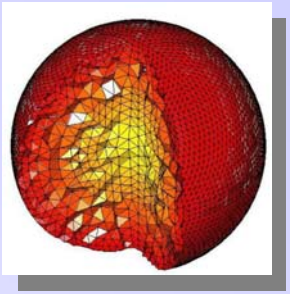
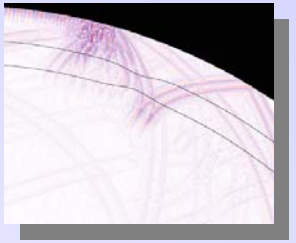
Numerical methods in the Earth Sciences:
seismic wave propagation

Heiner Igel, LMU Munich

III The latest developments, outlook

- Grenoble Valley Benchmark
- Waves on unstructured grids
- The SPICE library





3D numerical simulation of seismic wave propagation in the Grenoble valley (M6 earthquake)

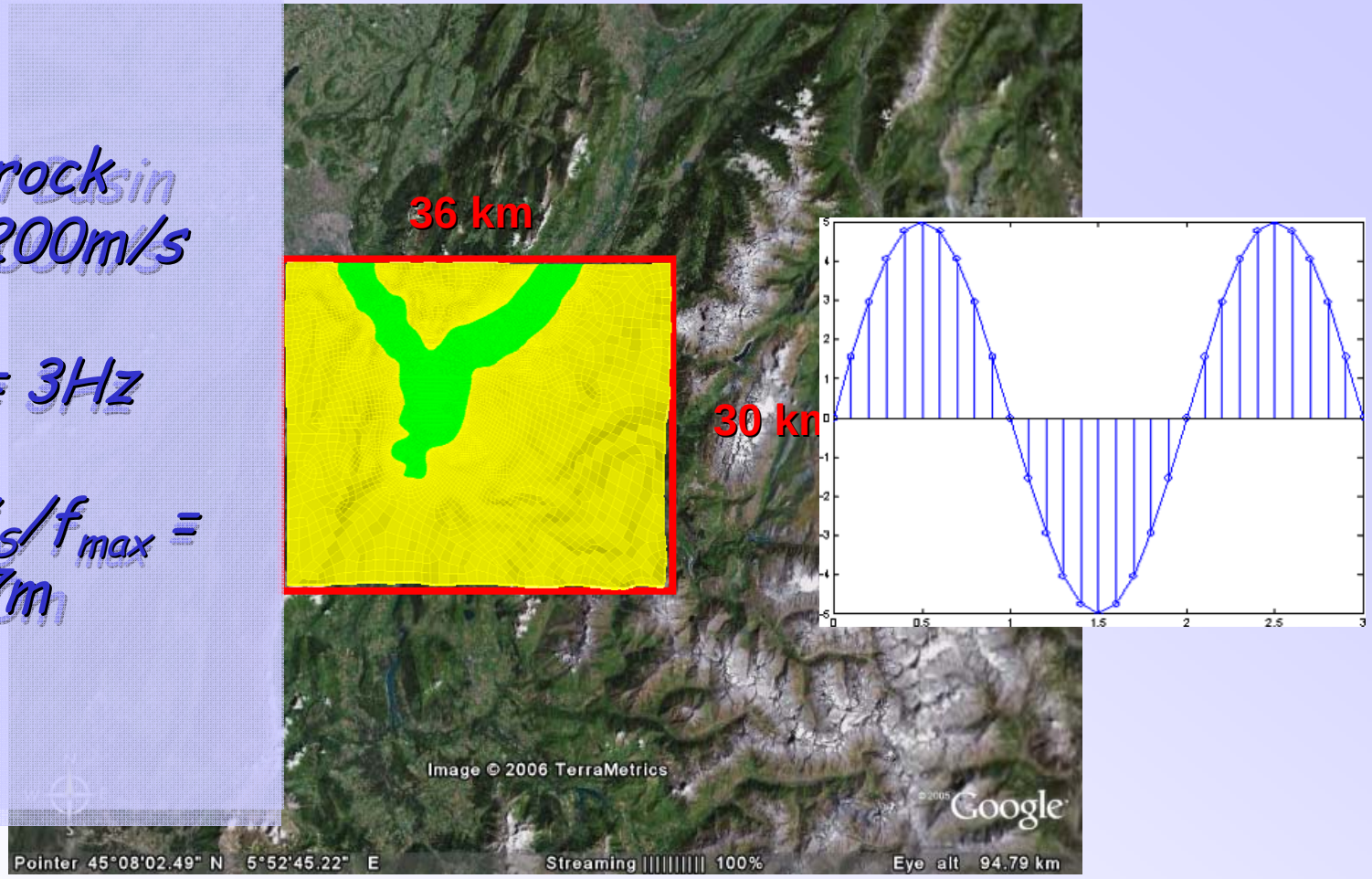
Forward modeling benchmark (Chaljub et al., 2006)

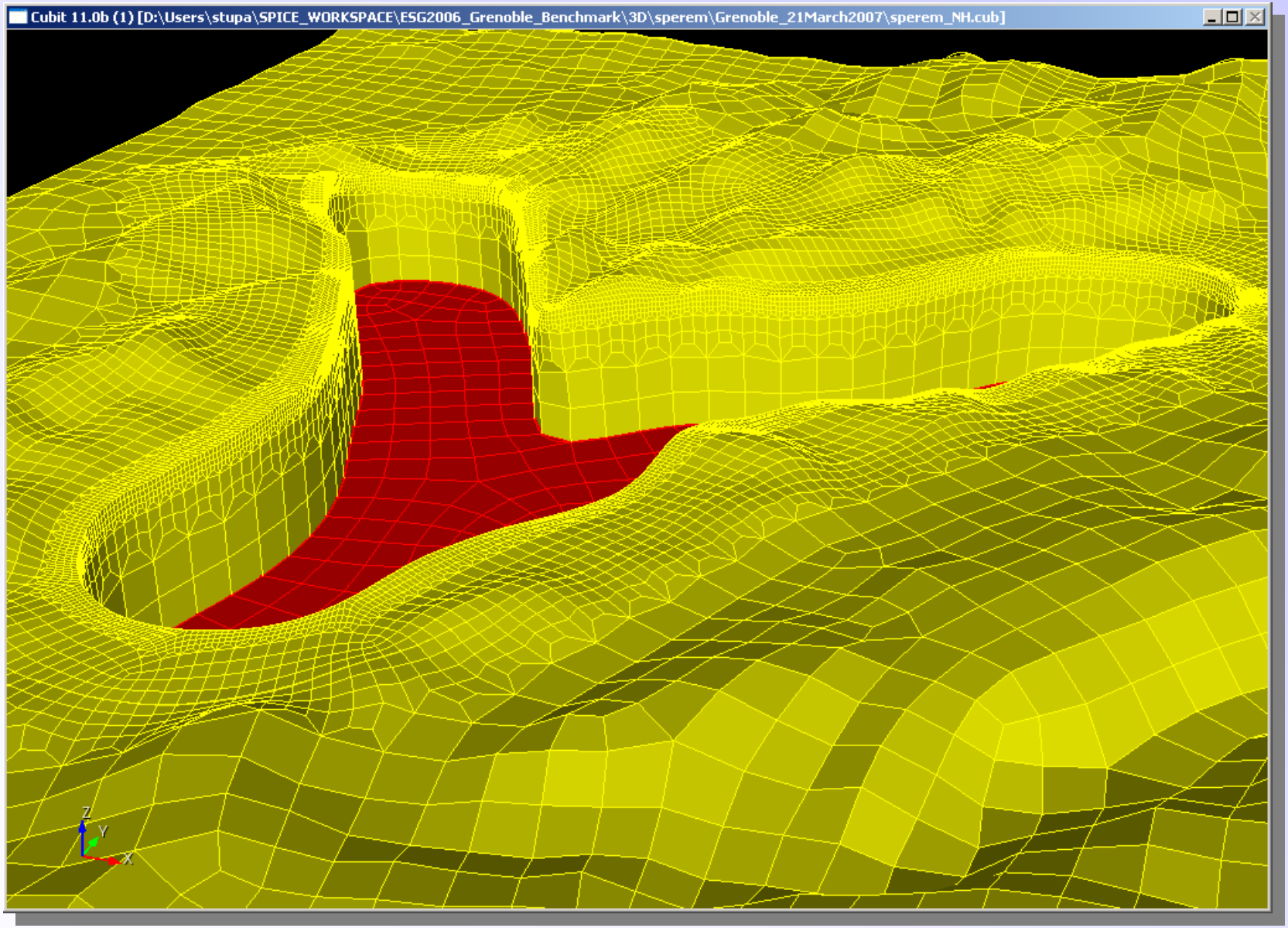
3D numerical simulation of seismic wave propagation in the Grenoble valley (M6 earthquake)

