The Finite – Difference Method 2

Theoretical Problems

I.

A seismometer consists of a spring with damping parameter ϵ, and eigenfrequency ω_0. The seismometer is excited by the (given) ground motion $\ddot{u}(t)$. The relative motion of the seismometer mass x(t) is governed by the following equation

$$\ddot{x} + 2\epsilon\dot{x} + \omega_0^2 x = \ddot{u}.$$

Replace the derivatives on the left-hand side with finite differences. Solve for $x(t + dt)$. Note: a good strategy in this example is to centre the differences at the same point in time. The dots denote time derivative.

II.

Certain isotopes (e.g. $_9$Be) are washed into the sea by rivers and then mixed by advection through ocean currents and diffusion. In addition, the isotopes are removed from the system through biomechanical processes (e.g. death). These processes can be described by the diffusion–advection–reaction equation (concentration $C(x, t)$, diffusivity k (const),

reactivity $R(x)$, source $p(x)$, advection velocity v). Substitute in the 1D equation below the partial differentials with finite differences and extrapolate to $C(t + dt)$:

$$\partial_t C = k\partial_x^2 C + v\partial_x C - RC + p.$$

How could a ring current be simulated with this 1D equation mimicking an oceanic gyre? What do you think is the best choice for the finite-difference formulation and why?

III.

You want to simulate 2-D acoustic wave propagation in a medium with size 1000km x 1000km. You want to model wave propagation up to a period of 10 seconds. The maximum velocity c is 8km/s, the minimum velocity is 4km/s. Your numerical algorithm requires 20 grid points per wavelength to be accurate for the propagation distances of interest. What space increment dx do you need for the simulation? The stability criterion says that maximum velocity c, space increment dx and time increment dt are related by $const = cdt/dx$. You want a seismogram length of 500 seconds. How many time steps do you have to simulate, when const=0.5?