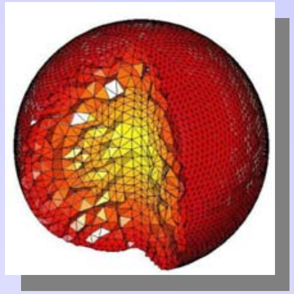
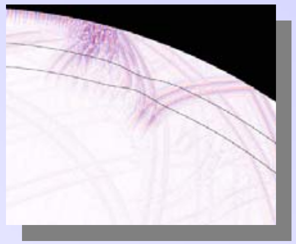
Numerical methods in the Earth Sciences:
seismic wave propagation

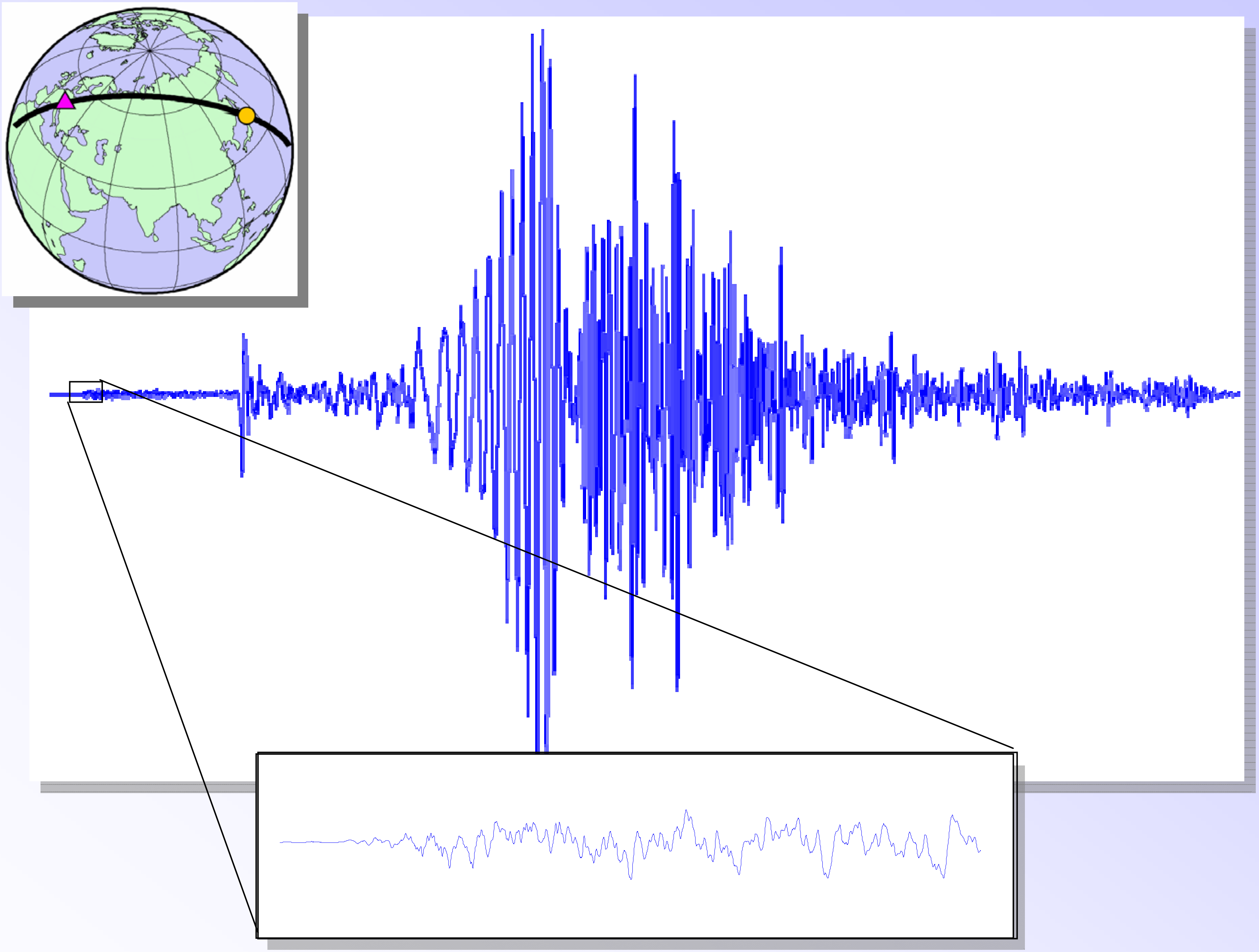
Heiner Igel, LMU Munich

I Waves and supercomputing

II Science with 3D wave propagation and rupture
- Understanding earthquake **rupture**
- Prediction of **strong ground motions**
- The **seismic signature of mantle convection**
- **Imaging** with 3-D methods – adjoint method



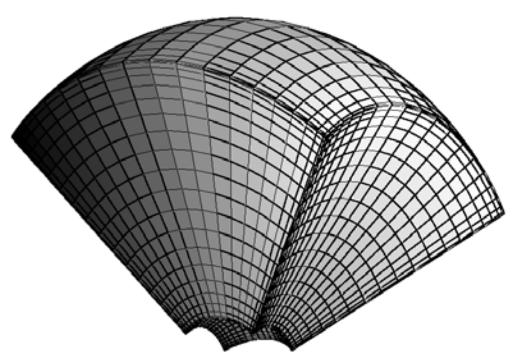


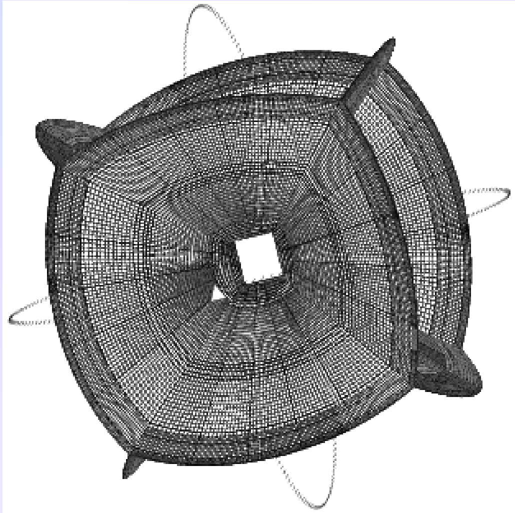


Spatial Scales and Memory

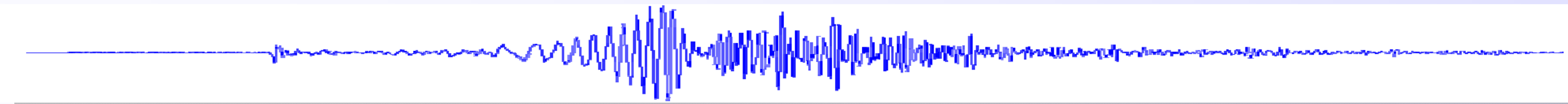
(back of the envelope)

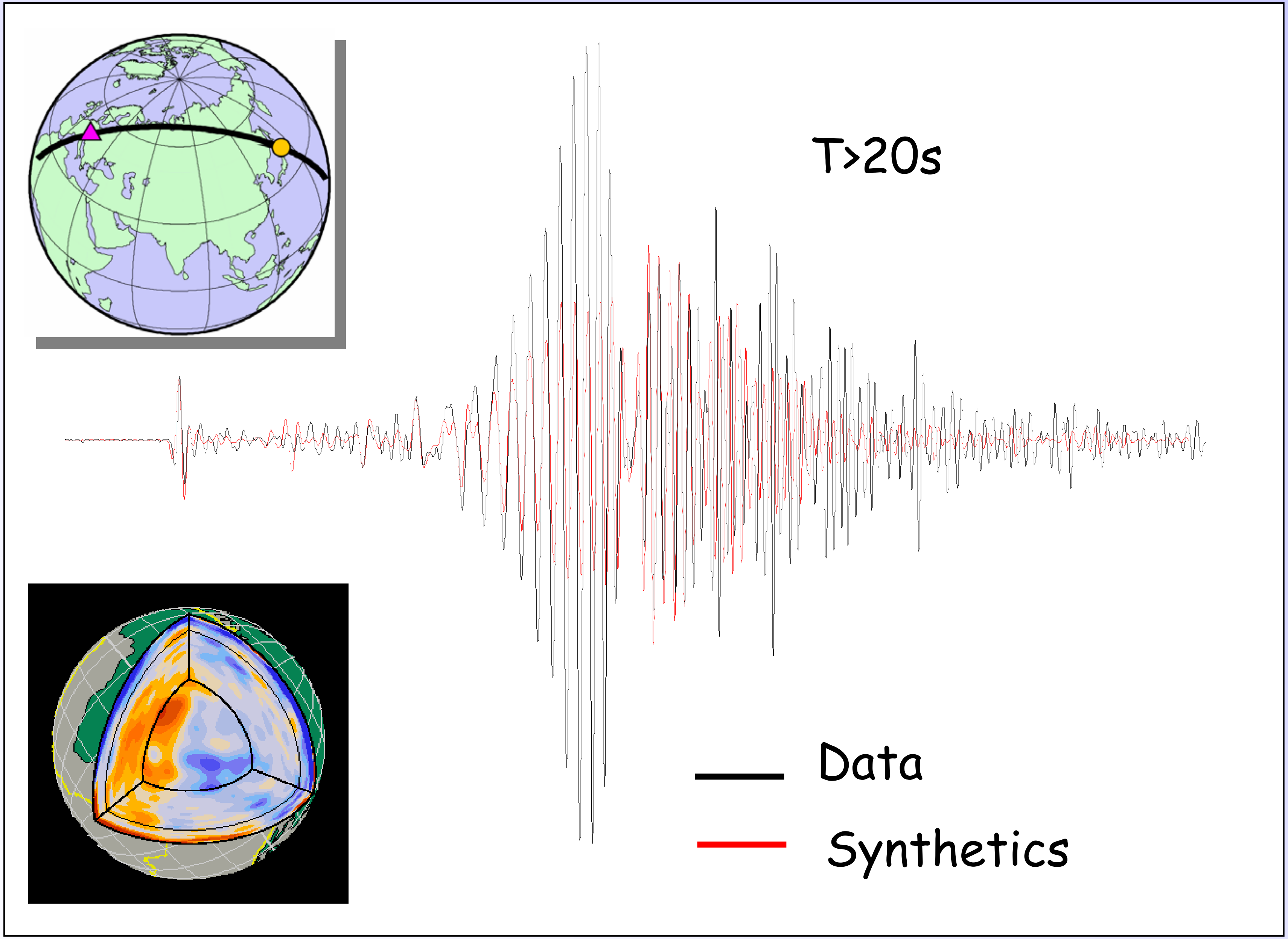
Highest frequency:	**0.1 Hz**
Shortest wavelength:	20 km (crust)
Shortest wavelength:	50 km (mantle)
Grid points per wavelength:	5
Grid spacing:	2000 m (crust)
Grid spacing:	5000 m (mantle)