Bedrock
$V_S = 3200 m/s$

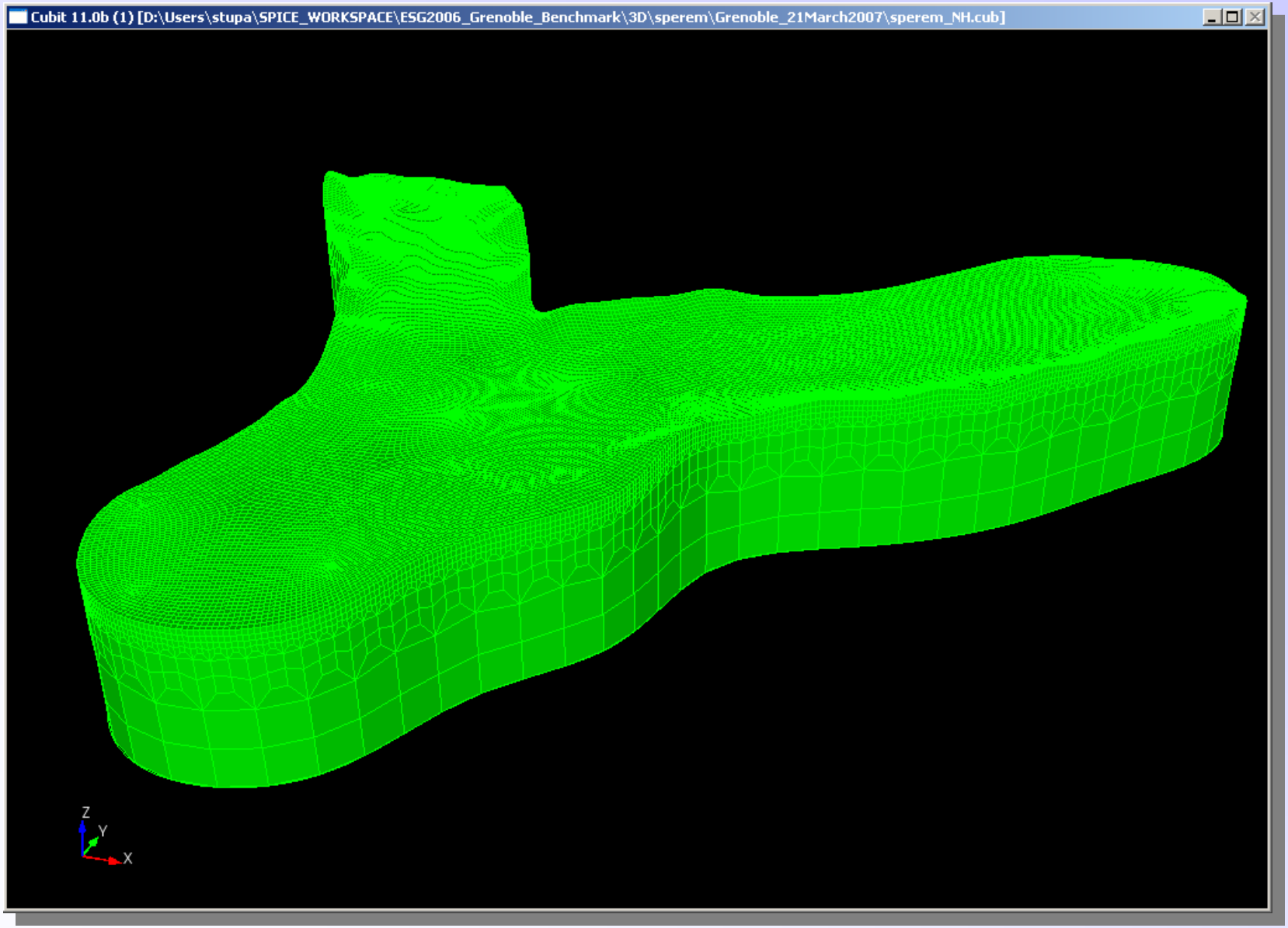
$f_{max} = 3Hz$

$\lambda_{min} = V_S / f_{max} = 1066.7m$



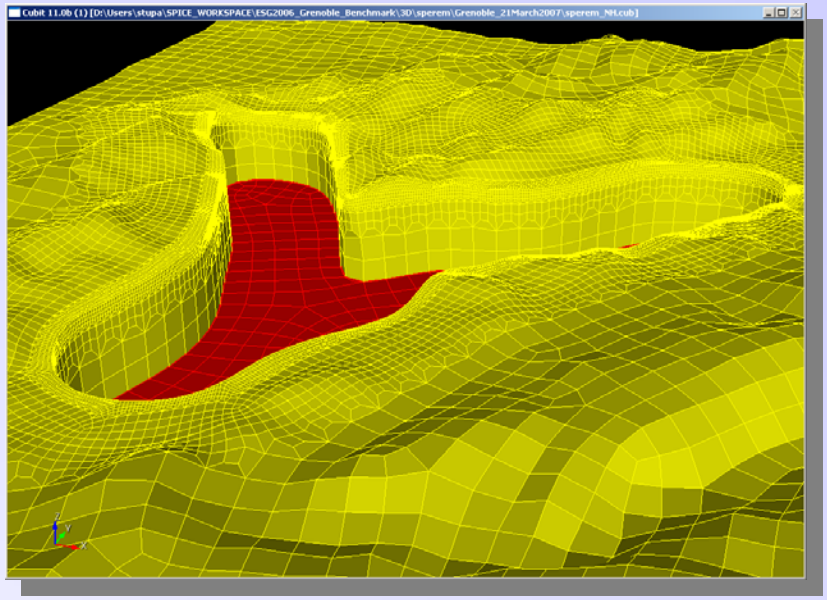


Stupazzini et al. (2006)


Cubit 11.0b (1) [D:\Users\stupa\SPICE_WORKSPACE\ESG2006_Grenoble_Benchmark\3D\sperem\Grenoble_21March2007\sperem_NH.cub]
Z
Y
X

