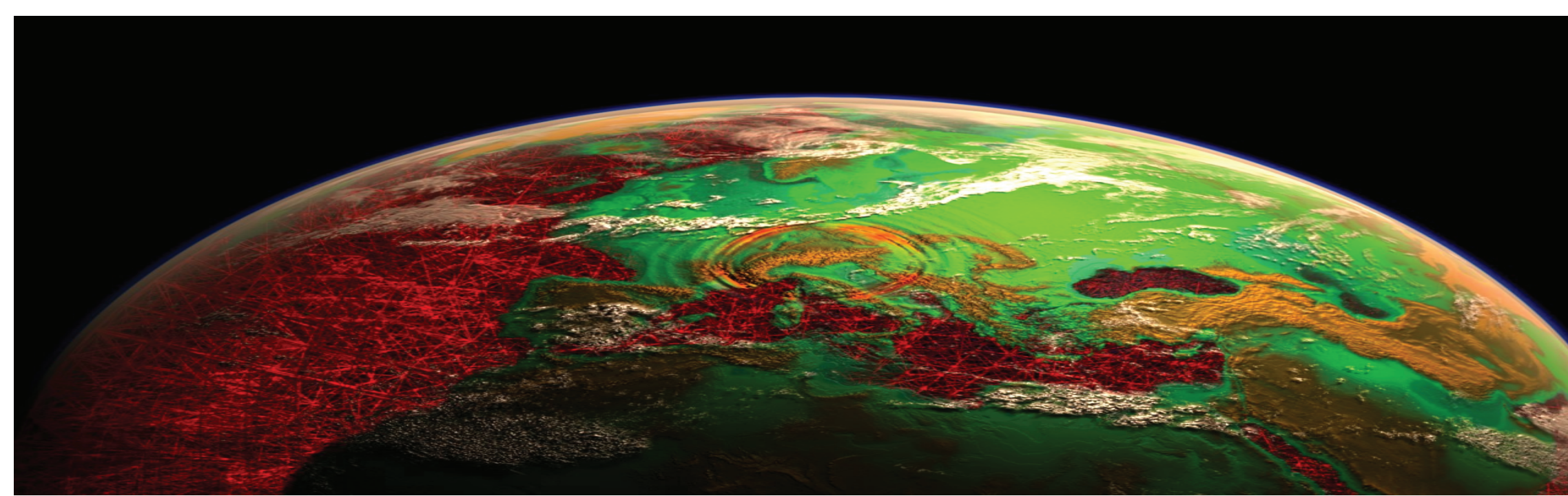


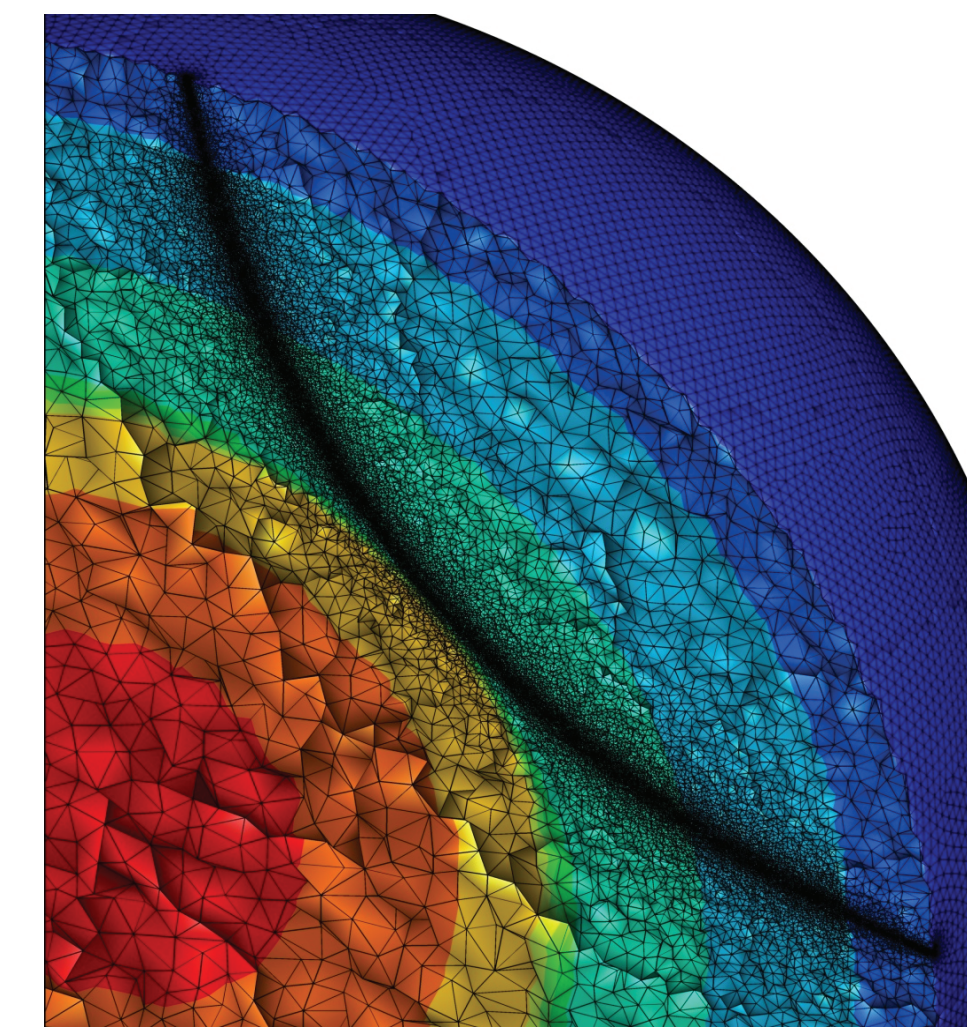
## 1 Introduction

We highlight features of the **Discontinuous Galerkin (DG)** approach applied to seismic wave propagation and earthquake rupture (e.g., **adapting polynomial order** in the elements and using **local time steps**). As the method is implemented for tetrahedral grids, the meshing of complex geometries – given appropriate surface representations – is straight forward. We show applications in reservoir problems, dynamic rupture simulations, as well as regional and global seismology.