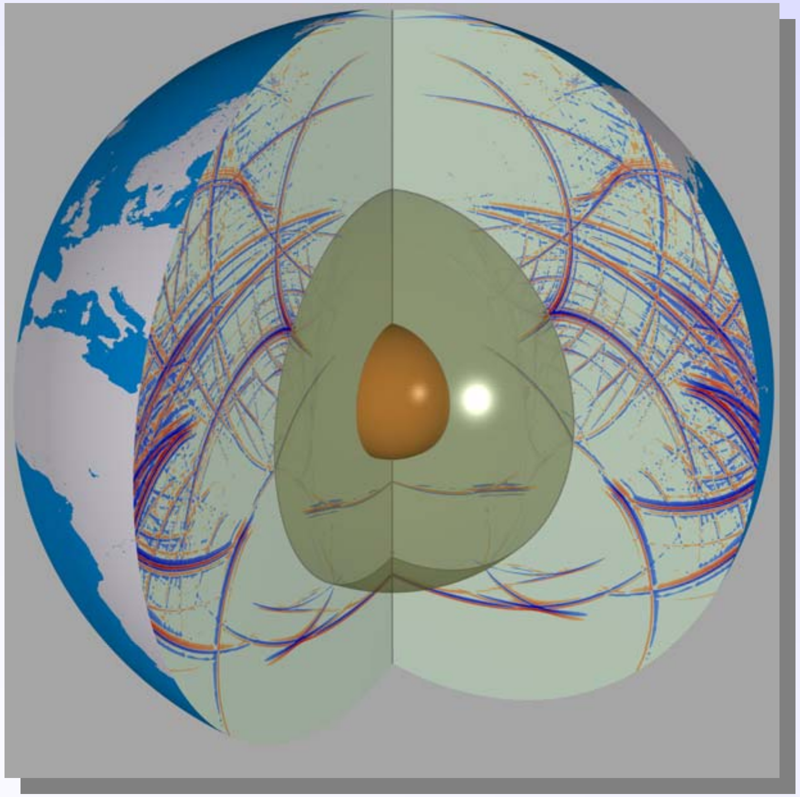


Required grid points: $O(10^9)$
Required memory: O(100 GBytes)

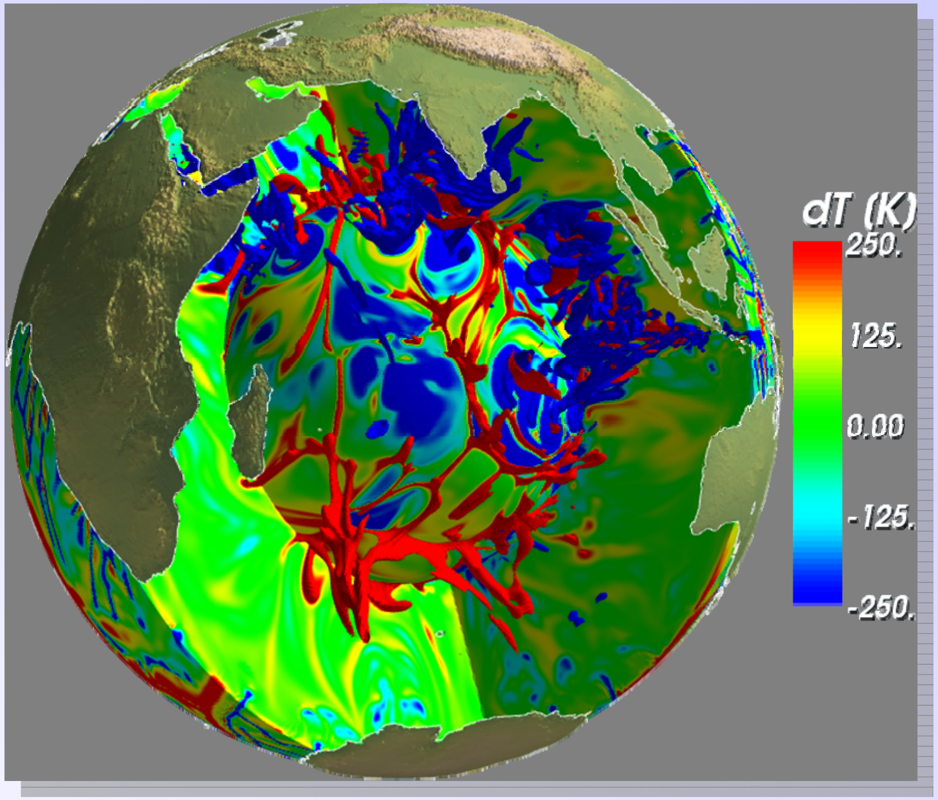



T>20s
Data
Synthetics

Seismology and Geodynamics



Courtesy: G. Jahnke



Courtesy: H.P. Bunge, B. Schuberth

Numerical simulation of seismic wave propagation

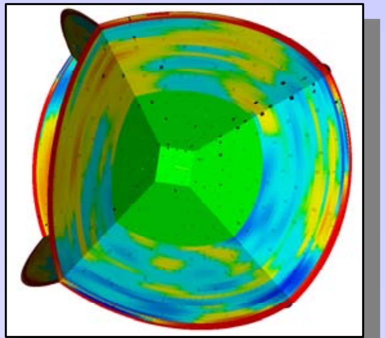
Elastic wave equations

$$\rho\partial_t^2 u_i = \partial_j(\sigma_{ij} + M_{ij}) + f_i$$

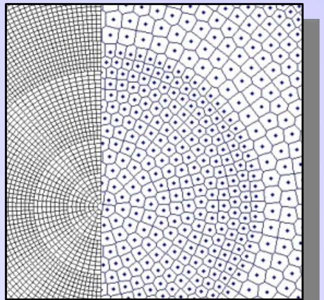
$$\sigma_{ij} = c_{ijkl}\varepsilon_{kl}$$