The Courant Criterion

$$v_P \left(\frac{dt}{dx} \right) \leq \varepsilon$$



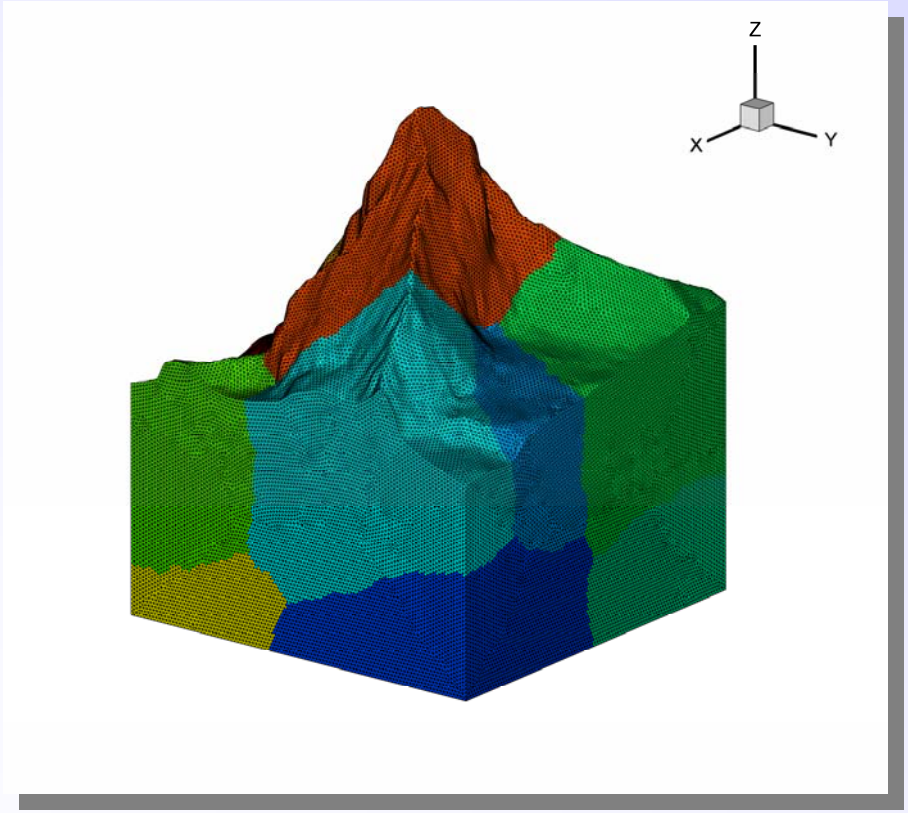
Largest velocity

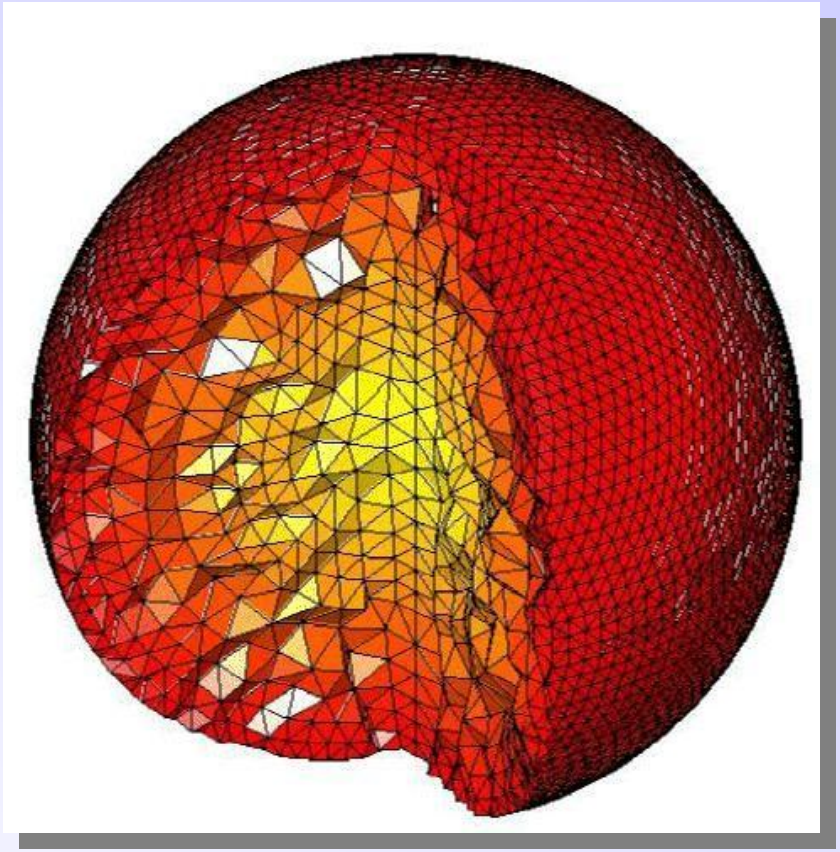
Smallest grid size

Problems …

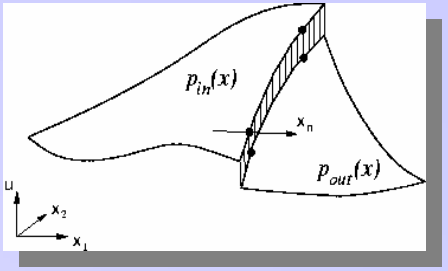
- … **grid generation** is cumbersome with hexahedra, trying to honor complex geometries and material heterogeneities …
- … large variations in seismic velocities (i.e. required grid size) lead to **very small time steps** – overkill in a large part of the model …

Waves on unstructured grids?
tetrahedral

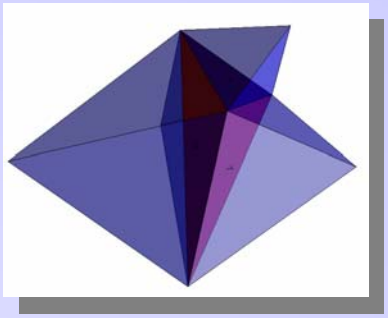


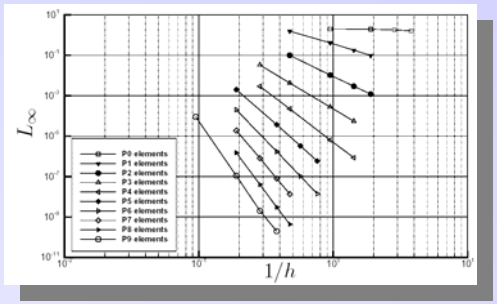


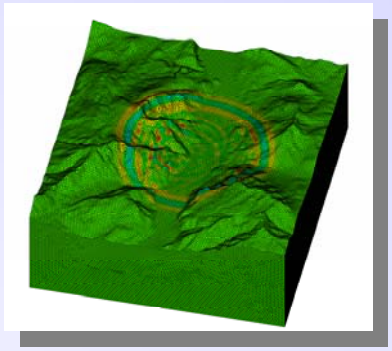
Arbirtrarily high-orDER - Discontinuous Galerkin



- Combination of a **discontinous Galerkin method** with ADER time integration
- Piecewise polynomial approximation combined with the fluxes across elements (**finite volumes**)
- **Time integration as accurate as space derivatives**, applicable also to strongly irregular meshes (not so usually for FD, FE, SE)
- Method developed in **aero-acoustics** and **computational fluid dynamics**
- The scheme is entirely local, not large matrix inversion -> **efficient parallelization**
- Algorithms on tetrahedral grids **slower** than spectral element schemes on hexahedra







ADER-DG in *Geophysical Journal International* a.o.

Käser, M., and M. Dumbser (2006), An Arbitrary High Order Discontinuous Galerkin Method for Elastic Waves on Unstructured Meshes I: **The Two-Dimensional Isotropic Case** with External Source Terms, *Geophysical Journal International*, *166*(2), 855-877.

Dumbser, M., and M. Käser (2006), An Arbitrary High Order Discontinuous Galerkin Method for Elastic Waves on Unstructured Meshes II: **The Three-Dimensional Isotropic Case**, *Geophysical Journal International*, *167*(1), 319-336.

Käser, M., M. Dumbser, J. de la Puente, and H. Igel (2007), An Arbitrary High Order Discontinuous Galerkin Method for Elastic Waves on Unstructured Meshes III: **Viscoelastic Attenuation**, *Geophysical Journal International*, *168*, 224-242.

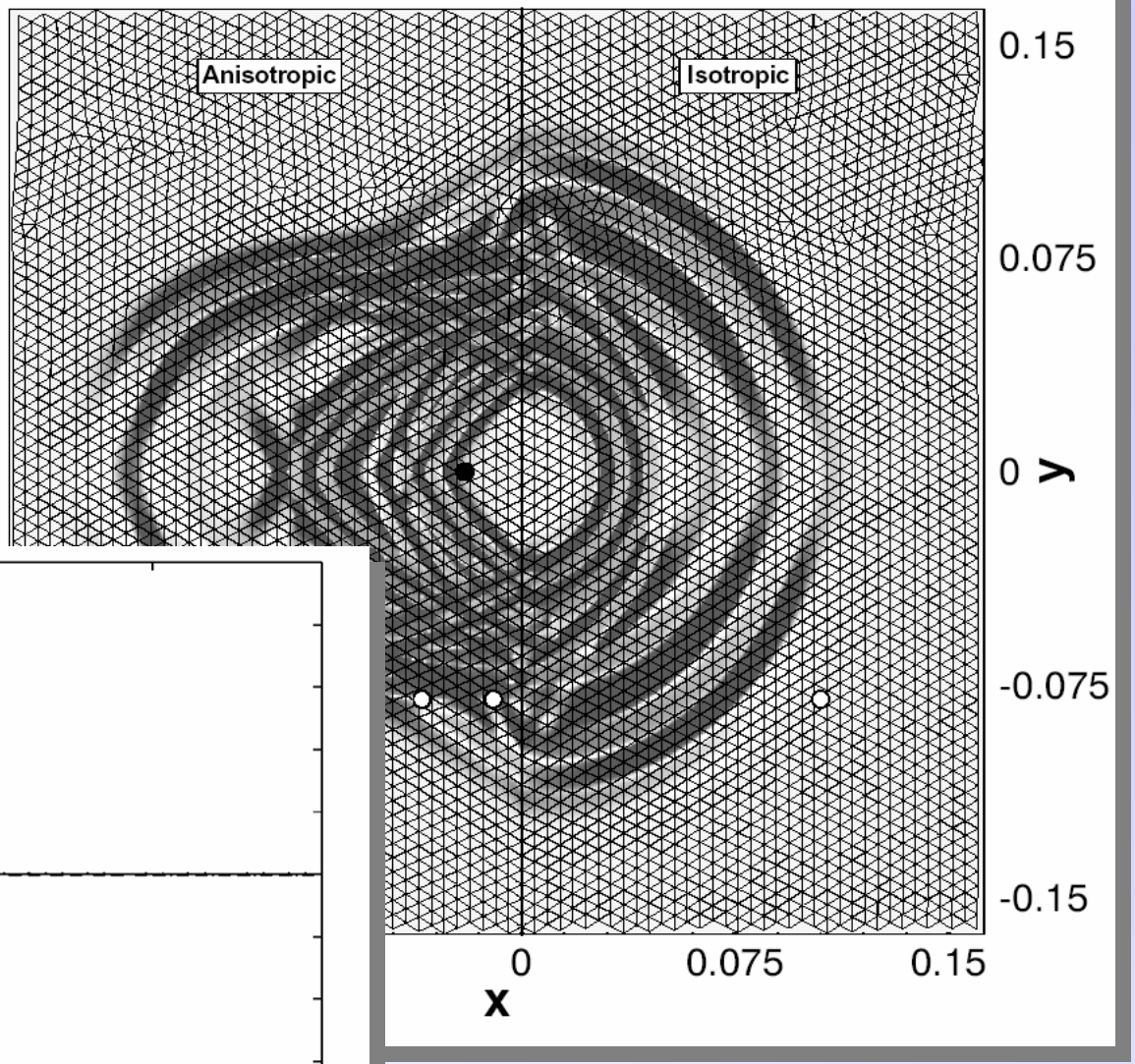
De la Puente, J., M. Käser, M. Dumbser, and H. Igel (2007), An Arbitrary High Order Discontinuous Galerkin Method for Elastic Waves on Unstructured Meshes IV: **Anisotropy**, *Geophysical Journal International*, in press.