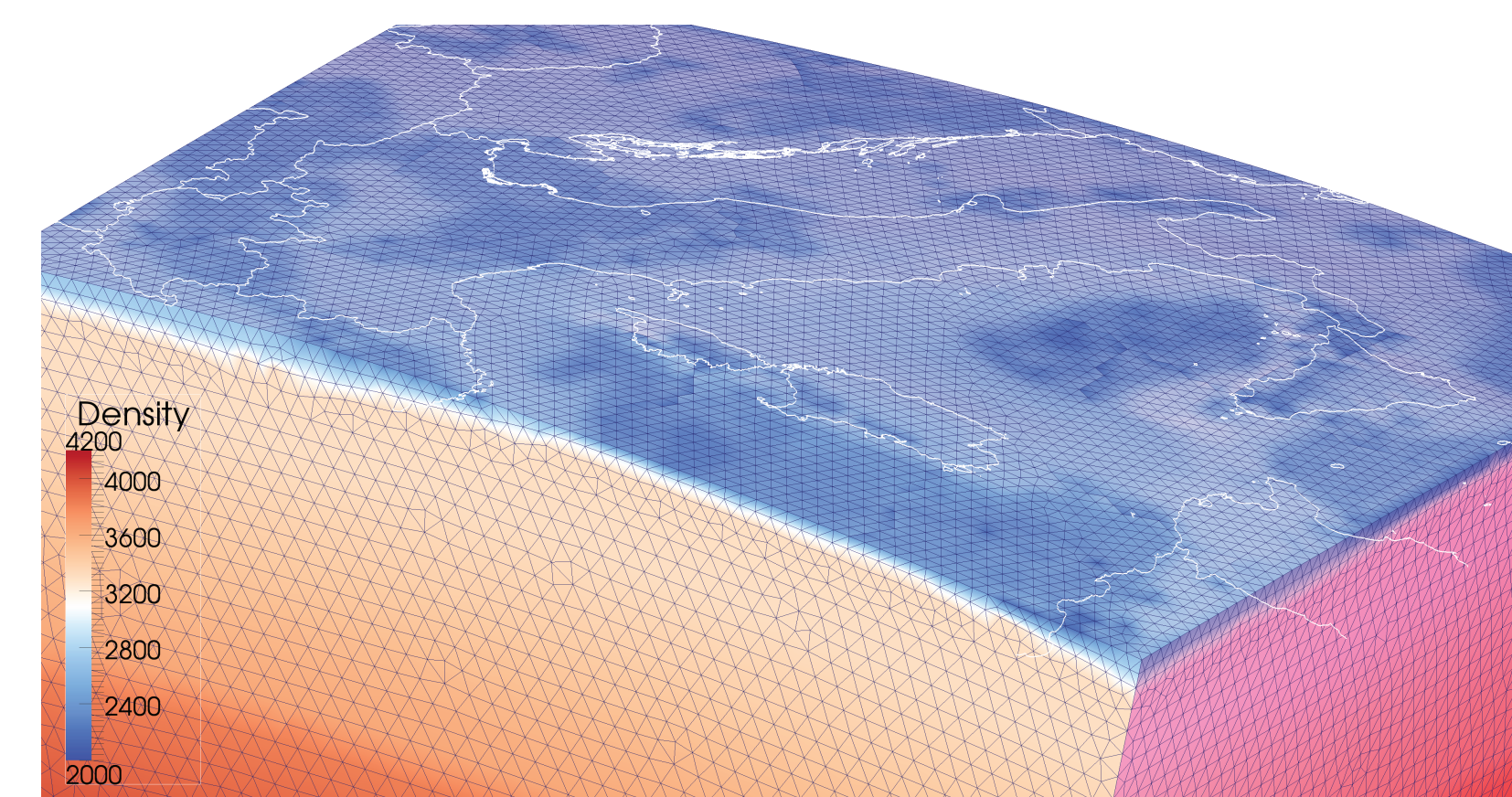


## 3 Global and regional wave propagation

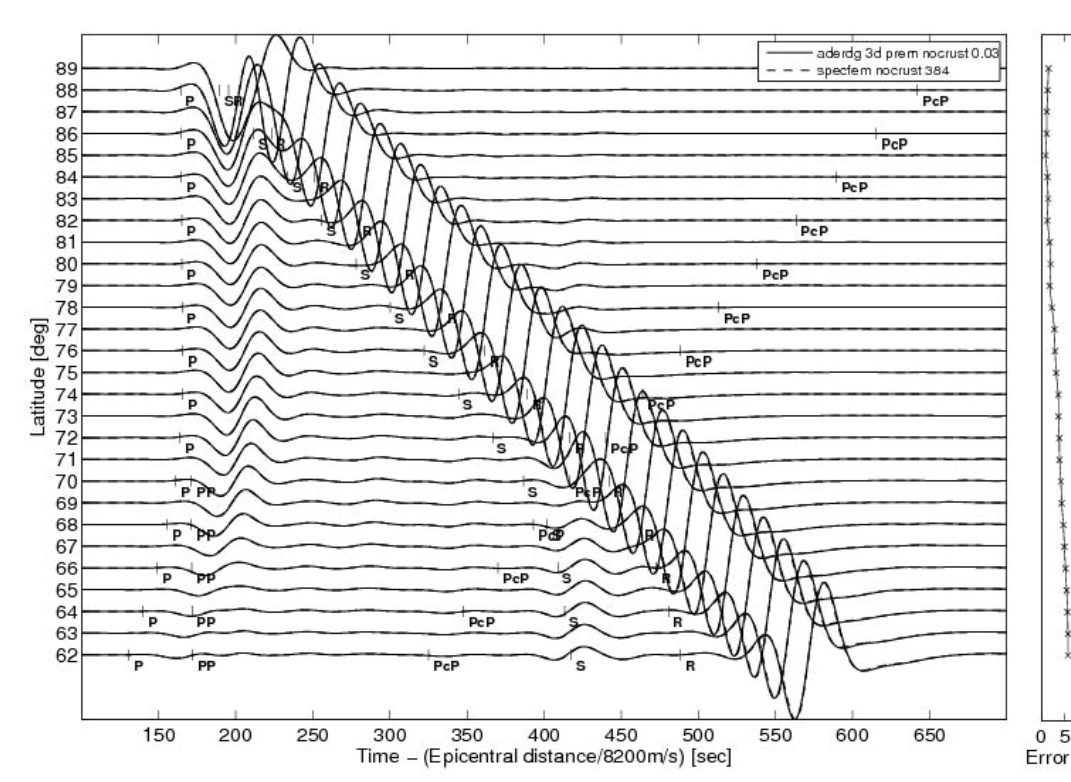
We investigate the applicability of DG to global and regional wave propagation. The tetrahedral grid density is easily adapted to specific requirements (e.g., velocity model, region of interest). We expect benefits of this approach when regions need to be meshed with very fine detail (e.g., subduction zones, crustal structures, topographic features).