$$\varepsilon_{kl} = 1/2(\partial_k u_l + \partial_l u_k)$$

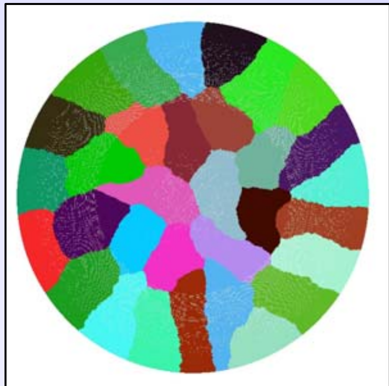
3D Model



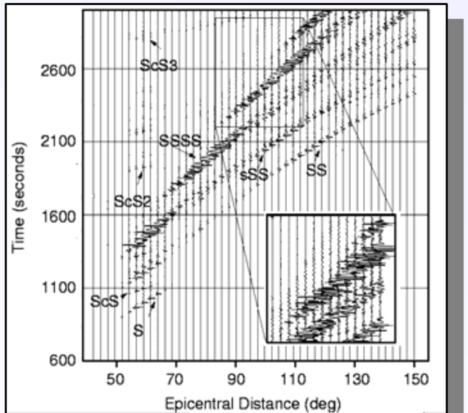
Grid



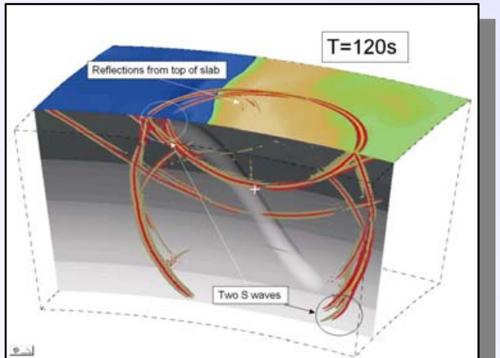
Parallelisation



Synthetic seismograms



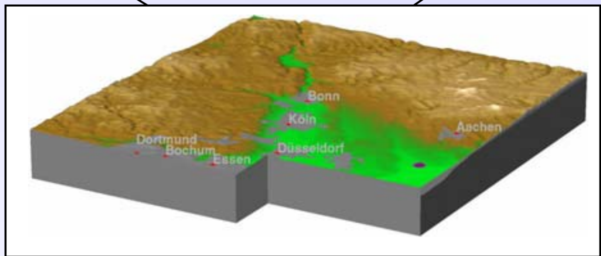
Simulation



The kernel

Earthquake scenarios
Shaking hazard

Phenomenological studies
Model space studies

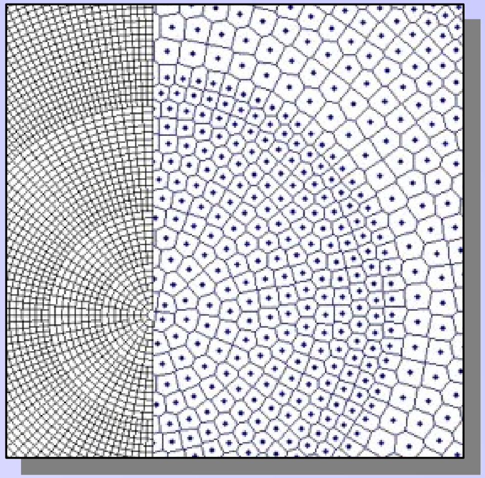


Dynamic rupture
Source physics

Sensitivities
Experiment design

Imaging (source and structure)
Adjoint methods

Numerical methods



- **Finite Differences** (**high order**, optimal operators)
- **Pseudospectral methods (Chebyshev, Fourier)**
- **Finite**/spectral elements on hexahedral grids
- Unstructured grids (finite volumes/elements, natural neighbours) or combinations
- Parallelization using MPI (message passing interface)

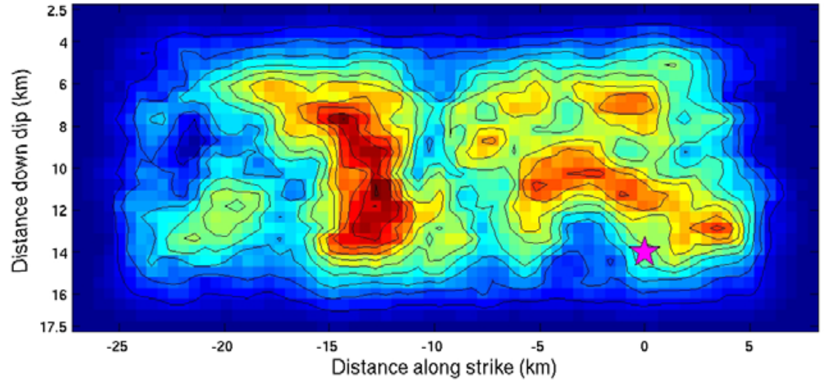
-> for rupture problems special internal boundary conditions apply

Fundamental concepts

- From the continuous to the discrete world
  - Function approximation
  - Collocation points
  - Stability
  - Numerical dispersion
- Methodologies
  - Finite differences
  - Pseudospectral methods
  - Finite elemtents

Dynamic rupture

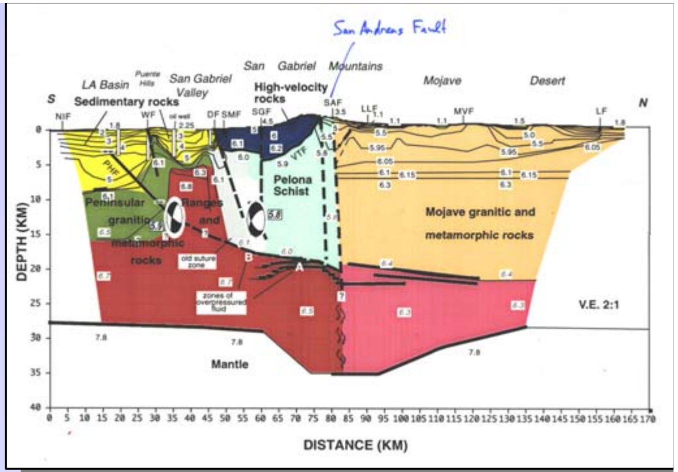
scientific objectives

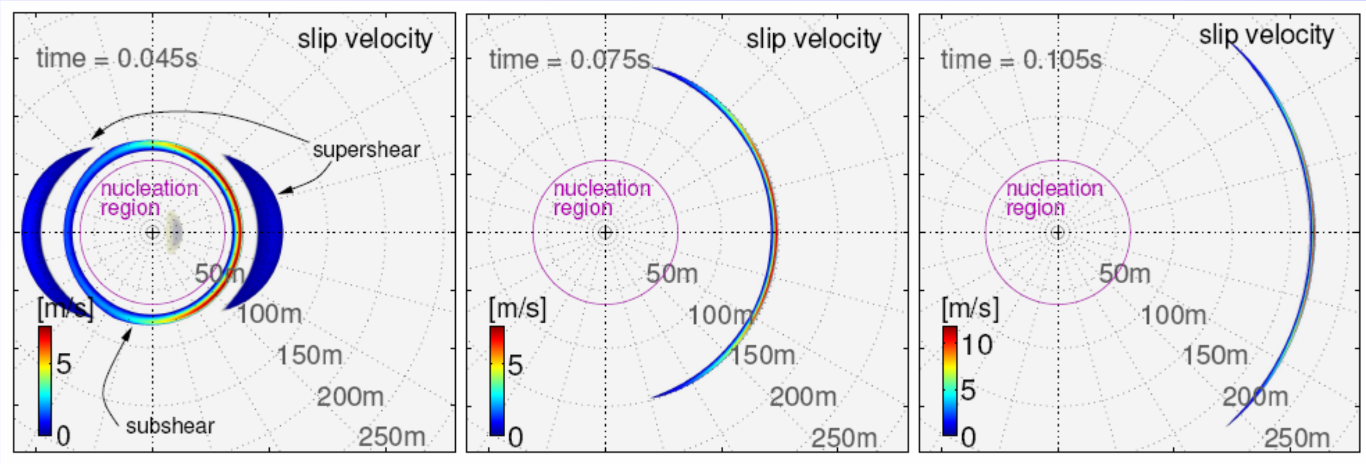


- Understanding the earthquake process
- Understanding the controlling mechanisms of earthquakes (frictional properties, strength heterogeneities, material interfaces, etc.)
- Resolving power of seismic observations with respect to (dynamic) source parameters
- Regional conditions (intraplate, interplate, subduction zones, normal, strike, etc.)