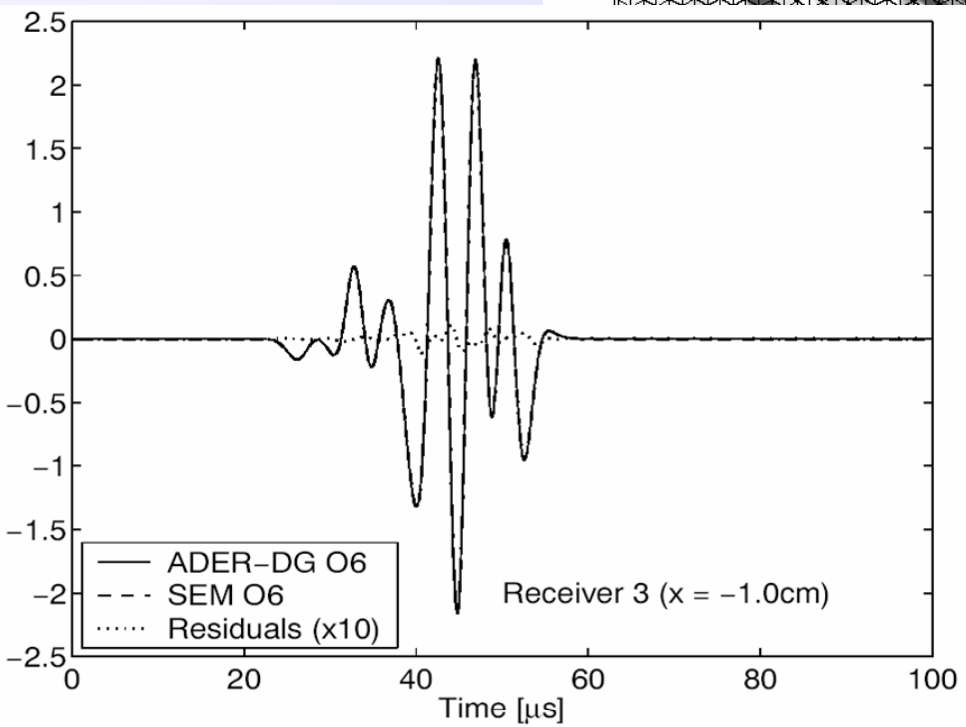
Dumbser, M, M. Käser, and E Toro (2007), An Arbitrary High Order Discontinuous Galerkin Method for Elastic Waves on Unstructured Meshes V: **Local Time Stepping and p-Adaptivity**, *Geophys. J. Int.*, in press

Käser, M., P. M. Mai, and M. Dumbser (2007), On the Accurate Treatment of **Finite Source Rupture Models** Using ADER-DG on Tetrahedral Meshes, *Bull. Seis. Soc. Am.*, in press.

Coming soon: poroelasticity, combined hexahedral and tetrahedral grids, dynamic rupture

