# The Finite – Difference Method

## Comprehensive Questions

Are finite-difference-based approximations of partial-differential equations unique (give arguments)?
What strategies are there to improve the accuracy of finite-difference derivatives? Give the procedures in words.

## Theoretical Problems

Show that

$$\frac{f(x+dx) - 2f(x) + f(x-dx)}{dt^2}$$

is an approximation for the second derivative of $f(x)$ with respect to $x$ at position $x$. Hint: Use Taylor series

$$f(x+dx) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x)}{n!} dx^n,$$

The source-free advection equation is given by

$$\partial_t u(x,t) = v \partial_x u(x,t),$$

where $u(x, t = 0)$ could be a displacement waveform at $t = 0$ (an initial condition) that is advected with velocity v (this will become important in Chapters 8 and 9 on finite volumes and the discontinuous Galerkin method, respectively). Replace the partial derivatives by finite differences. Which approach do you expect to work best? Turn it into a programming exercise and write a simple finite-difference code and play around with different schemes (centred vs. non-centred finite differences). What do you observe?

## Programming exercise

Go to www.seismo-live.org -> Computational seismology -> Finite Difference method

- Click “Open” on “fd_first_derivative” , extend to 5 point operator
- Click “Open” on “fd_ac1d”, extend to 5 point operator