Adaptive mesh for global kernel



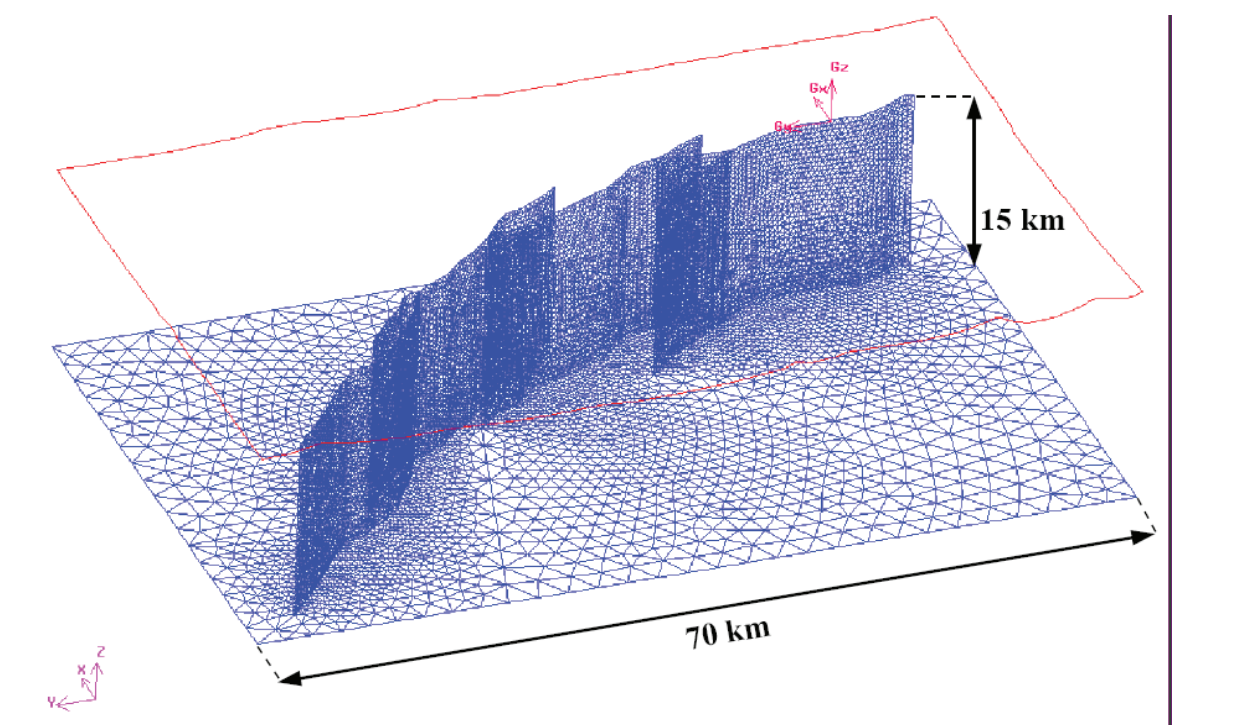
Model with 3-D European crust



Comparisons with specfem3d

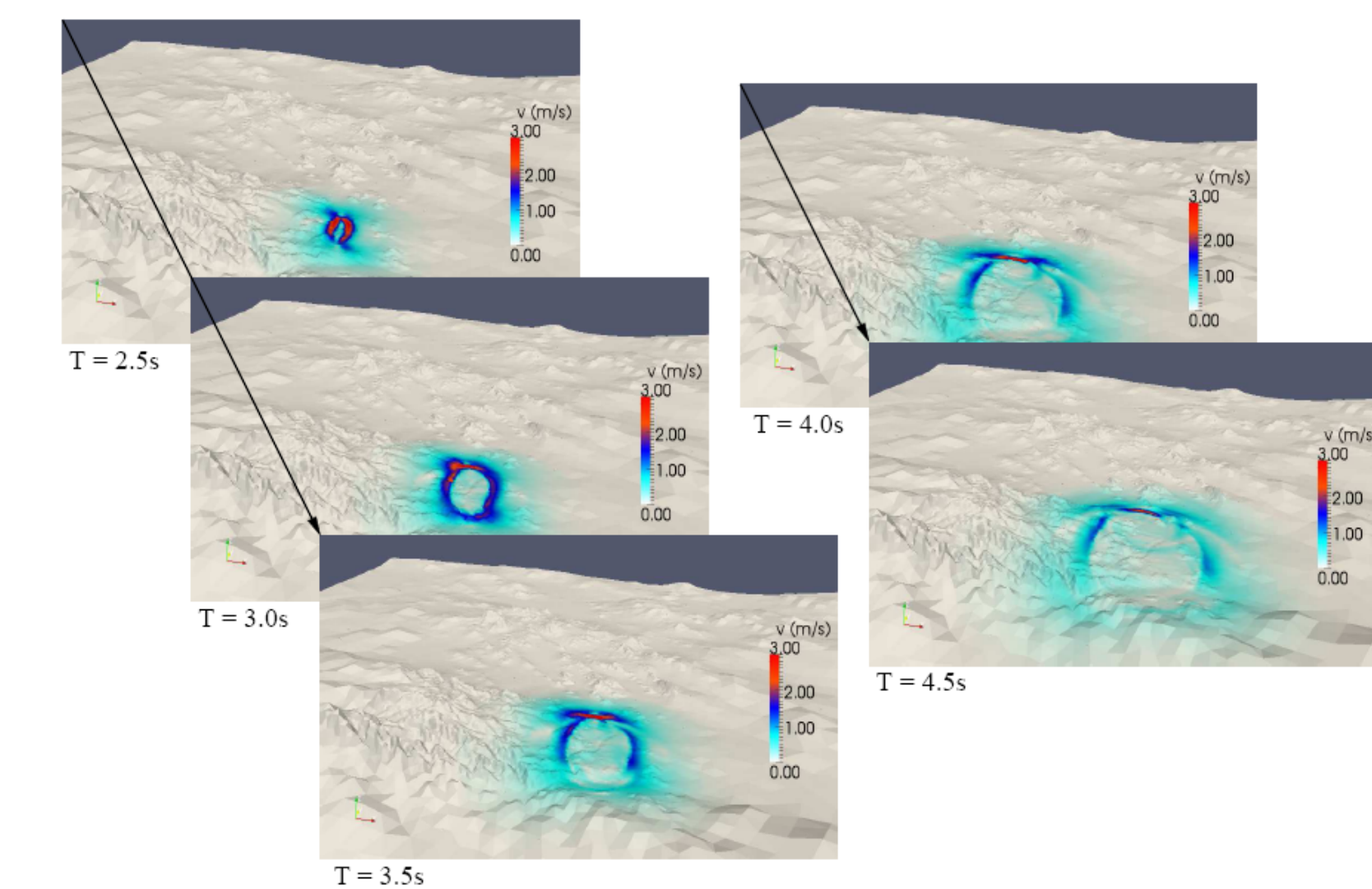
## 5 Dynamic rupture simulations

DG is extremely well suited for the numerical solution of the dynamic rupture problem. Recent results show that the scheme is superior to other numerical approximations as spurious oscillations in the solutions can be omitted without further damping term.



Mesh for Landers earthquake

The Landers simulation demonstrates the benefits of the proposed method based on unstructured tetrahedral meshes that can be aligned into merging faults under shallow angles. Areas of interest - here the topography and the fault - can be modeled adequately by small elements while mesh coarsening can be applied elsewhere.

