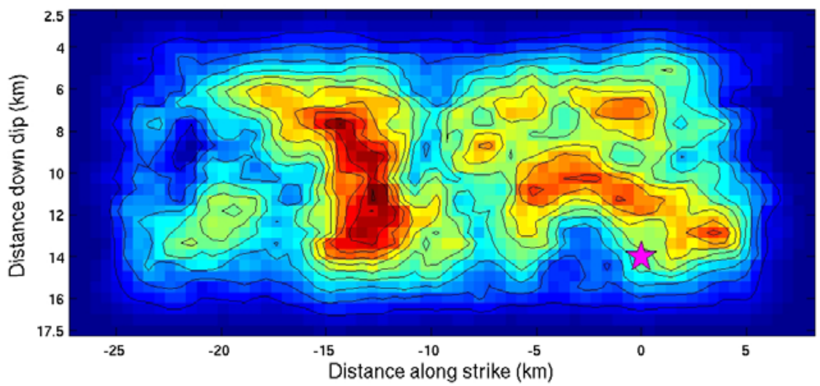
- Finite Differences (high order, optimal operators)
- Pseudospectral methods (Chebyshev, Fourier)
- Finite/**spectral elements** on hexahedral grids
- **Unstructured (tetrahedral) grids** (finite volumes/elements, natural neighbours, **discontinuous Galerkin**) or combinations
- Parallelization using MPI (message passing interface)