→ phenomenological studies

Rupture at a bi-material interface

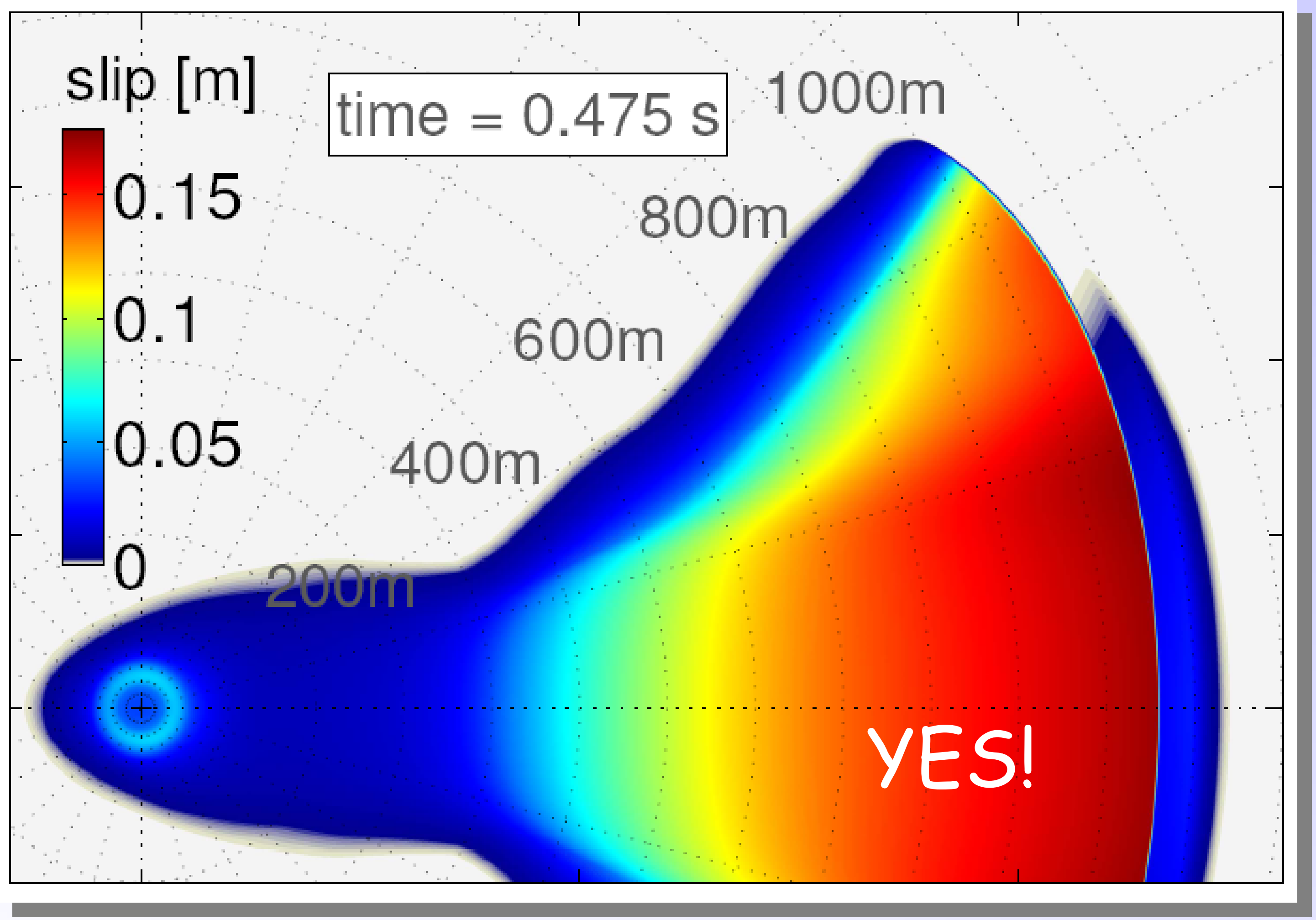




Convergence tests with high-resolution models

- Grid size 500x3200x3200
- 12.5 cm grid spacing
- High-order staggered-grid finite differences

Self-sustained pulse in 3D?

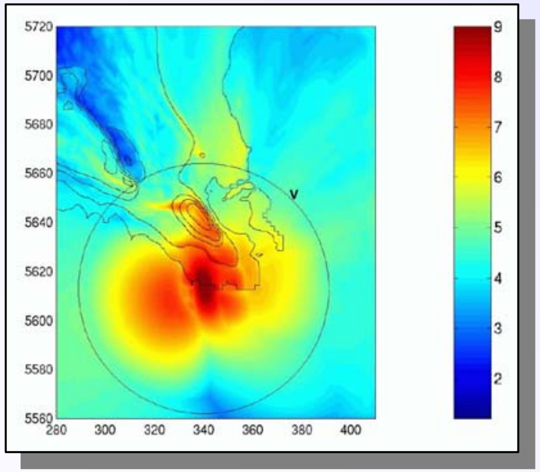


Brietzke, Cochard, Igel, GRL 2007

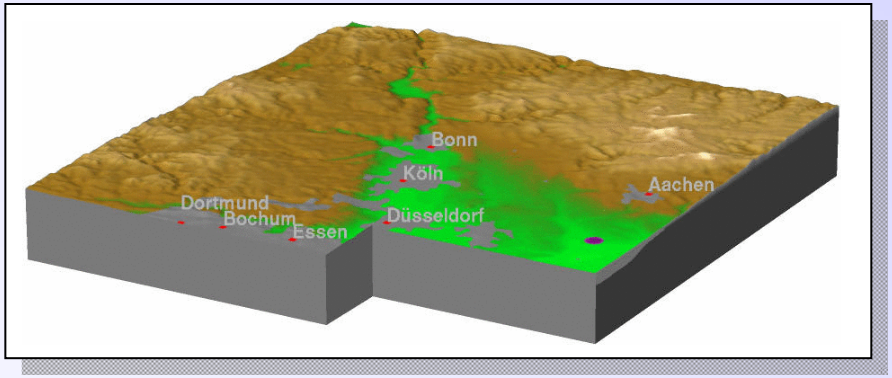
Earthquake scenarios
scientific objectives

- Accurate forecasting of hazard and risk scenarios for specific regions and time intervals
- Incorporation of earthquake scenario simulations into probabilistic hazard analysis