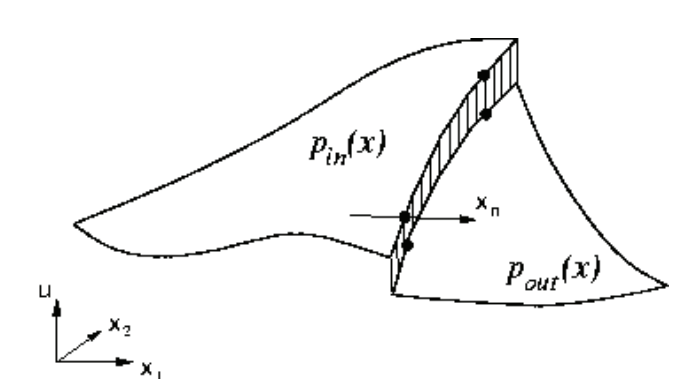


Snapshots for Landers simulation

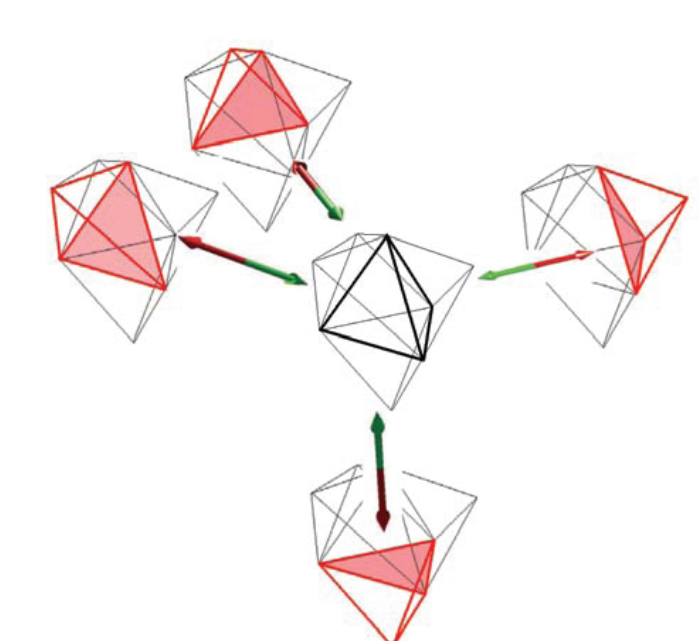
See also talk by C. Pelties et al. S54C-08, Friday, 5:45, Room 2005

## 2 Discontinuous Galerkin Method

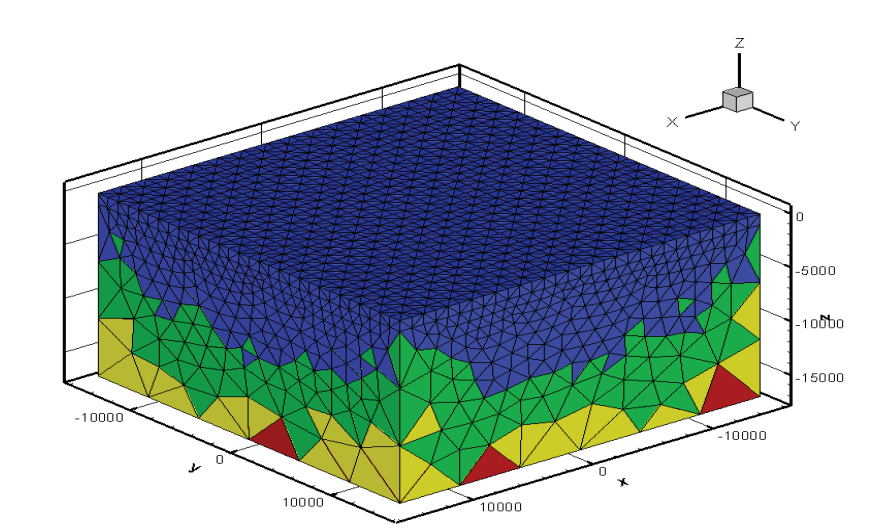
In contrast to classic finite element schemes the fields are allowed to be discontinuous across the element edges. This discontinuities are corrected for by a flux scheme. In the case of rupture propagation this inherent discontinuous behaviour allows accurate implementation of frictional boundary conditions. The scheme is local (excellent scaling on parallel hardware), the polynomial degree can vary in each cell (p-adaptivity) and the time step can be locally adapted. This creates a challenge to load balance the algorithm.