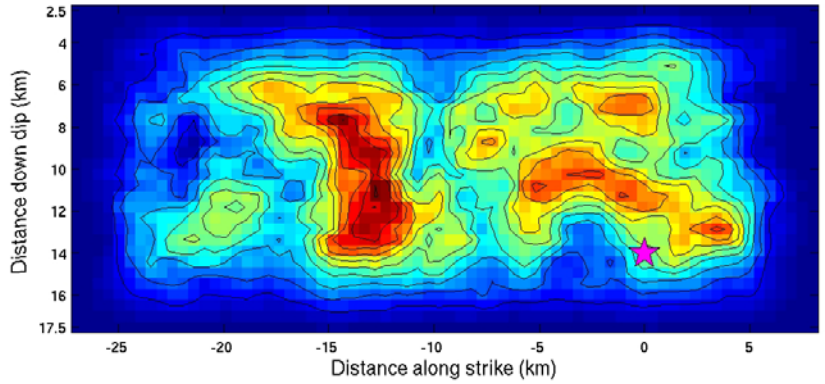
Anisotropic Material



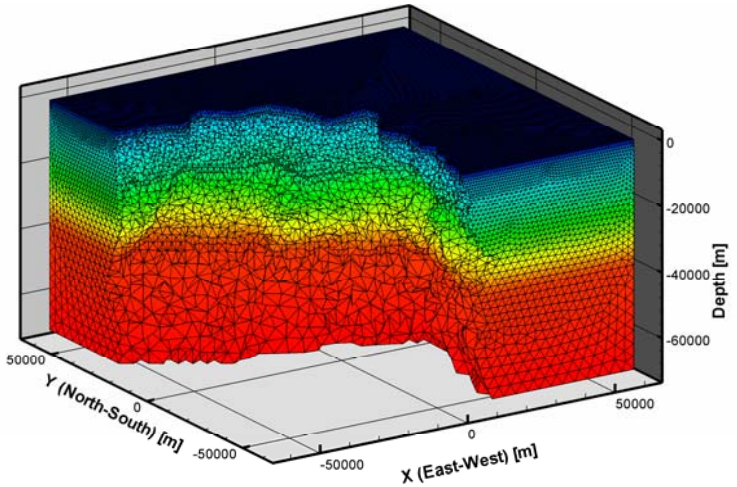


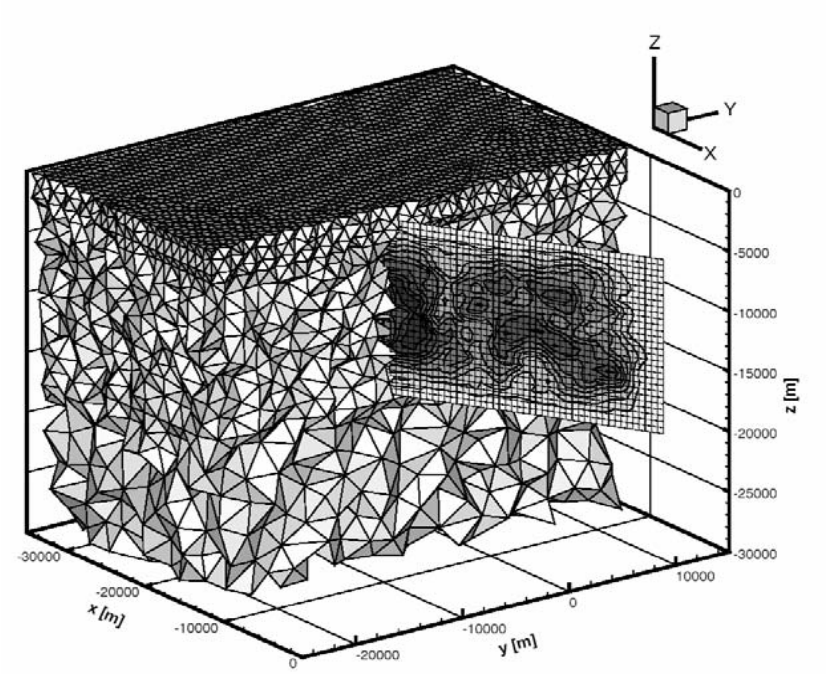
Arbitrarily shaped finite sources

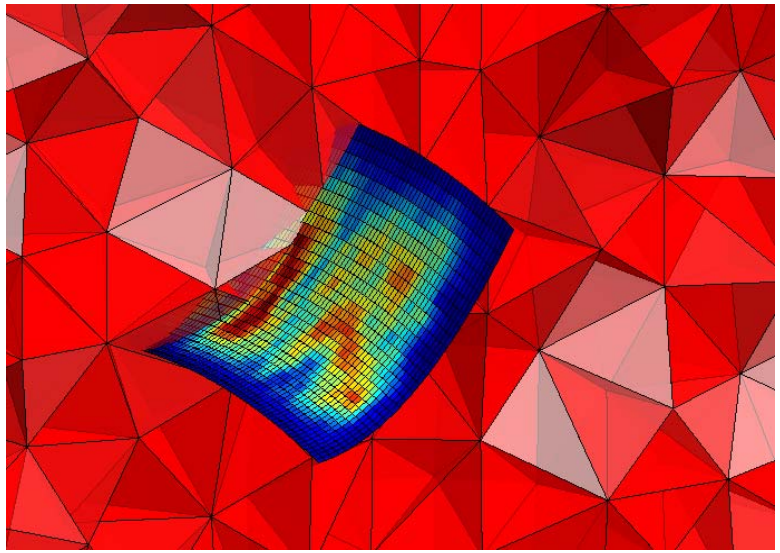
Slip map of an earthquake fault



Mesh spacing is proportional to P-wave velocity



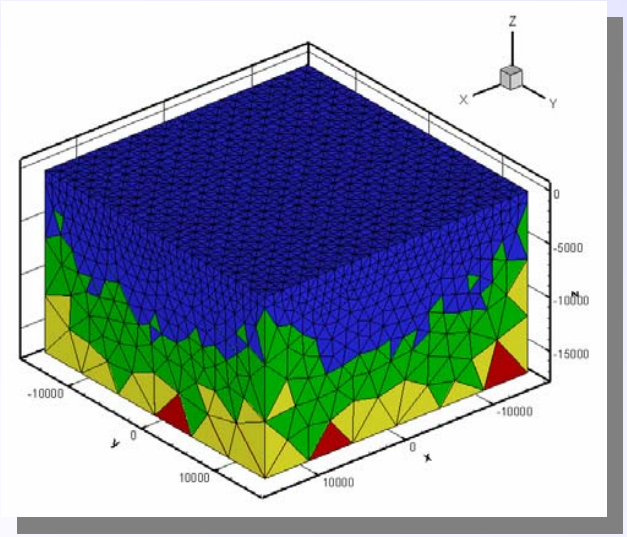


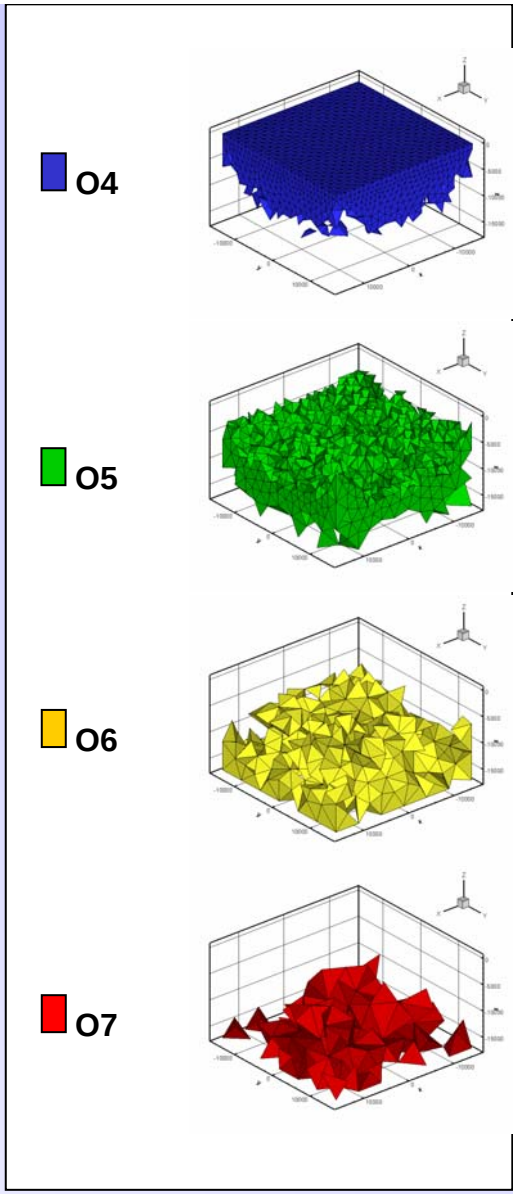


Käser, Mai, Dumbser, 2007

Local precision

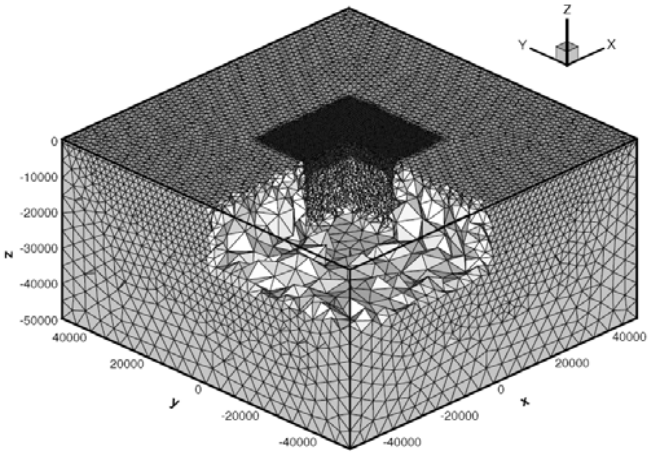
- Use high precision (i.e., high-order polynomials) only where necessary
- High precision where cells are large (high velocities)
- Low precision where cells are small (because of structural heterogeneities)

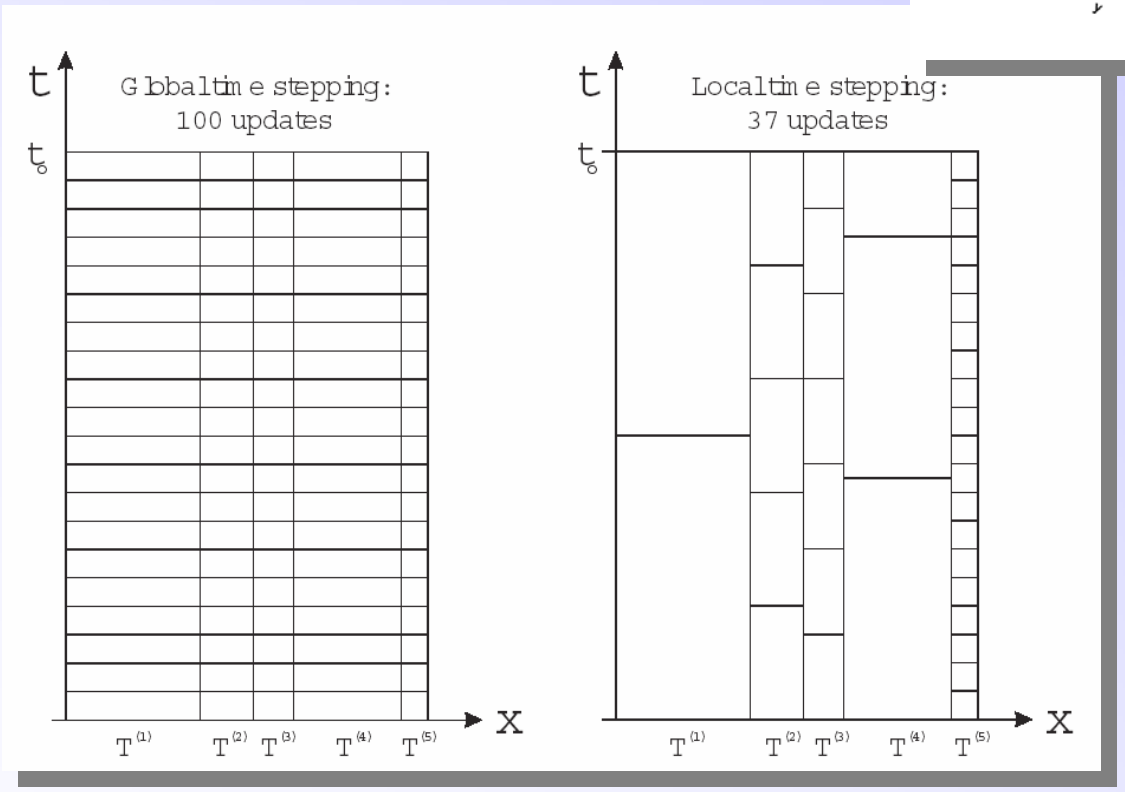




Käser et al. (2006)

