Computational seismology – *Finite element method*

1. **Finite Element: static problem.** Calculate with the help of the finite-element formulation in Matlab® the following problem:

$$-\Delta u(x) = f(x)$$

Follow the lecture material and implement the calculation of the stiffness matrix A as:

```
% assemble matrix Aij
A=zeros(nx);

for i=2:nx-1,
   for j=2:nx-1,
      if i==j,
         A(i,j)=1/h(i-1)+1/h(i);
      elseif i==j+1
         A(i,j)=-1/h(i-1);
      elseif i+1==j
         A(i,j)=-1/h(i);
      else
         A(i,j)=0;
      end
   end
end
```

The calculation should be made in the interval [0,1]. The space is discretised with 5 points: 1/10 * [0, 2, 3, 8, 10]. The boundary conditions are u(0)=0.5, u(1) =3. The forcing is given as $f(x) = \delta(x - x_s)$, x_s=3. Complement the above Matlab code part to the complete solution. Note that through the Delta function the elements of the source vector are simply the boundary value divided by the element size.

2. Finite element, time-dependent problem

We want to compare the numerical solutions of the acoustic wave equation using finite differences and finite elements. Download the program femfd.m and the circular shift function csh.m and run it (>> femfd). Inspect the source code and appreciate the entirely different approach to solving the acoustic wave equation (by matrix inversion in the FEM case and by differencing in the FD case). We want to demonstrate the flexibility of the FEM approach by modifying the regular grid spacing. Options are (1) to add a random perturbation to the regular grid spacing, (2) to make a sinusoidal perturbation of the grid spacing, (3) Gaussian perturbation, (4) Chebyshev collocation points. Implement one or several of those perturbations and investigate the influence on the solution.

Access to codes here:
http://www.geophysik.uni-muenchen.de/~igel/Utrecht07/practical.html