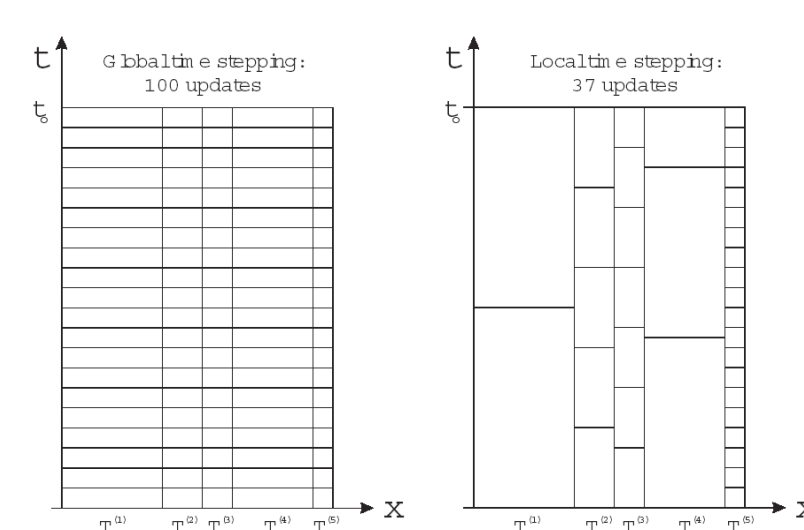
Discontinuous fields



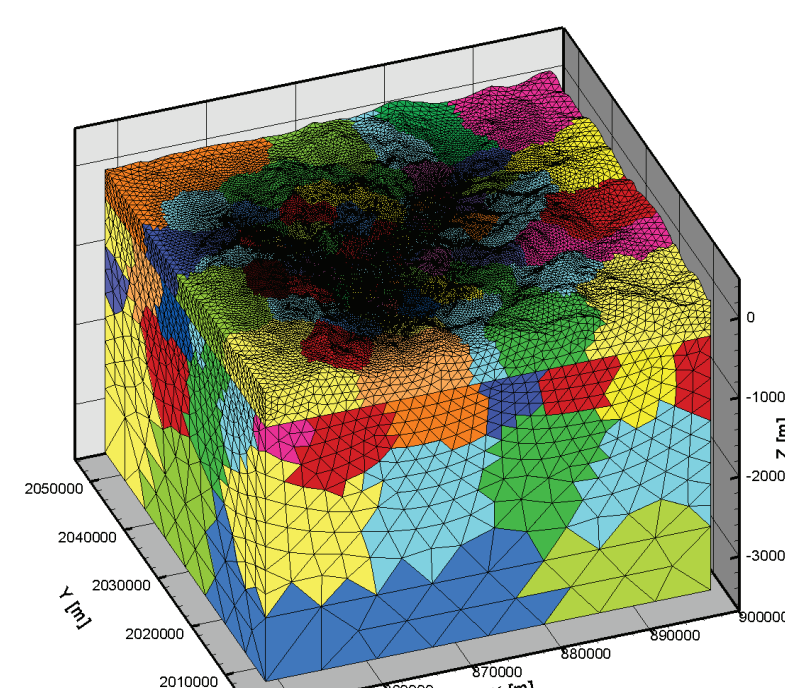
Local flux scheme



P-adaptivity



Local time stepping

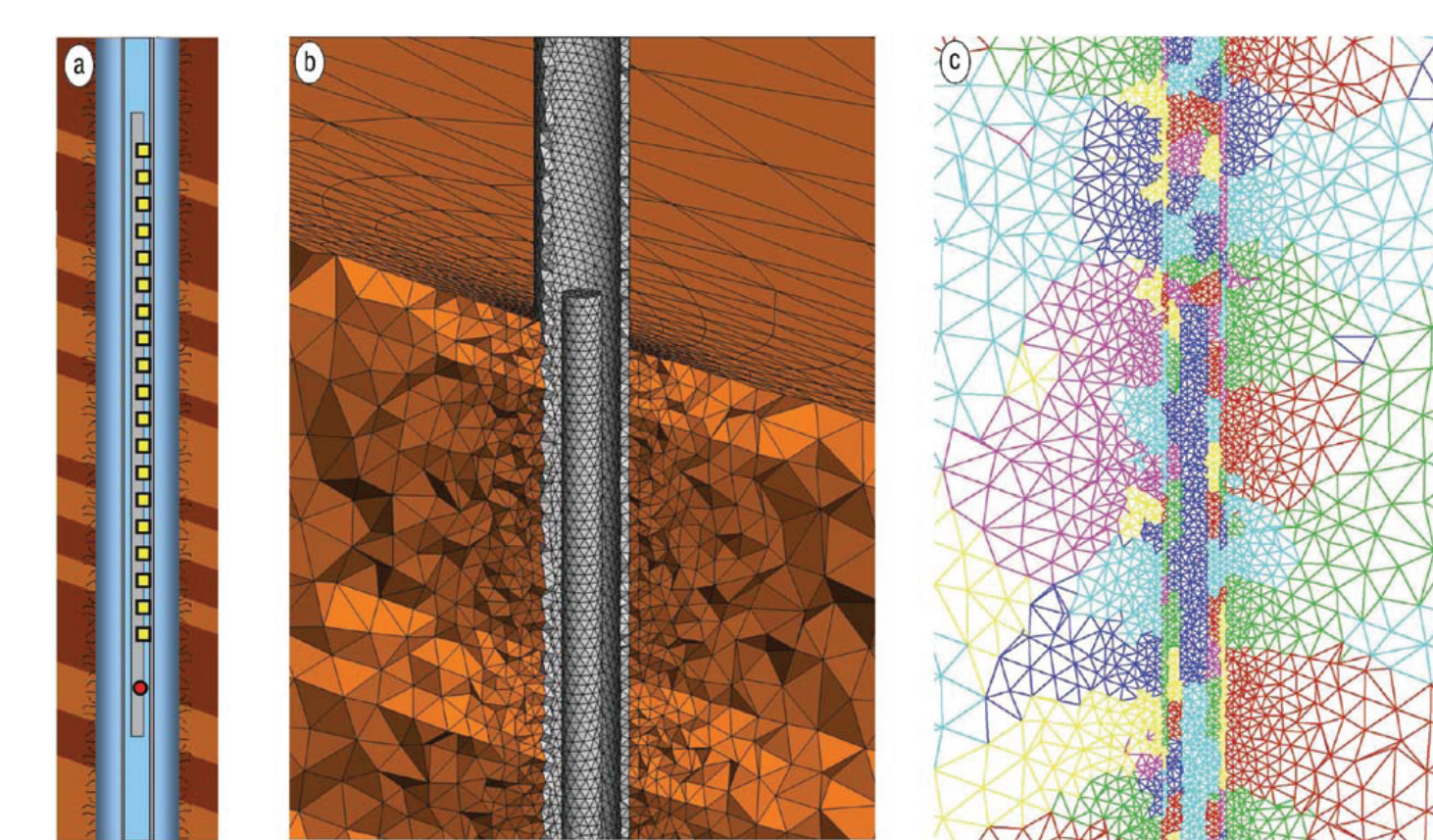


Mesh partitioning

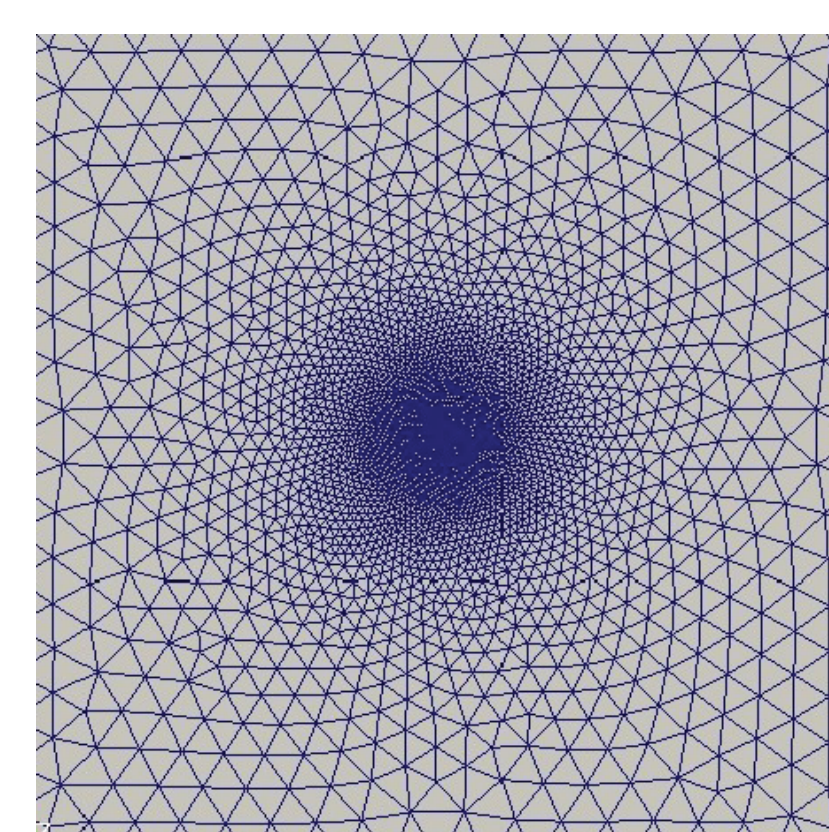
## 4 Industrial applications

One of the attractive features of tetrahedral grids is that spherical or cylindrical geometries can be easily meshed inside 3-D structures.

This led to applications in connection with wave propagation originating from downhole seismic sources. The size of the tetrahedra varies by two orders of magnitude.