Local time-stepping





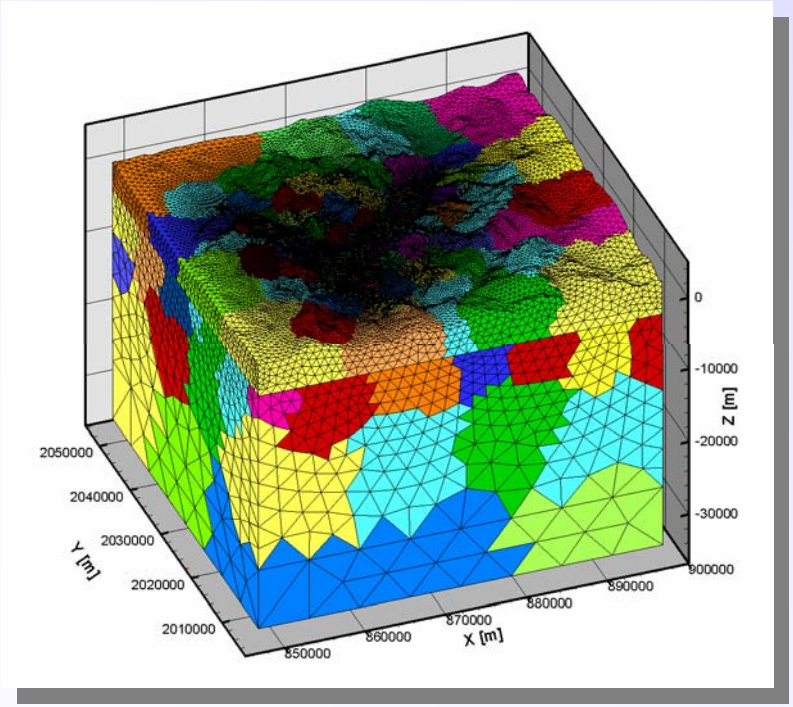
global ***local***

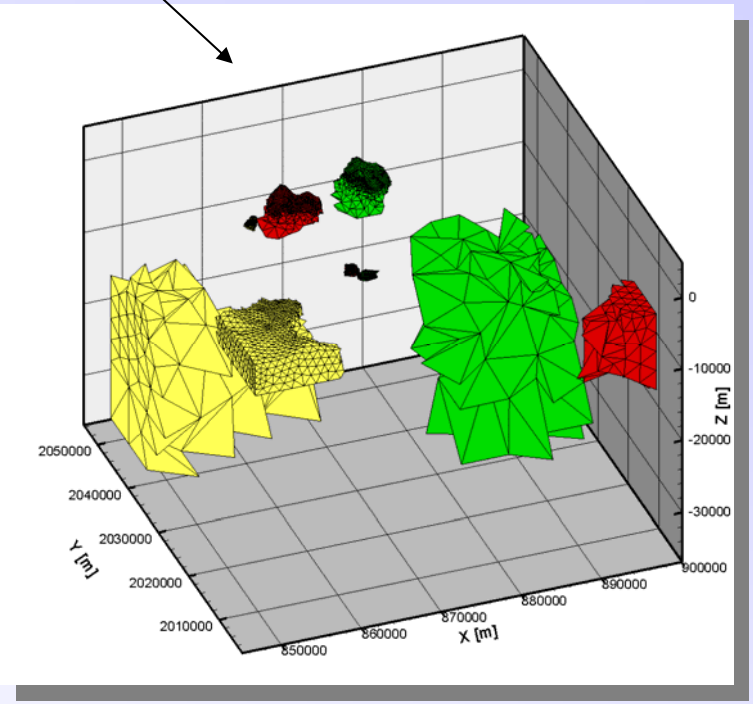
Local time-stepping is possible without loosing the accuracy of the scheme

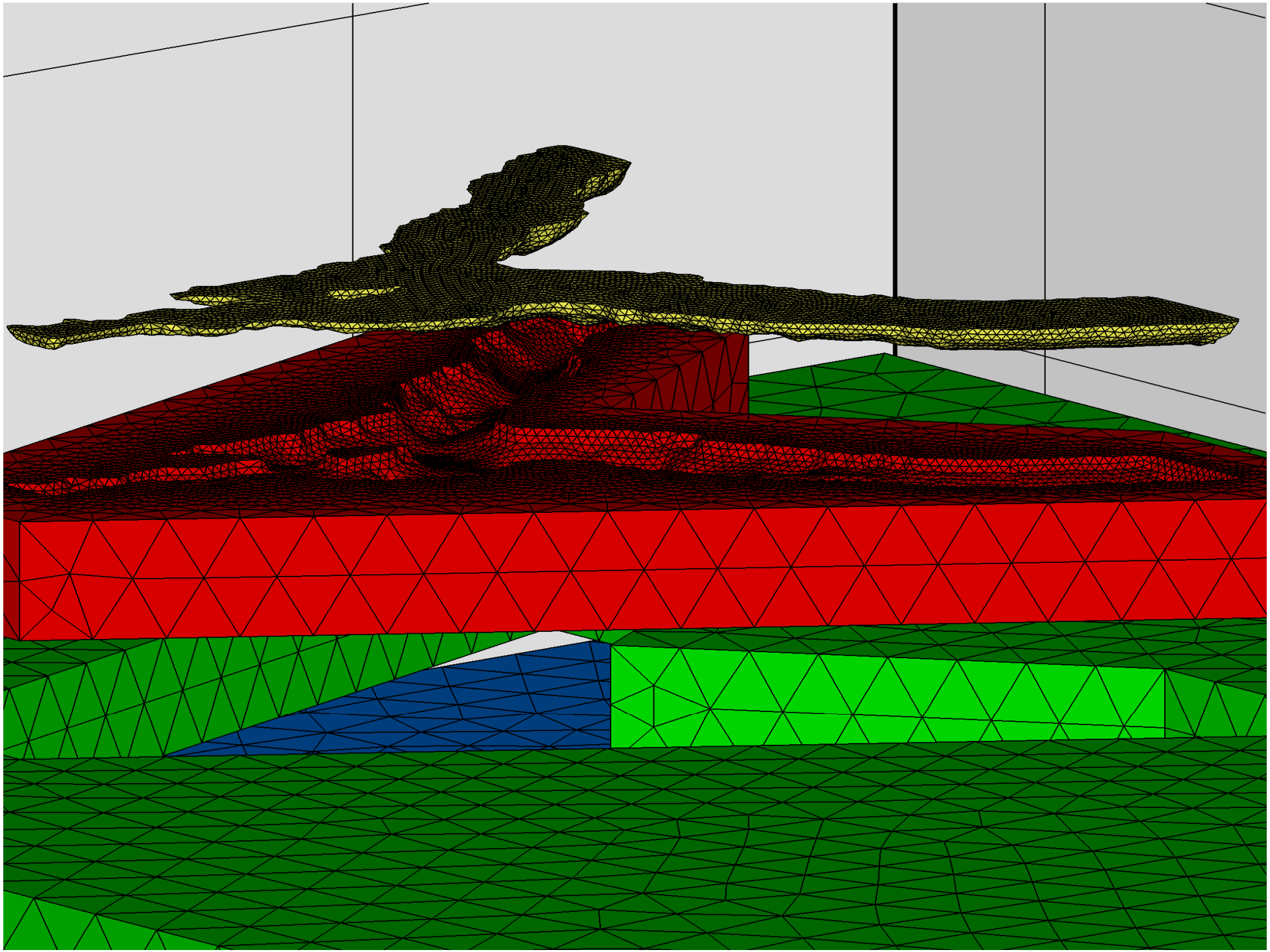
Mesh Partitioning and Parallel Computing

