Introduction to seismology **Exercise 2**

1) Show that $\Phi(r,t)=\frac{1}{r}f(r\pm\alpha t)$, where α is the wave velocity, is a solution to the 1-D wave equation in spherically symmetric media:

$$\frac{1}{r^2}\partial_r\left[r^2\partial_r\phi\right]-\frac{1}{\alpha^2}\partial_t^2\Phi=0$$

2) Computational seismology: You want to simulate wave propagation on a discrete regular grid with physical dimensions $(1000\text{km})^3$ with a dominant period of 10s. The maximum velocity in the medium is 8km/s, the minimum 5km/s. Your numerical algorithm requires 20 points per dominant wavelength. How many grid points do you need? The so-called stability criterion (or Courant criterion) is const=c dt/dx where c is the maximum velocity and const=1. You want to simulate 500s. Determine dt and the number of required time steps for the simulation?

3) Assume a longitudinal plane wave propagating in x-direction. Show that the time derivative of displacement divided by the longitudinal strain (ε_{xx}) is proportional to phase velocity. Any applications?

4) The Fourier transform (FT) brings a function f(x or t) into its representation in the Fourier domain (k or ω). Show that the FT of the first derivative is given as

   $F\{\partial_x f(x)\}=-ikF(k)$ or $F\{\partial_t f(t)\}=-i\omega F(\omega)$

   Where F(k or ω) is the spectrum of f(x or t). Can you generalise to the n-th derivative? As a consequence, what does the wave equation (acoustic, 1D) look like in the ω-k domain?

   Hint: The definitions of the Fourier transform are (integration from -∞ to ∞):
   Time domain: $f(t)=\int F(\omega)e^{2i\omega t}d\omega \quad F(\omega)=\int f(t)e^{-2i\omega t}dt$
   Space domain: $f(x)=\int F(k)e^{2ikx}dk \quad F(k)=\int f(x)e^{-2ikx}dk$

   Replace f(x) with d/dx f(x) (or d/dt f(t)) in the above equations and integrate by parts:
   $$\int_a^b f(x)g'(x)dx=[f(x)g(x)]_a^b-\int_a^b f'(x)g(x)dx$$