Introduction to seismology: Maths fundamentals

Show – using index notation - that

a) u is a scalar field

$$\Delta u = \nabla \bullet \nabla u$$

b) u is vector field

$$\nabla \bullet (\nabla \times \mathbf{u}) = 0$$

c) u is a vector field, show for one component (e.g., x)

$$\Delta \mathbf{u} = \nabla \nabla \bullet \mathbf{u} - \nabla \times \nabla \times \mathbf{u}$$

Note that

$$\nabla = \begin{pmatrix} \partial_x \\ \partial_y \\ \partial_z \end{pmatrix}$$

and

$$\Delta = \nabla \bullet \nabla$$