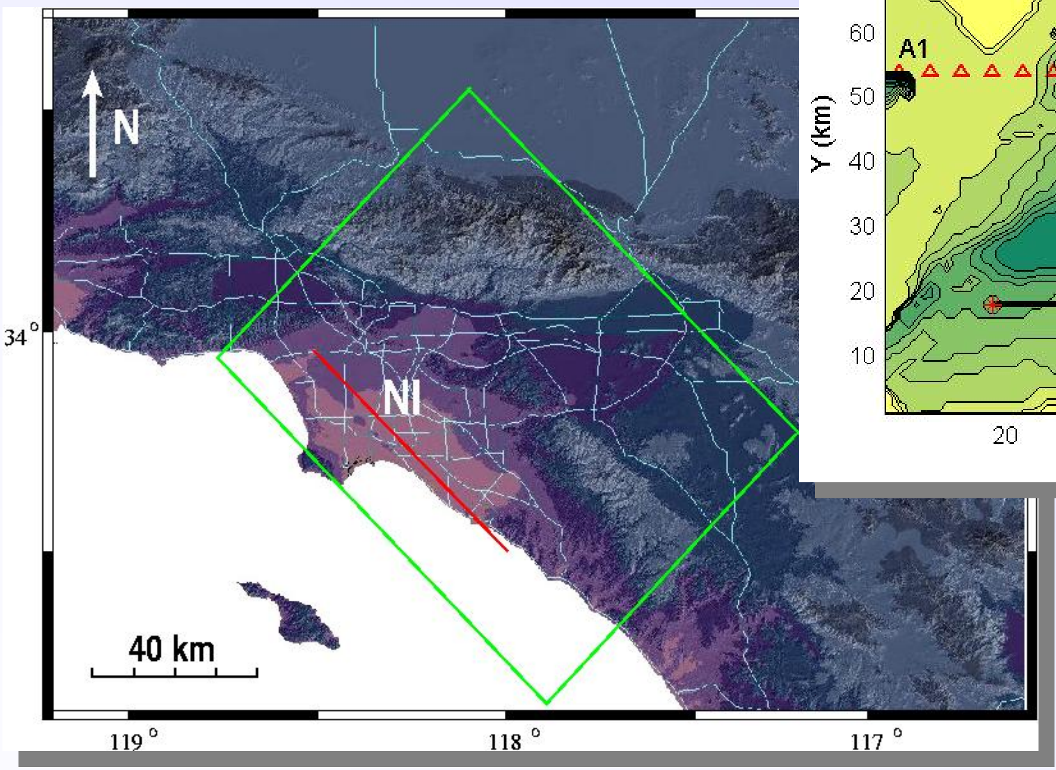
Shaking hazard

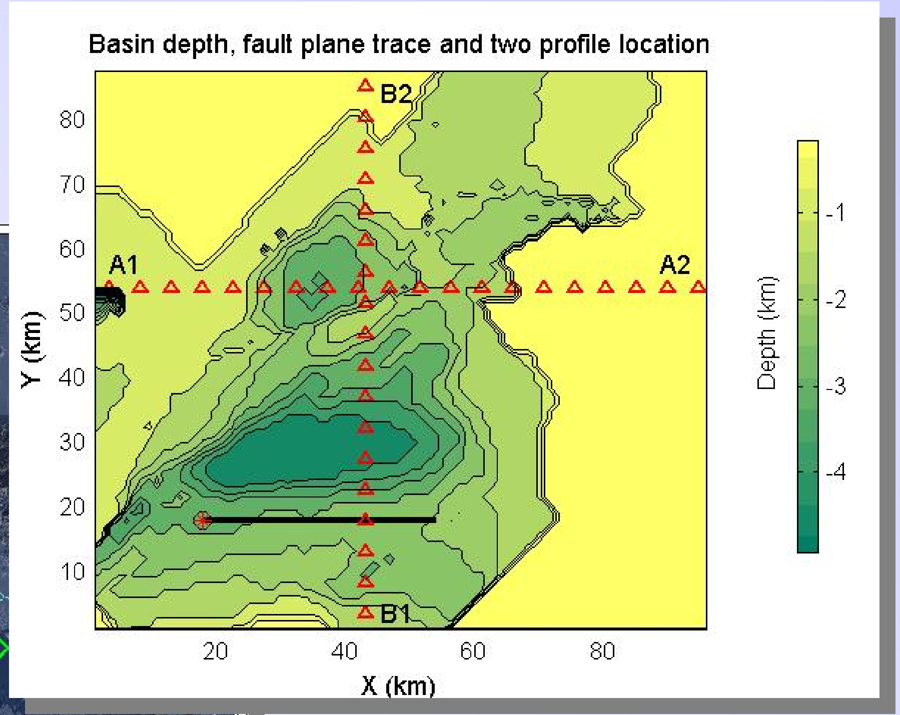


M5.9 Roermond 1992



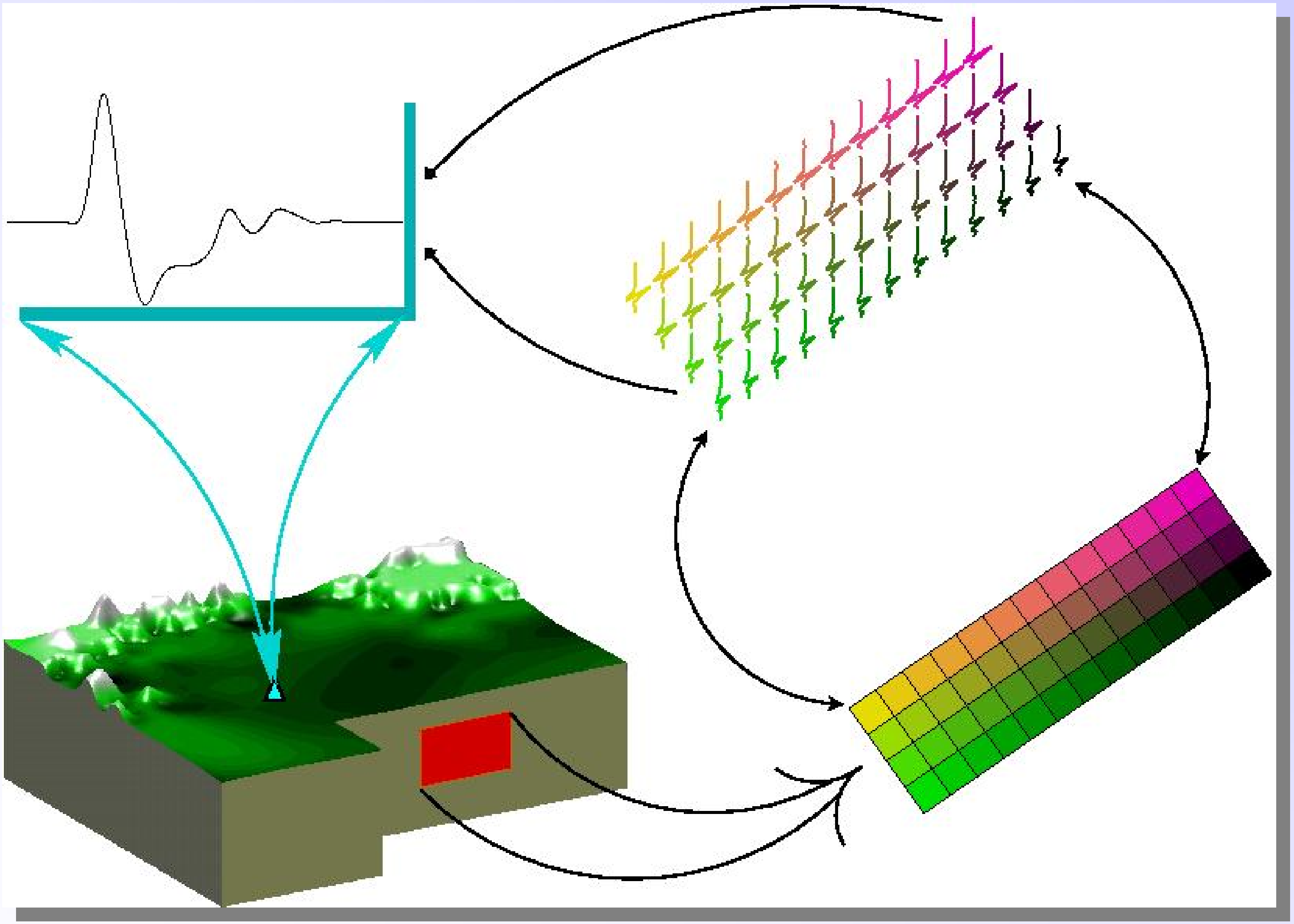
Example: Newport-Ingelwood Fault, Los Angeles Basin



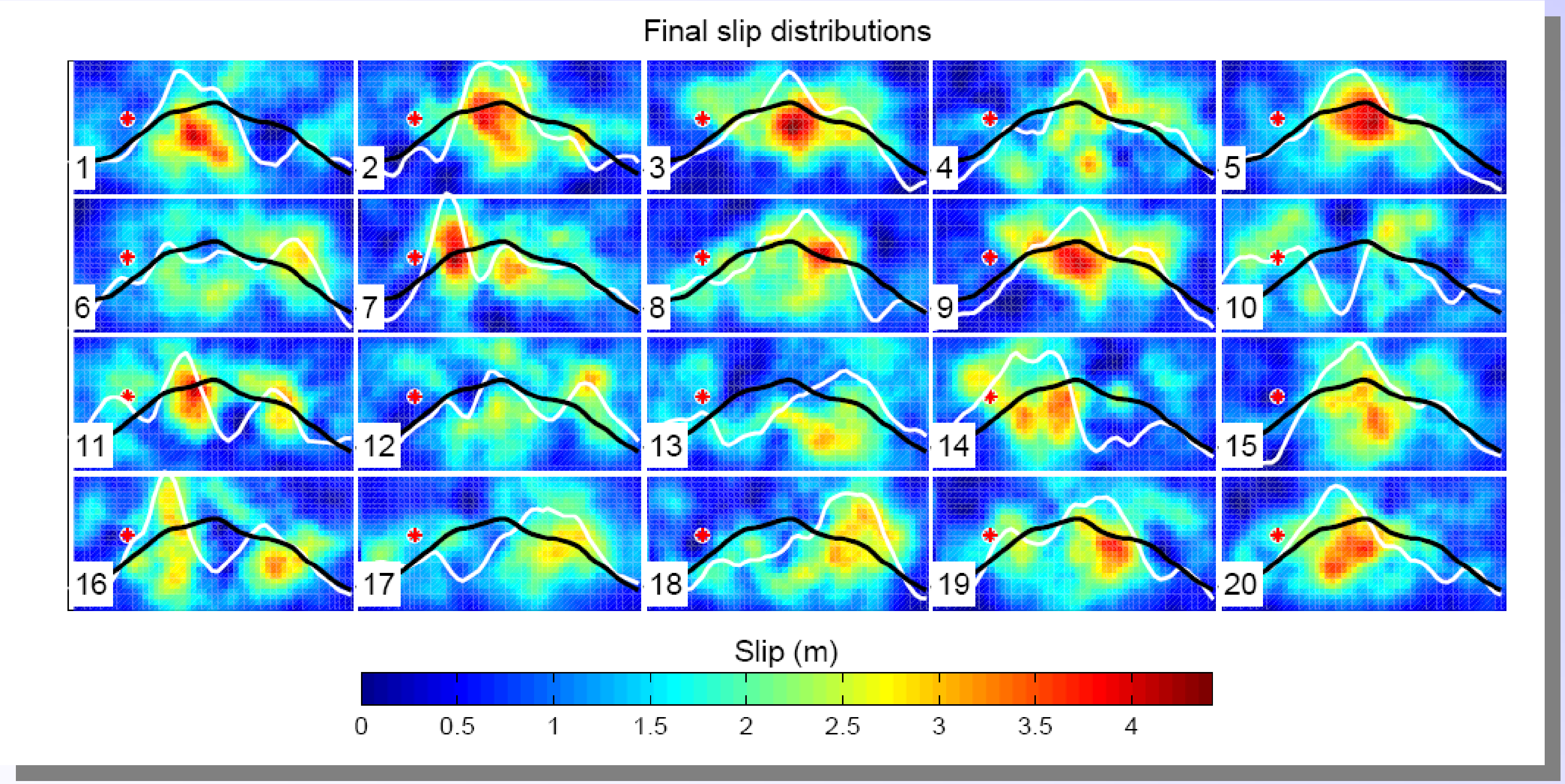


Wang, Igel, Cochard, Ewald (2006)