Mesh of a borehole, and surrounding structure

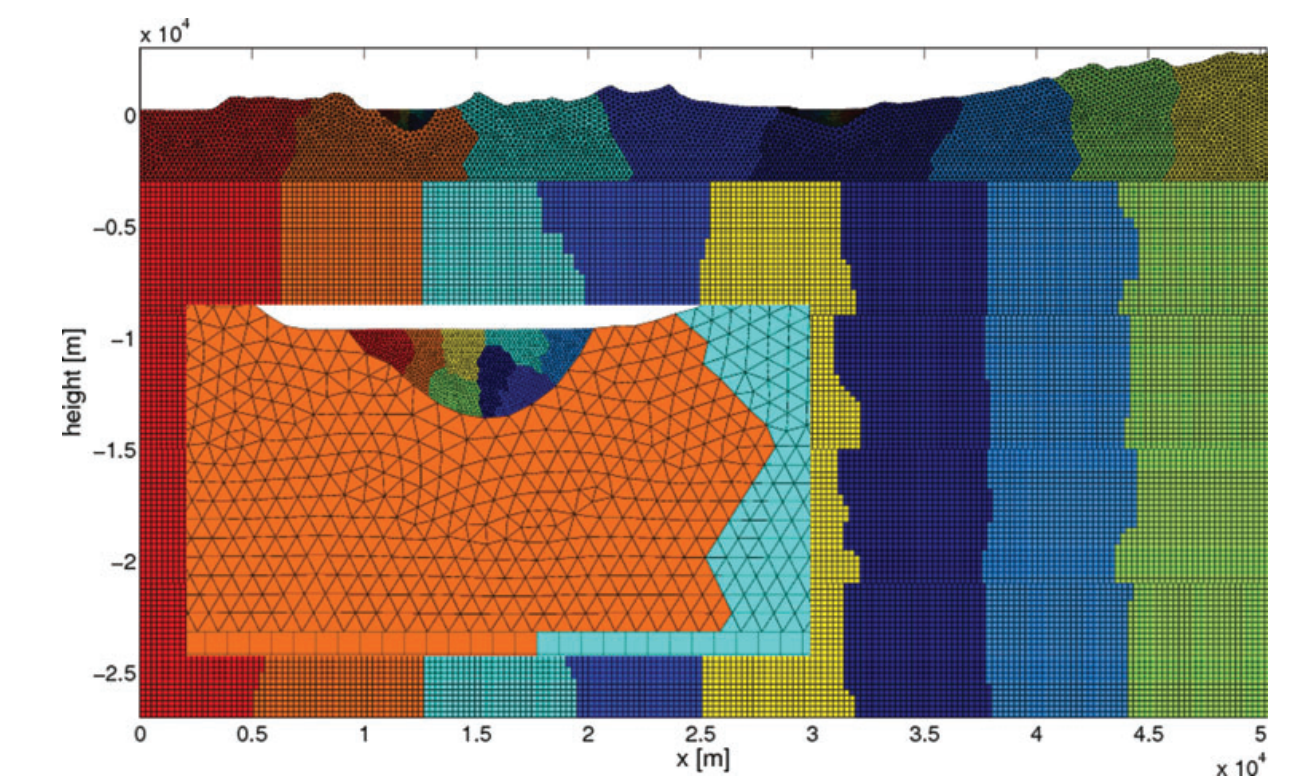
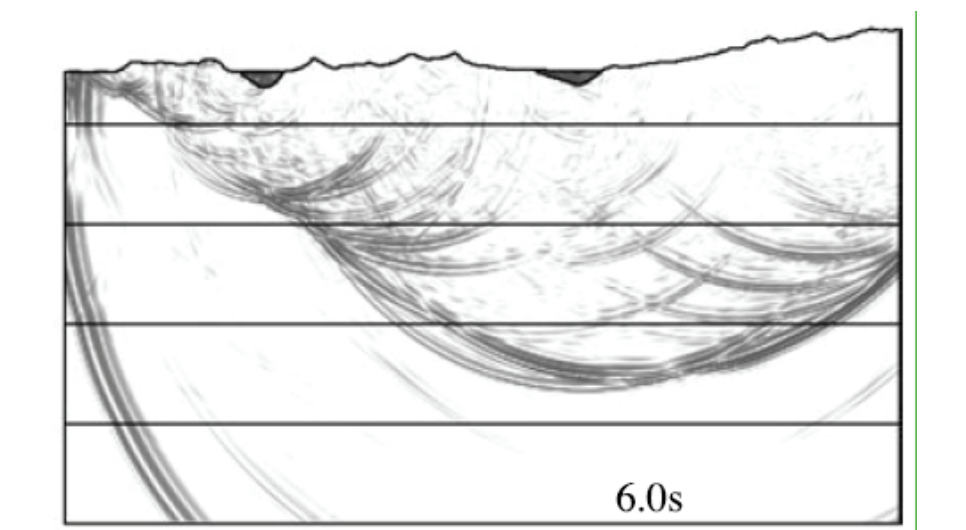


Borehole refinement

The rheologies incorporated in the DG-based programme SEISSOL included viscoelastic, fully anisotropic behaviour and poroelasticity.

## 6 Combining mesh types

In many situations it would make sense to have tetrahedral meshes in the complex regions of interest and hexahedral meshes elsewhere. This concept was tested in 2-D. It could be shown that the accuracy can be preserved and that there are no artefacts due to the mesh type transitions. This shall be extended to 3-D in the future.



Mixing tetrahedral and hexahedral meshes

**Recent Papers using Seissol (selection)**  
 Käser, M. et al. (2010), Wave Field Modeling in Exploration Seismology Using the Discontinuous Galerkin Finite Element Method on HPC-Infrastructure, *The Leading Edge*, 29, 76-85.  
 Pelties, C. et al. (2010), Regular versus irregular meshing for complicated models and their effect on synthetic seismograms, *Geophys. J. Int.*, doi:10.1111/j.1365-246X.2010.04777.x.  
 De la Puente, J. et al. (2009), Dynamic Rupture Modeling on Unstructured Meshes Using a Discontinuous Galerkin Method, *J. Geophys. Res.*, 114, B10302, doi:10.1029/2008JB006271.  
 Hermann, V. et al. (2011), Non-conforming hybrid meshes for efficient 2D wave propagation using the Discontinuous Galerkin method, *Geophys. J. Int.*, 184(2), 746-758, doi:10.1111/j.1365-246X.2010.04858.x.  
 Pelties, C. et al. (2012), Three-Dimensional Dynamic Rupture Simulation with a High-order Discontinuous Galerkin Method on Unstructured Tetrahedral Meshes, *J. Geophys. Res.*, submitted.