the problem of load blancing

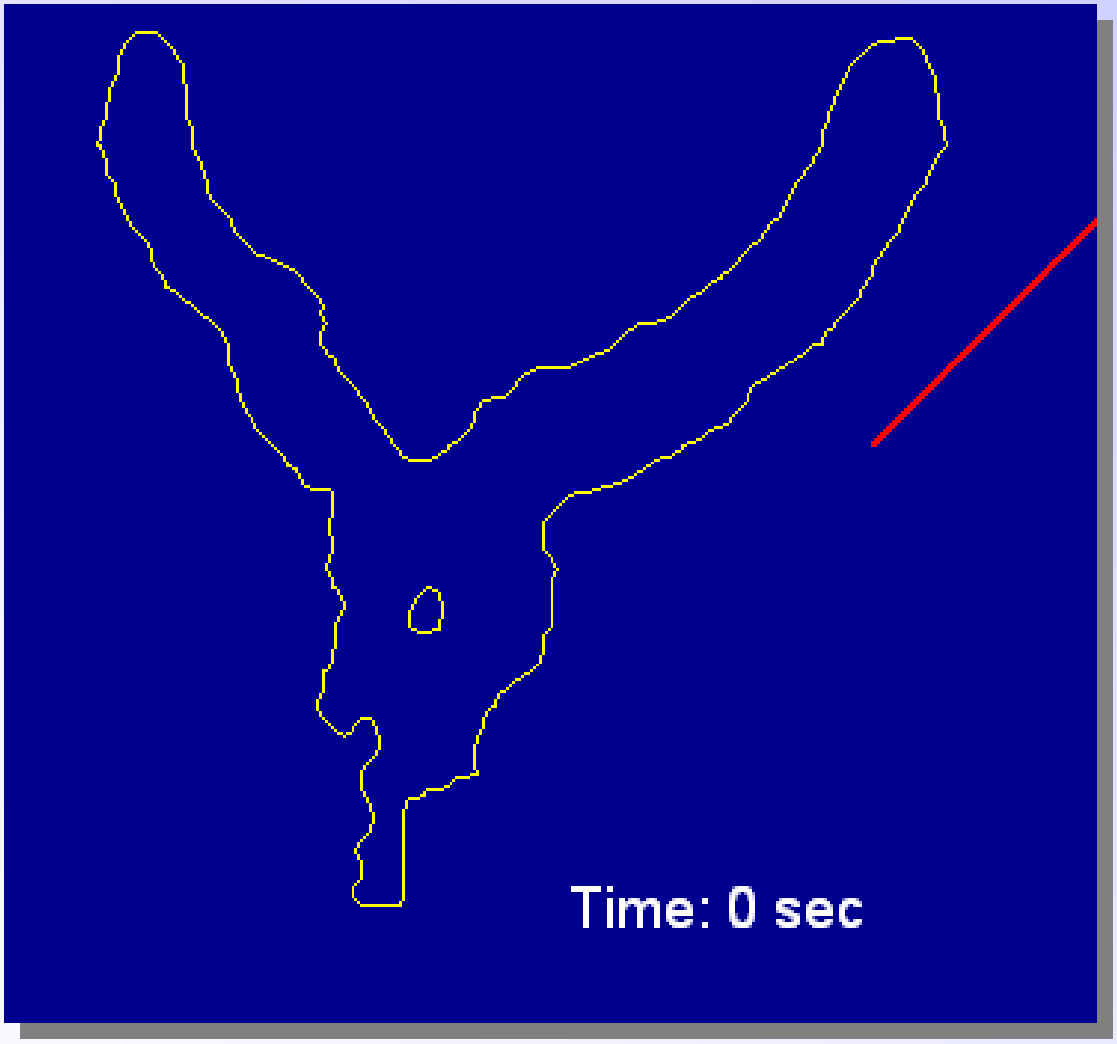
Same color means same processor

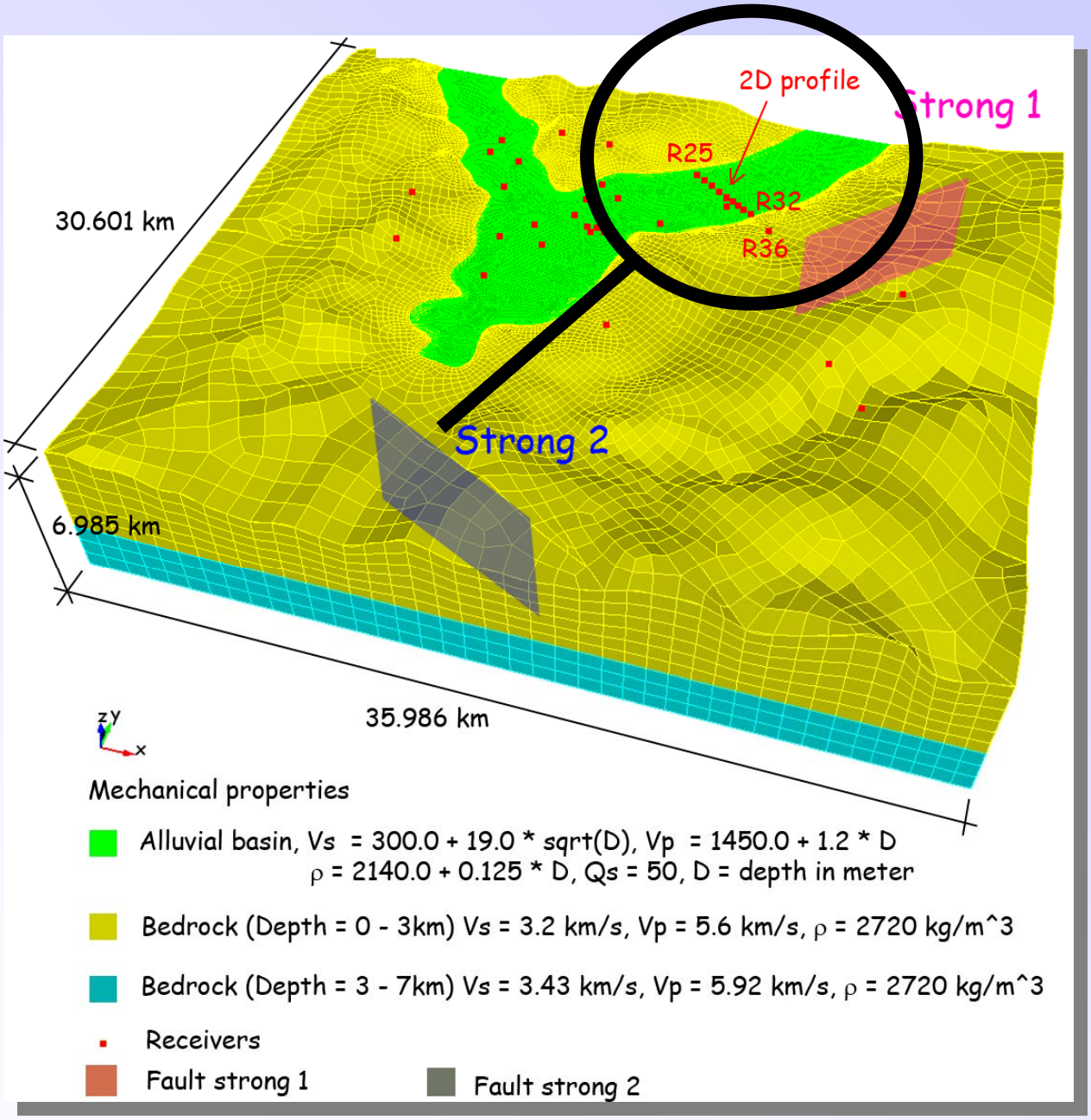




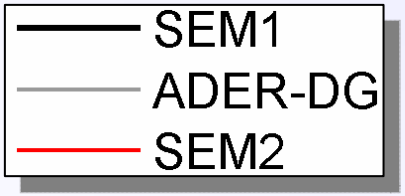


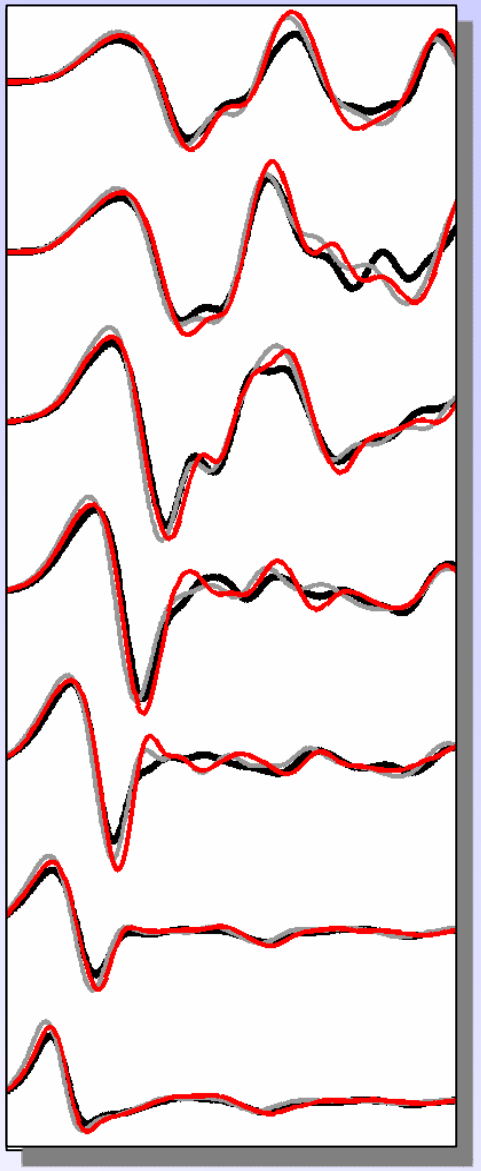
Grenoble Basin Simulation



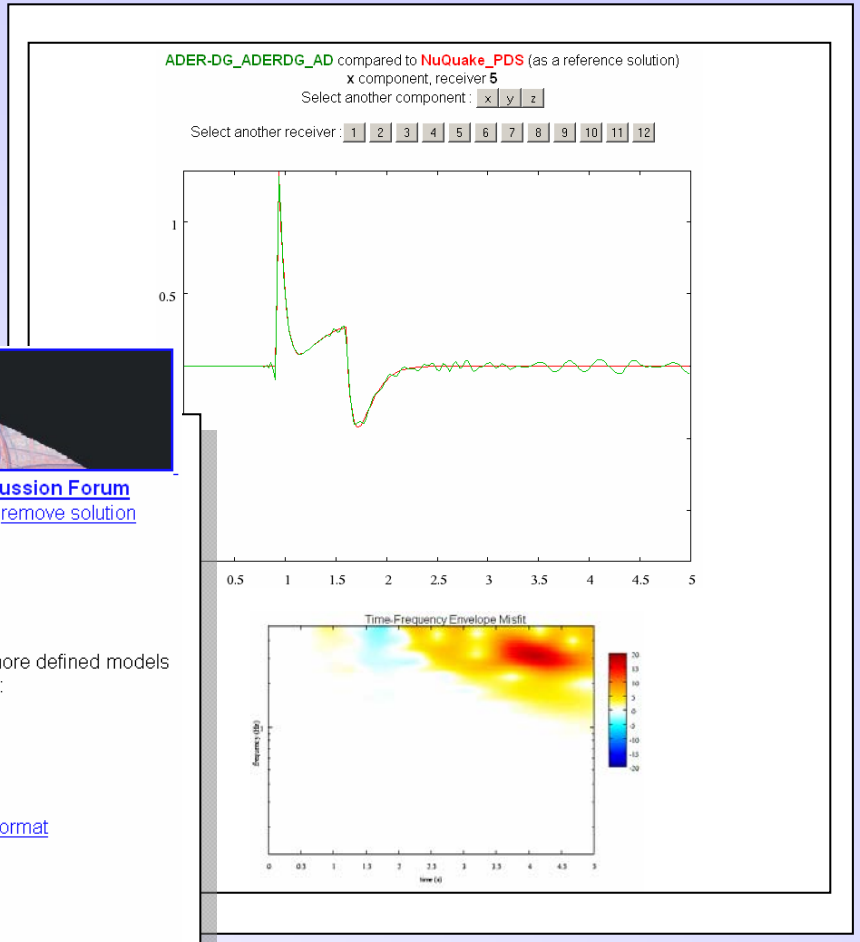

2D profile
Strong 1
R25
R32
R36
30.601 km
Strong 2
6.985 km
35.986 km
z
y
x
Mechanical properties
Alluvial basin, Vs = 300.0 + 19.0 * sqrt(D), Vp = 1450.0 + 1.2 * D
ρ = 2140.0 + 0.125 * D, Qs = 50, D = depth in meter
Bedrock (Depth = 0 - 3km) Vs = 3.2 km/s, Vp = 5.6 km/s, ρ = 2720 kg/m^3
Bedrock (Depth = 3 - 7km) Vs = 3.43 km/s, Vp = 5.92 km/s, ρ = 2720 kg/m^3
Receivers
Fault strong 1
Fault strong 2

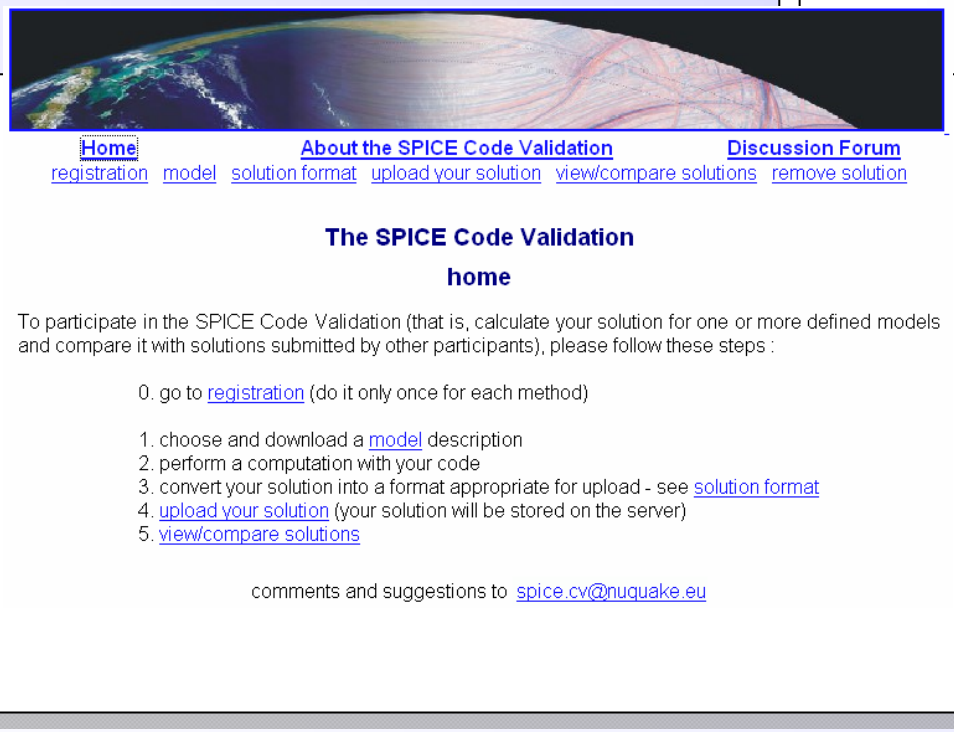
Seismogram Comparison





Interactive Benchmarking

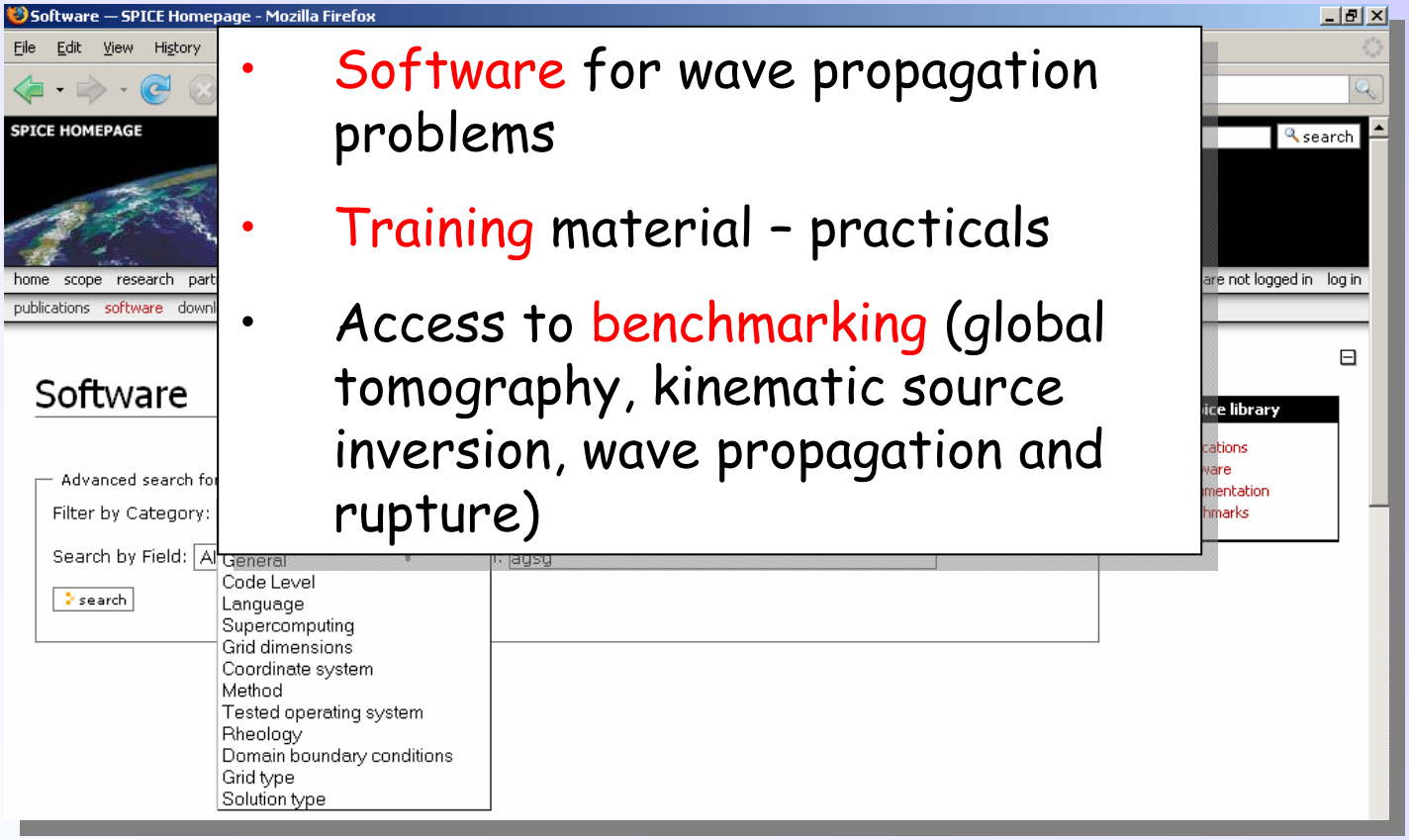




Moczo et al., 2006

www.spice-rtn.org

SPICE Digital Library



- Software for wave propagation problems
- Training material – practicals
- Access to benchmarking (global tomography, kinematic source inversion, wave propagation and rupture)

www.spice-rtn.org

… more info on the SPICE stand …

Conclusions – Technical Challenges

- Strongly heterogeneous structures (or complex surfaces) still pose problems particularly when using hexahedral grids (e.g. oversampling, instabilities)
- Unstructured grids (triangles, tetrahedra) have advantages concerning grid generation but numerical operators often are less accurate, or expensive
- Efficient parallelization algorithms with heterogeneous time steps, accuracy and grid density requires substantial interaction with software engineers.

What's missing?

… easy access for data modellers to well tested simulation tools …

… easy (e.g., hidden) access to HPC infrastructure (GRIDs, EU-HPC)

… community codes for wave propagation problems

… software engineering support