Numerical Green's Functions



Varying slip histories
M7 earthquakes



… while keeping the hypocenter location fixed …

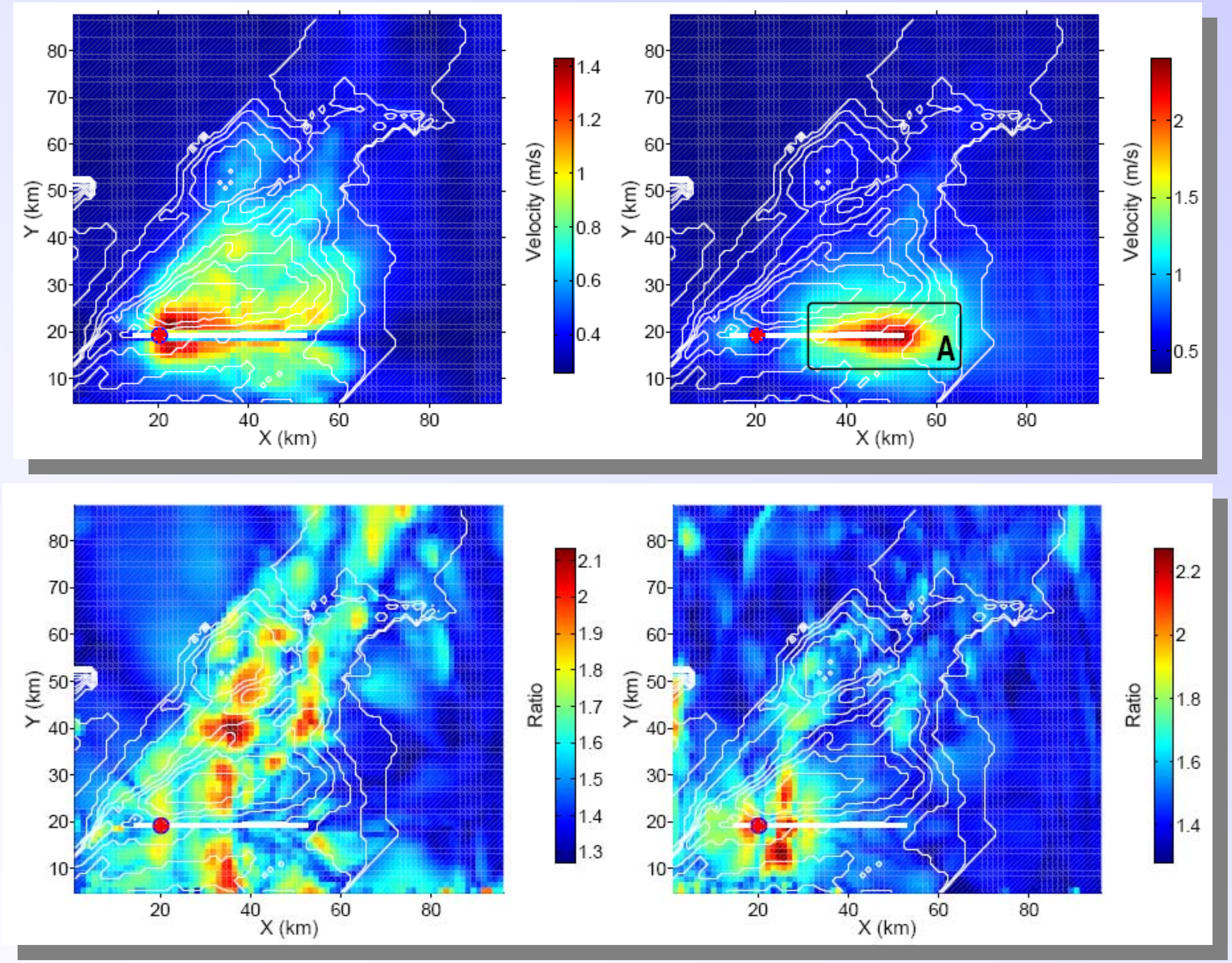
Variations due to slip history

20 scenarios

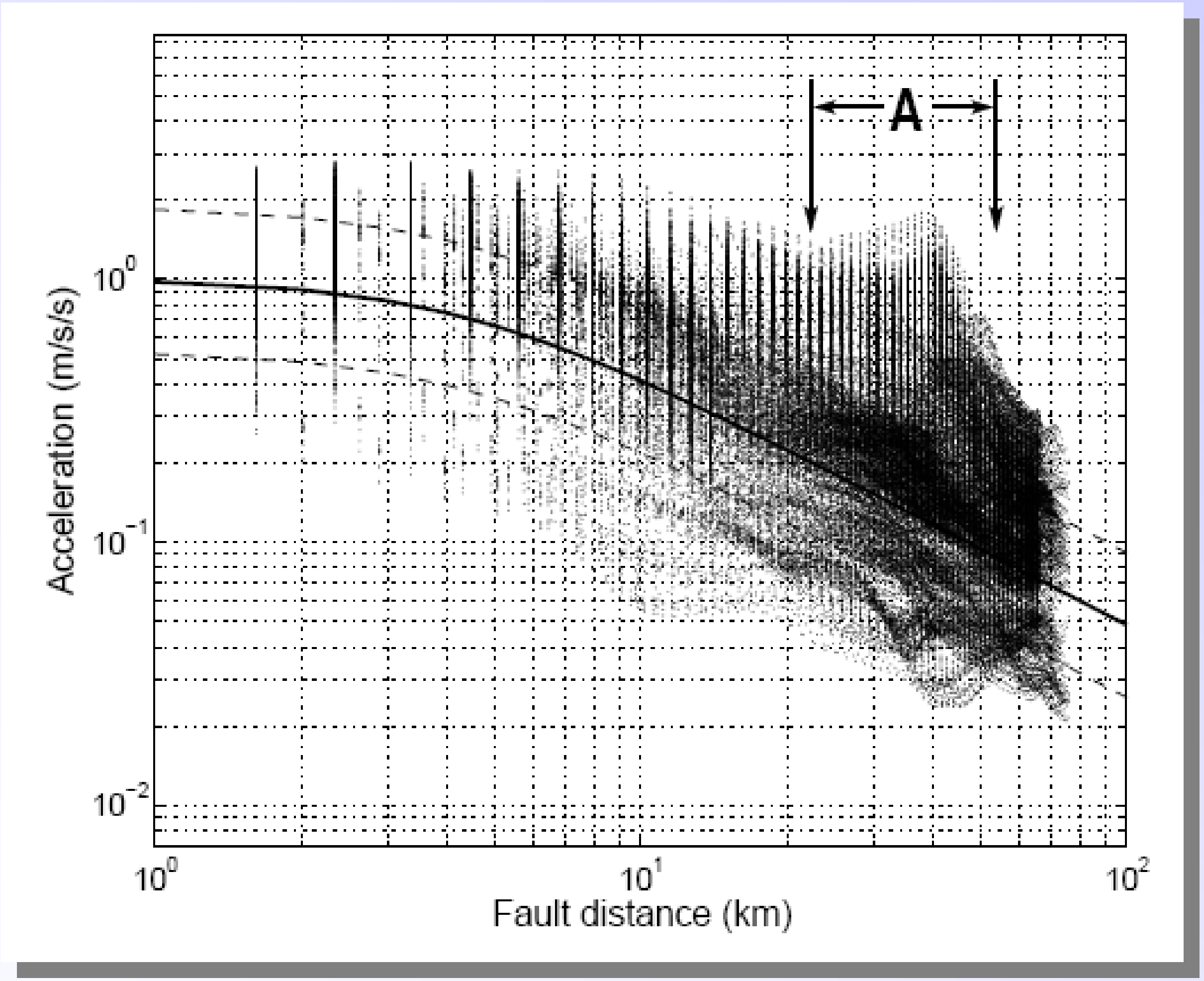
Fault || Fault ⊥

PGV

Max / Mean



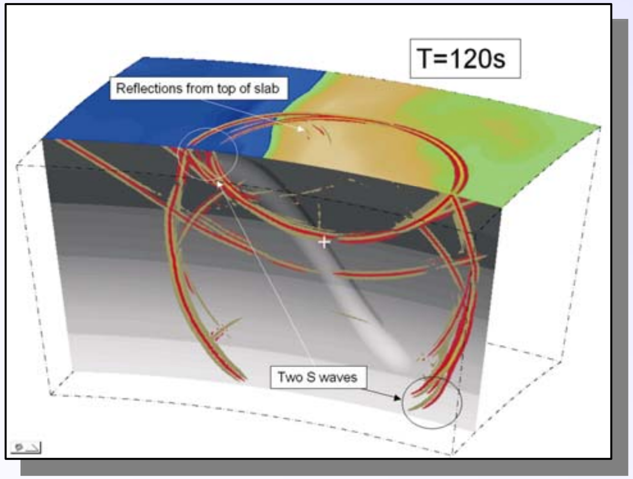
Compatible with Attenuation Relations?