Spectral element techniques

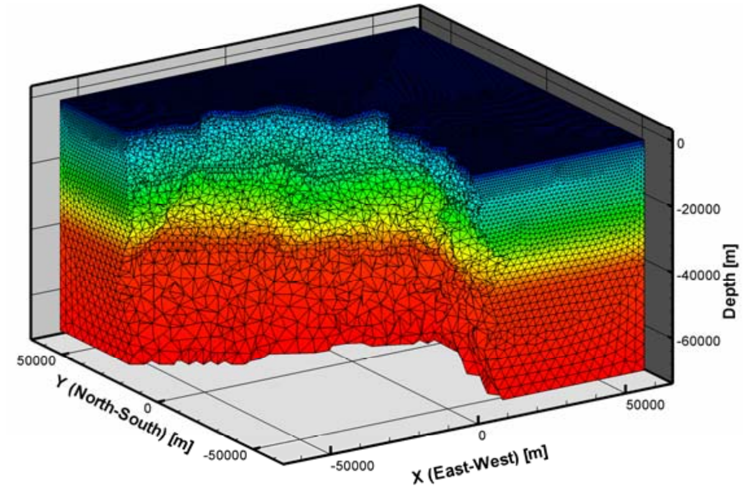


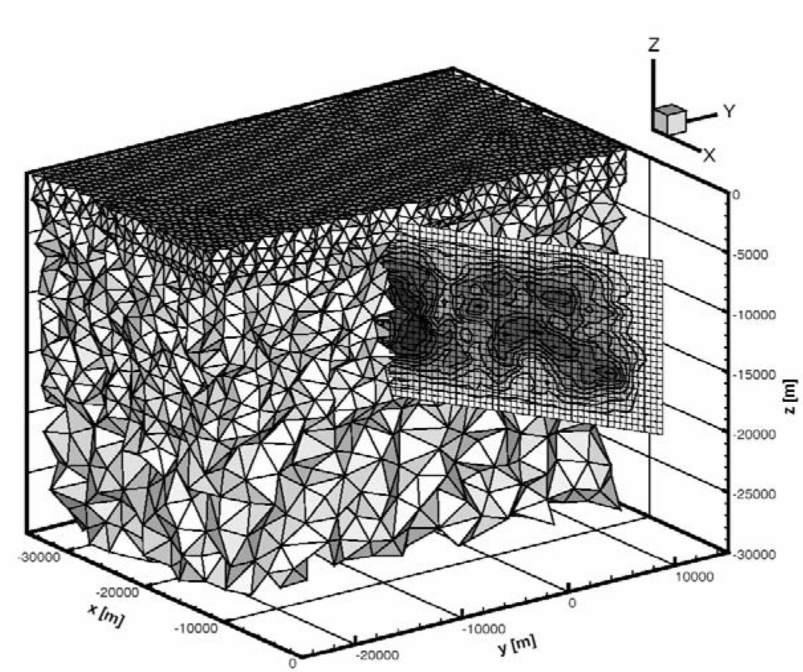
Discontinuous Galerkin Method

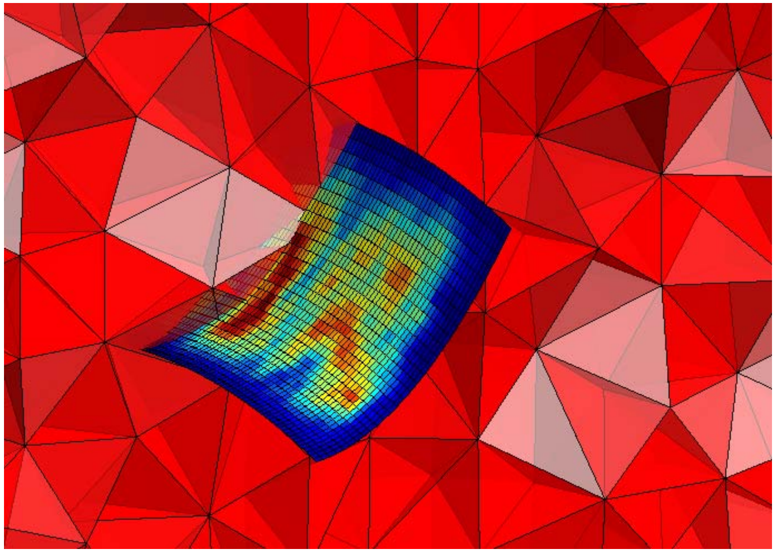
Slip map of an earthquake fault



Mesh spacing is proportional to P-wave velocity

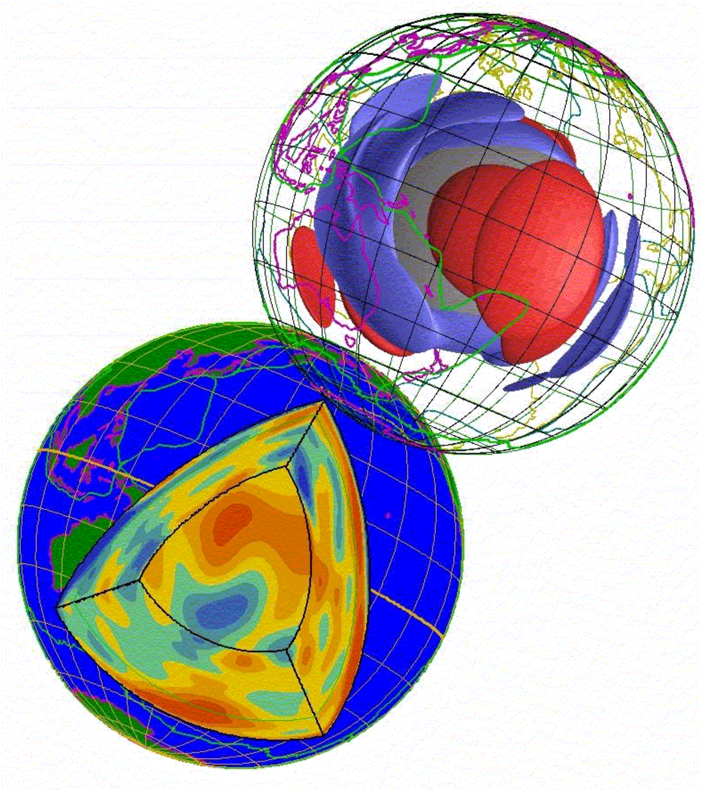






Käser, Mai, Dumbser, 2007

The (structural) inverse problem



Data vector d:

Traveltimes of phases observed at stations of the world wide seismograph network, **now we move to complete waveforms**

Model m:

