

1)

Inject the trial solution $p(x, t) = p_0 e^{i(kx - \omega t)}$ into the source-free 1D acoustic wave equation $\partial_t^2 p = c^2 \partial_x^2 p$. Discuss the solution.

Show that $p(x, t) = f(x - ct) + f(x + ct)$ is a general solution to the wave equation $\partial_t^2 p = c^2 \partial_x^2 p$. Discuss the result.

2) Show that $\Phi(r, t) = \frac{1}{r} f(r \pm \alpha t)$, where α is the wave velocity, is a solution to the 1-D wave equation in spherically symmetric media:

$$\frac{1}{r^2} \partial_r [r^2 \partial_r \Phi] - \frac{1}{\alpha^2} \partial_t^2 \Phi = 0$$

3) Express the vp/vs ratio as a function of Poisson's ratio defined as: $\sigma = \frac{\lambda}{2(\lambda + \mu)}$.

Calculate the vp/vs ratio for $\sigma=0.3$.

4) Got to seismo-live -> Computational Seismology -> Wave propagation -> Acoustic Green's function

- Run the program and investigate the results
- What is the meaning of the Green's function?
- Discuss the results in 1D, 2D, and 3D, what are the implications?
- Explain the concept of the convolution theorem in connection with Green's functions and the wave equation

5) Computational seismology: You want to simulate wave propagation on a discrete regular grid with physical dimensions $(1000\text{km})^3$ with a dominant period of 10s. The maximum velocity in the medium is 8km/s, the minimum 5km/s. Your numerical algorithm requires 20 points per dominant wavelength. How many grid points do you need? The so-called stability criterion (or Courant criterion) is $\text{const} = c \, dt/dx$ where c is the maximum velocity and $\text{const}=1$. You want to simulate 500s. Determine dt and the number of required time steps for the simulation?