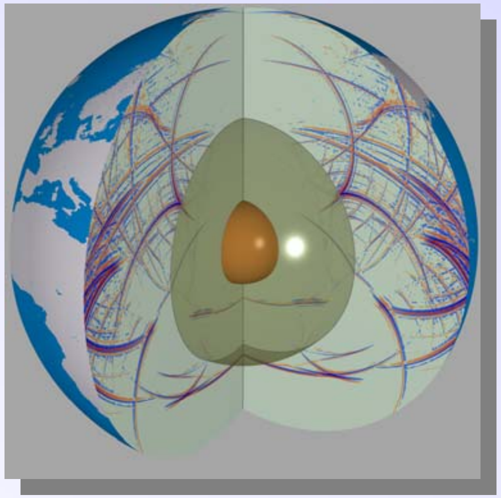


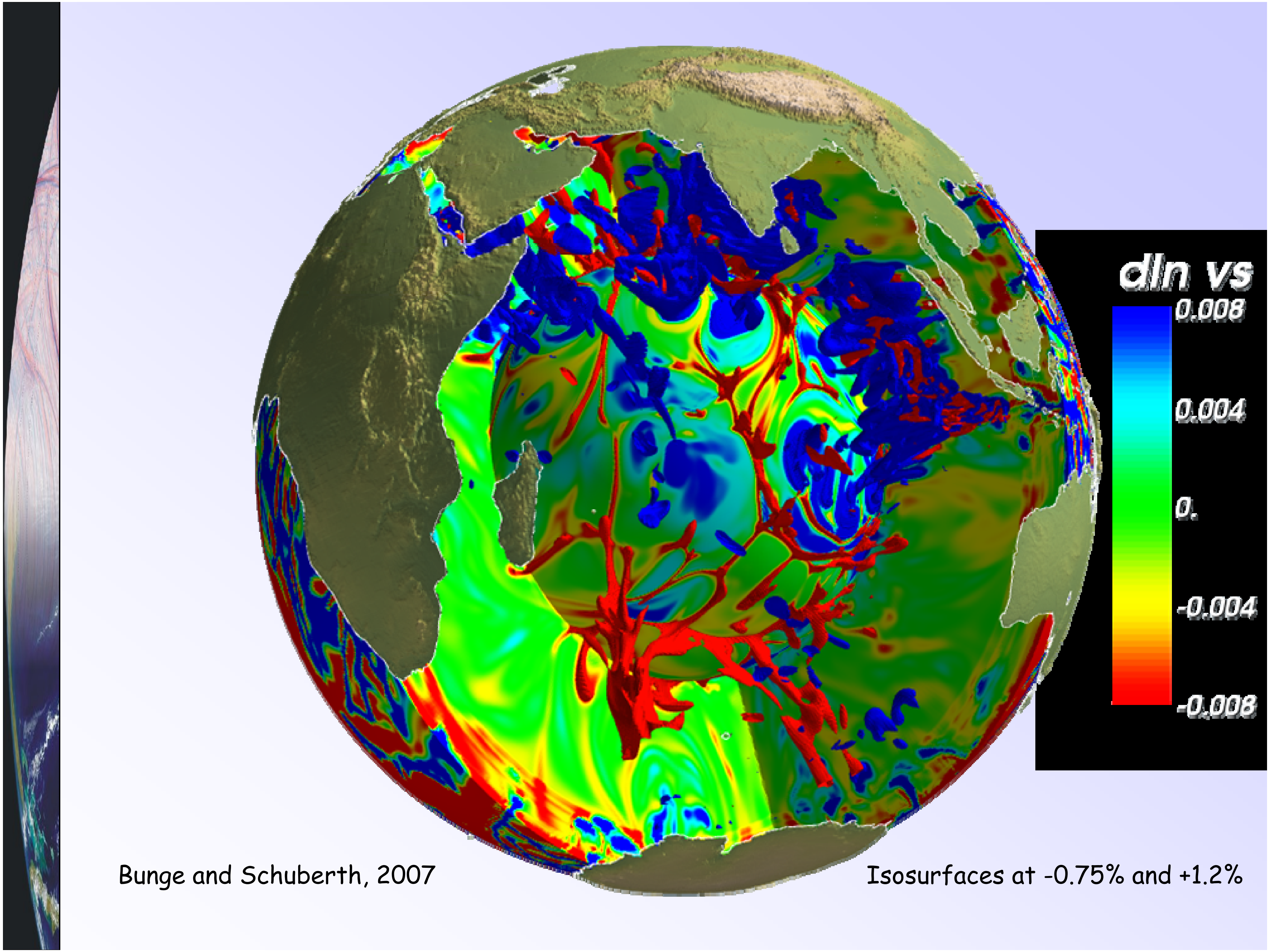
Global and regional seismology

scientific objectives

- High resolution imaging (diffration tomography) of global earth structure (geodynamics)
- 3D wave effects of structures like plumes, subduction zones, D" -> geodynamic issues
- Development of 3D reference models (e.g. European reference model)



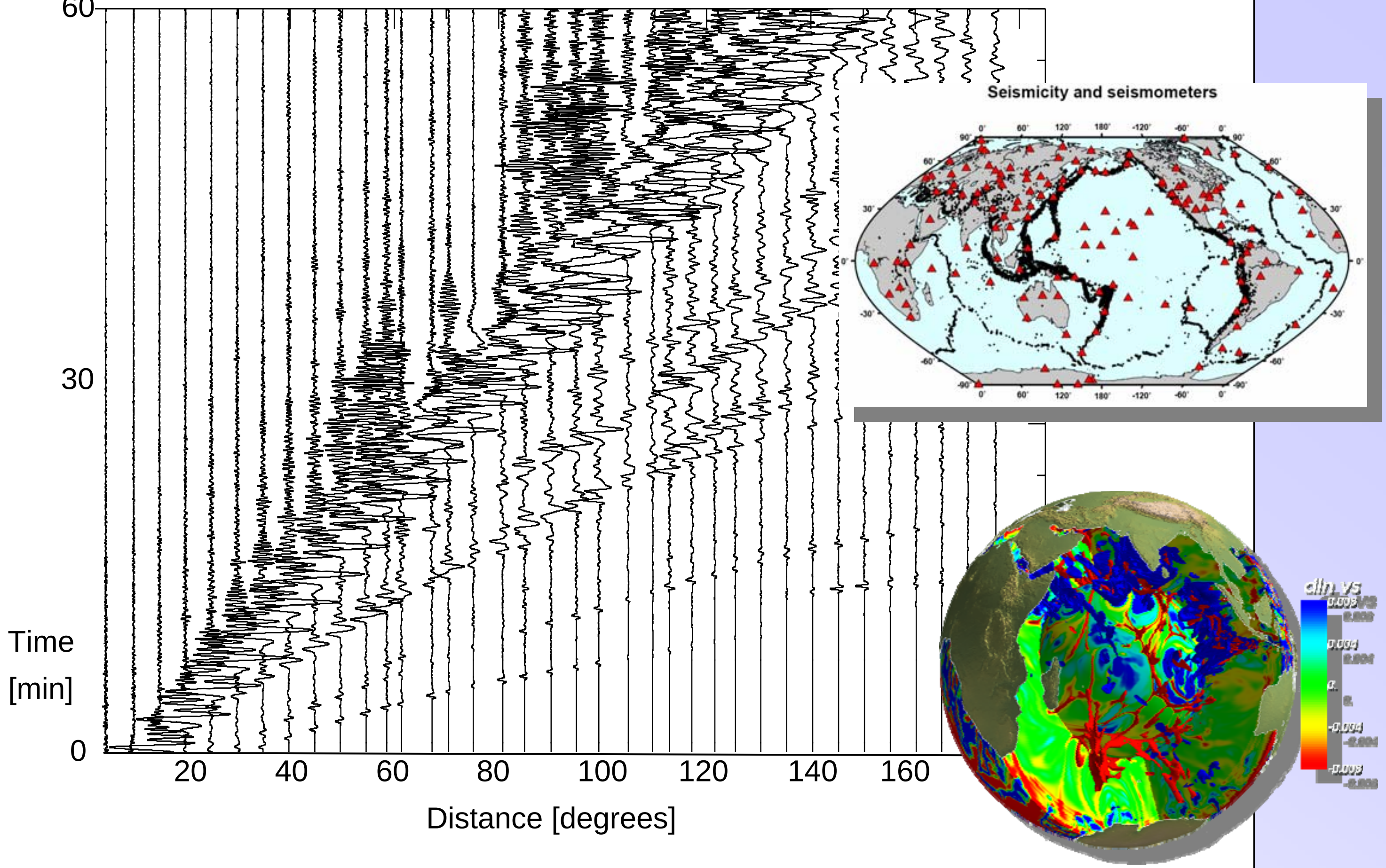




Bunge and Schuberth, 2007

Isosurfaces at -0.75% and +1.2%

Spectral Element Simulations

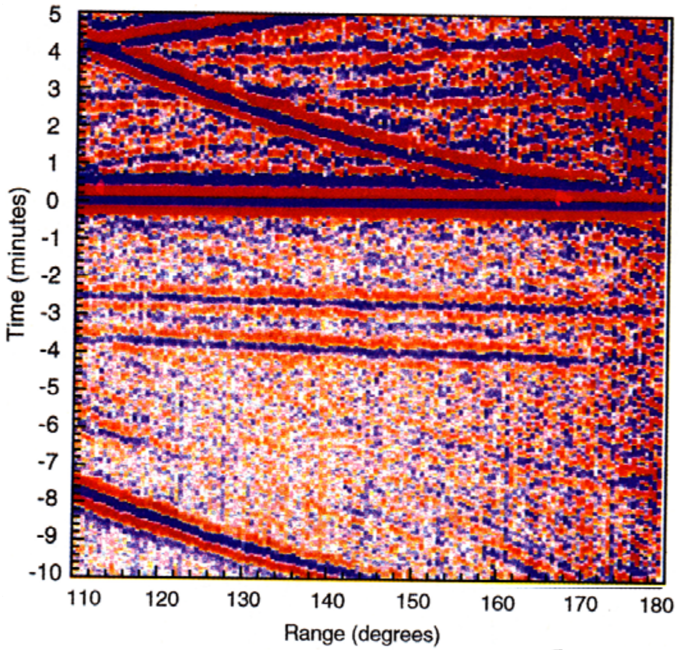


(SPECFEM3D, Komatitsch and Tromp)