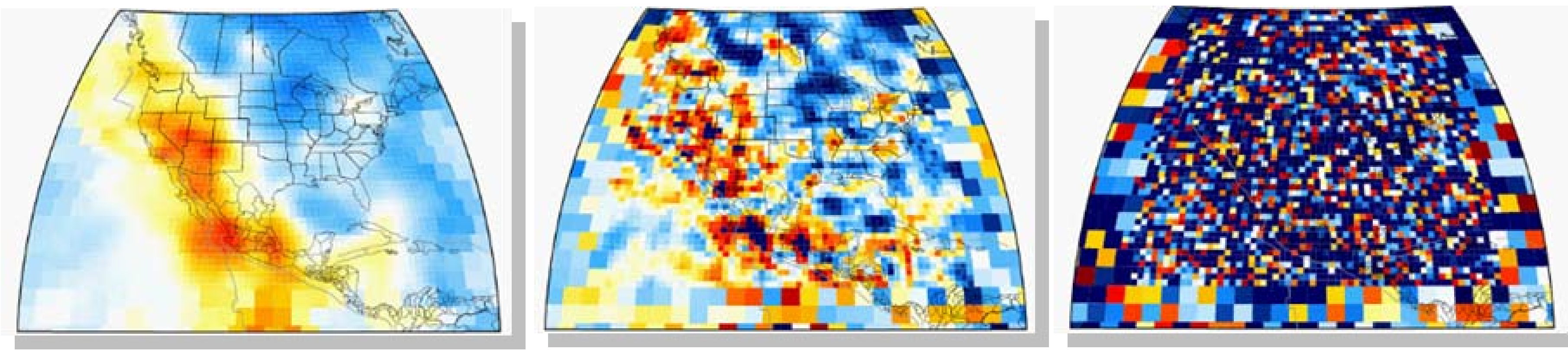
3-D seismic velocity model in the Earth's mantle. Discretization using splines, spherical harmonics, Chebyshev polynomials or simply blocks.

Sometimes 10000s of travel times (**millions of seismogram samples**)