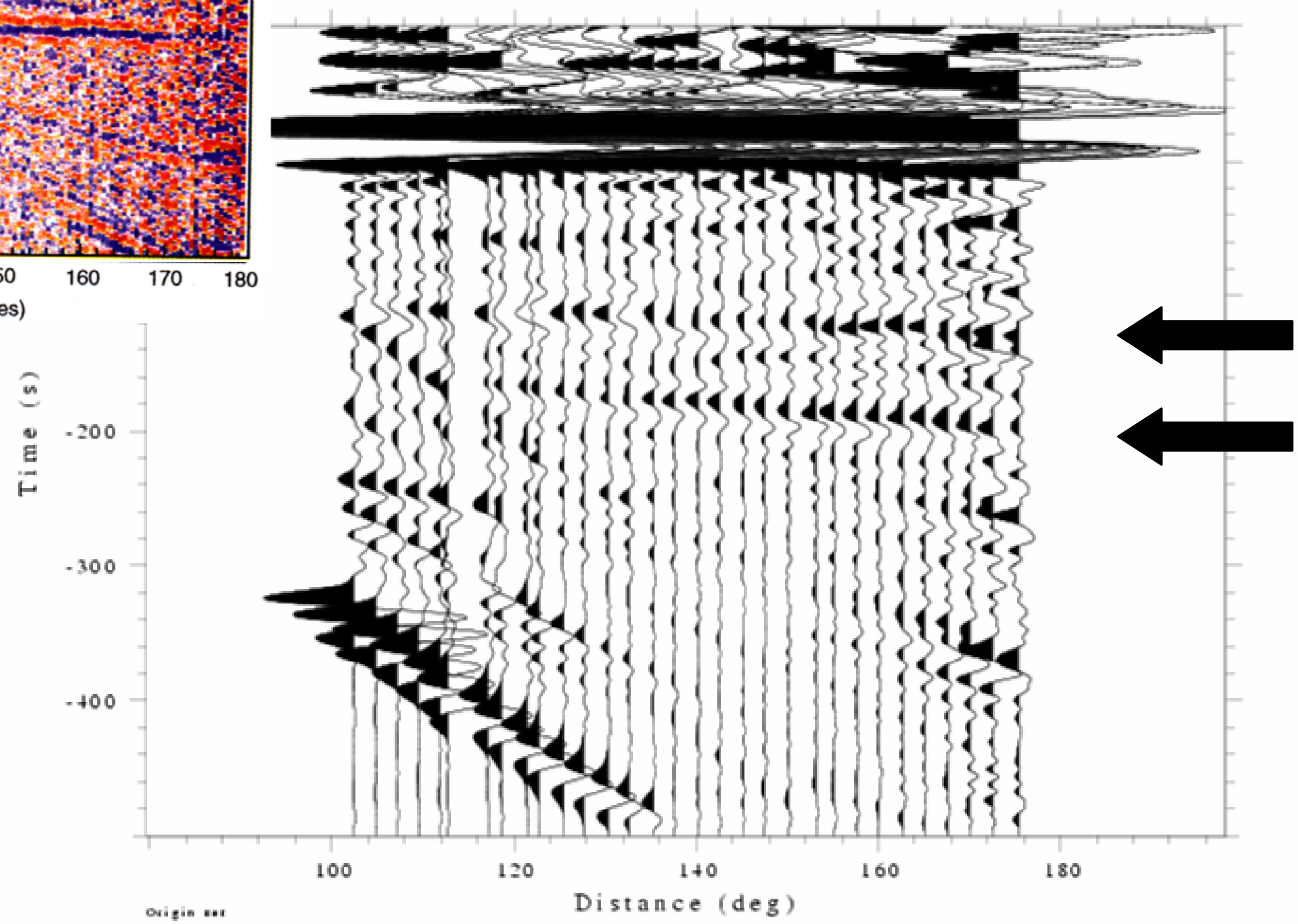
14.5 billion DOF on 1944 procs, down to 5 secs period! 50 h runtime

Study of SS-precursors
Mantle discontinuities

From Flanagan & Shearer JGR 1998

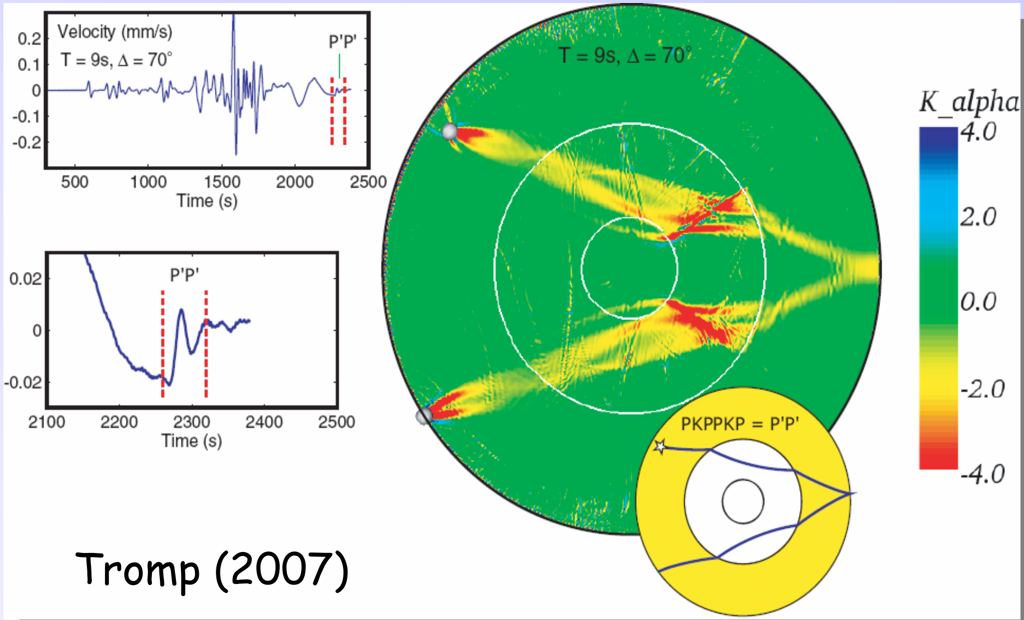


3-D synthetics for Model Earth

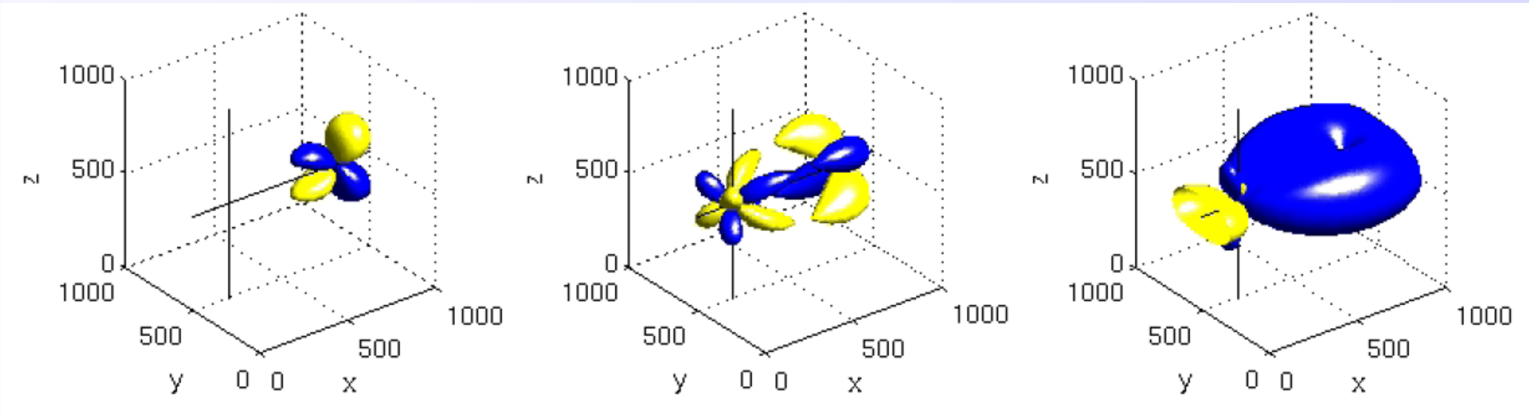


Adjoint methods - sensitivities

Quantification of sensitivities with 3D simulation technology



Tromp (2007)



Fichtner et al. (2007)

Conclusions

- Numerical methods are now widely used for the *forward problem* in many modelling studies
- Young Earth scientists are often not well trained in computational/numerical methods
- Some *fundamentals* should be known when using community software as black boxes