and a large number of model blocks: **underdetermined** system

Model Uncertainties – Degrees of Freedom

Decreasing misfit



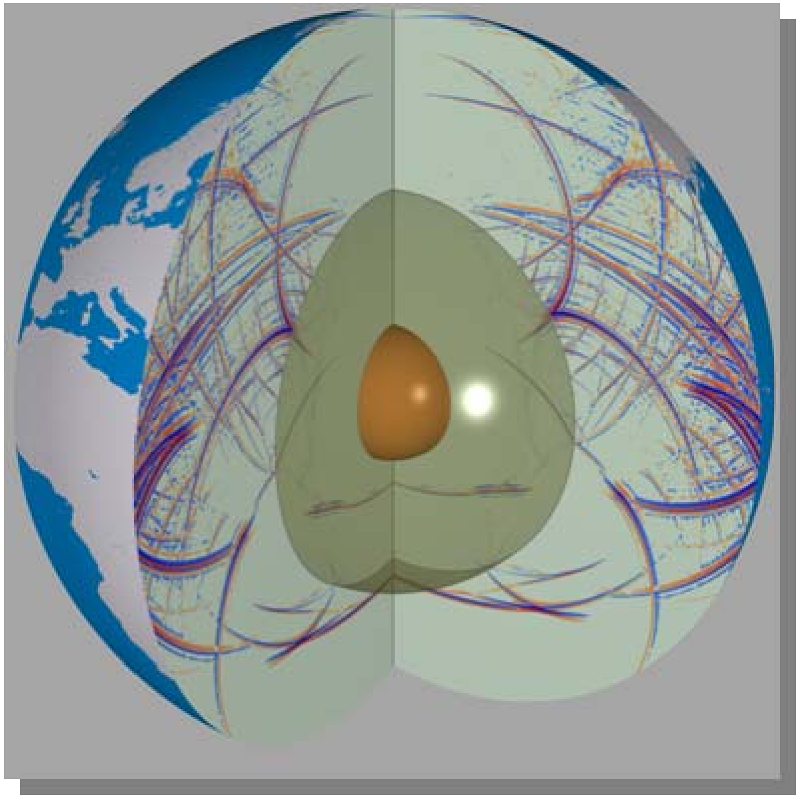
Increasing model complexity

Increasing number of degrees of freedom

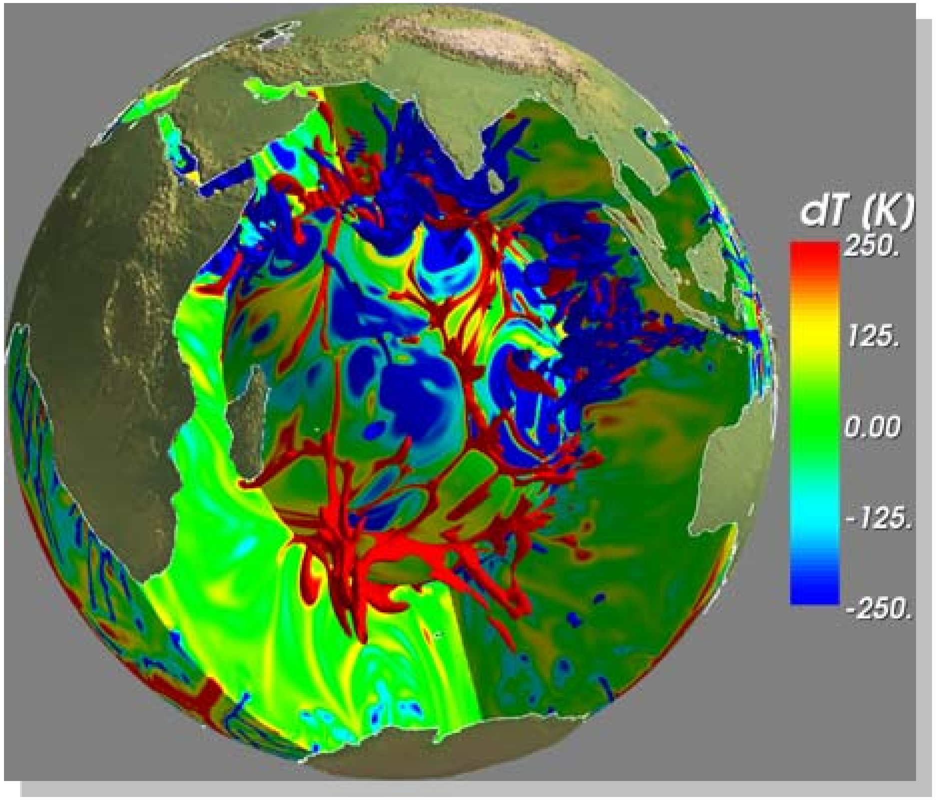
after L. Boschi (2007)

Scientific problems

Geodynamics



Courtesy: G. Jahnke



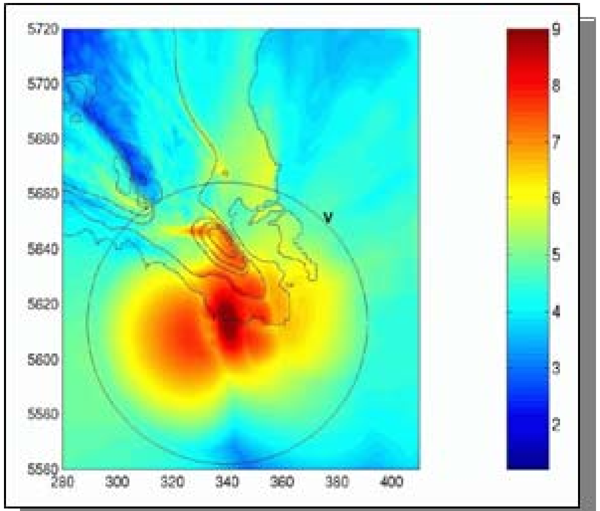
Courtesy: H.P. Bunge, B. Schuberth

scientific problems

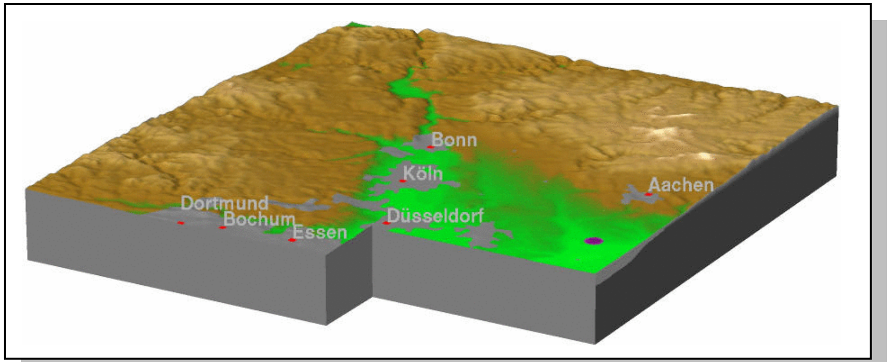
Earthquake scenarios

- Accurate forecasting of hazard and risk scenarios for specific regions and time intervals
- Incorporation of earthquake scenario simulations into probabilistic hazard analysis

Shaking hazard



M5.9 Roermond 1992



… and now …

- Andreas Fichtner: seismic waveform inversion as an adjoint problem
- Heiner Igel: probabilistic desccription of inverse problems – Monte Carlo Methods
- Karin Fichtner: finite frequency tomography and relevance for geodynamic issues

… some key questions …

related to waveform inversion

- How can we properly quantify uncertainties of the inverted model(s)
- How can we properly visualize uncertainties in large-dimensional model spaces
- Can we quantitatively describe prior information on Earth‘s structure?
- Should we find optimal parameterizations of Earth models?