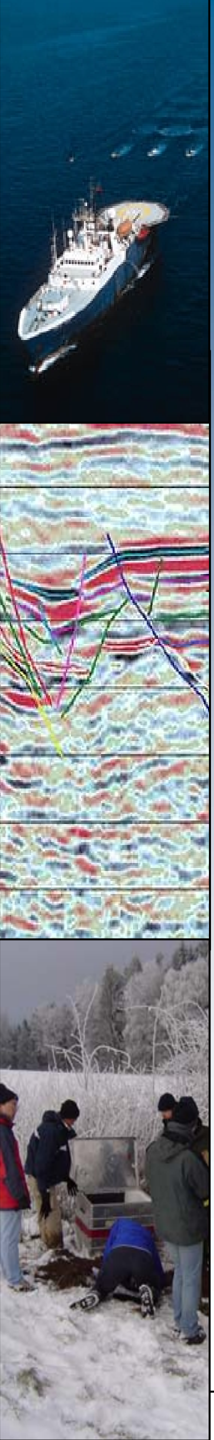


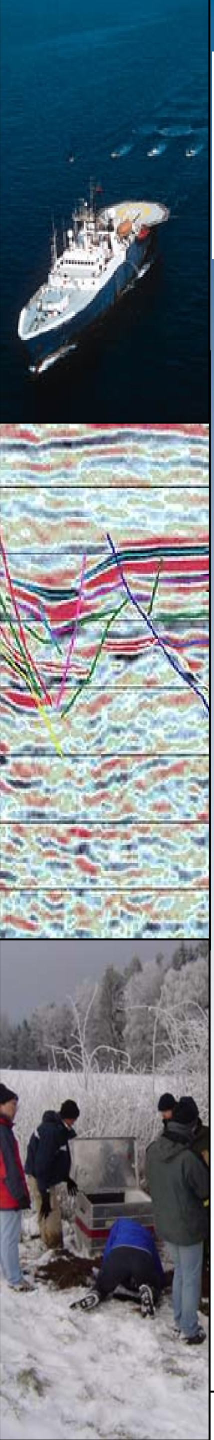
Seismic Tomography: Example of a geophysical inverse problem

Dr. Karin Sigloch (karin.sigloch@lmu.de)
Theresienstr. 41, Zi. 445

1. What is an inverse problem?

2. A real-world example: tomography of the Earth's mantle under North America

3. How does it work?



Q: What is an „inverse problem"?

A: An **indirect measurement.**

We want to measure some important „EARTH_PROPERTY" (e.g., seismic velocity v(x)), and have no tools to do it.

Instead we know how to measure some other property called „DATA" (e.g., traveltime delays dT)

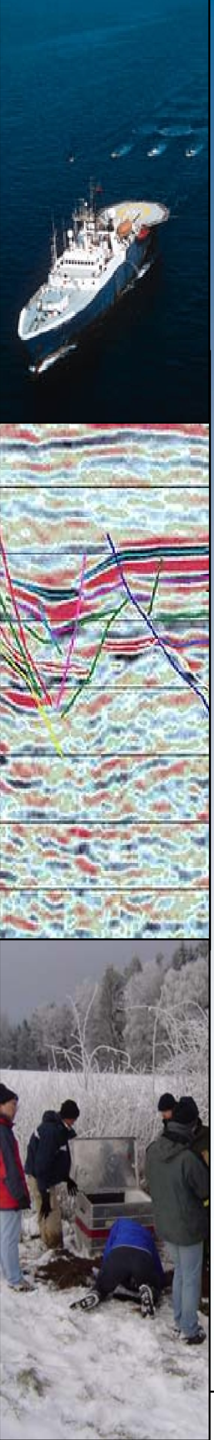
And we know some phys./math. relationship „MAPPING_FCT", so that:

DATA = MAPPING_FCT(EARTH_PROPERTY)

If we are able to find an „inverse function" MAPPING_FCT^{-1} so that

EARTH_PROPERTY = MAPPING_FCT^{-1}(DATA),

then the problem is solved.



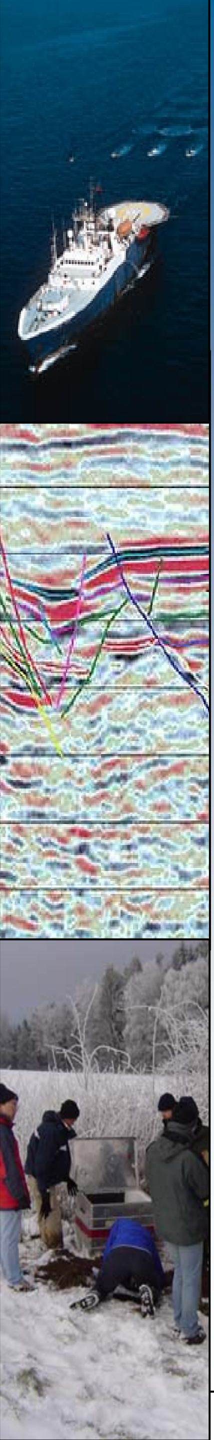
Inverse problems are common

Seismology

EARTH_PROPERTY: as a function of space (x,y,z), e.g., P-velocity or intrinsic attenuation, or rock composition

DATA: Seismograms (and data dreived from them, like traveltimes, amplitudes...) at discrete points at the surface

MAPPING_FCT: wave equation (or some approximation to it, like rays from Snell´s law)



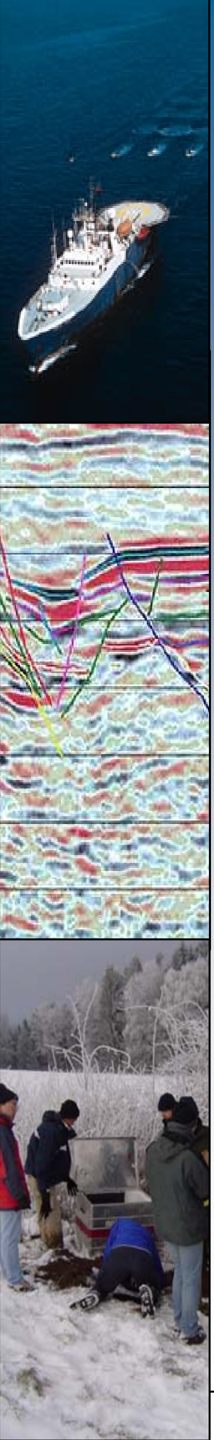
Inverse problems are common

Medical Imaging: Computed Tomography

EARTH_PROPERTY: structure of tissue in the human body

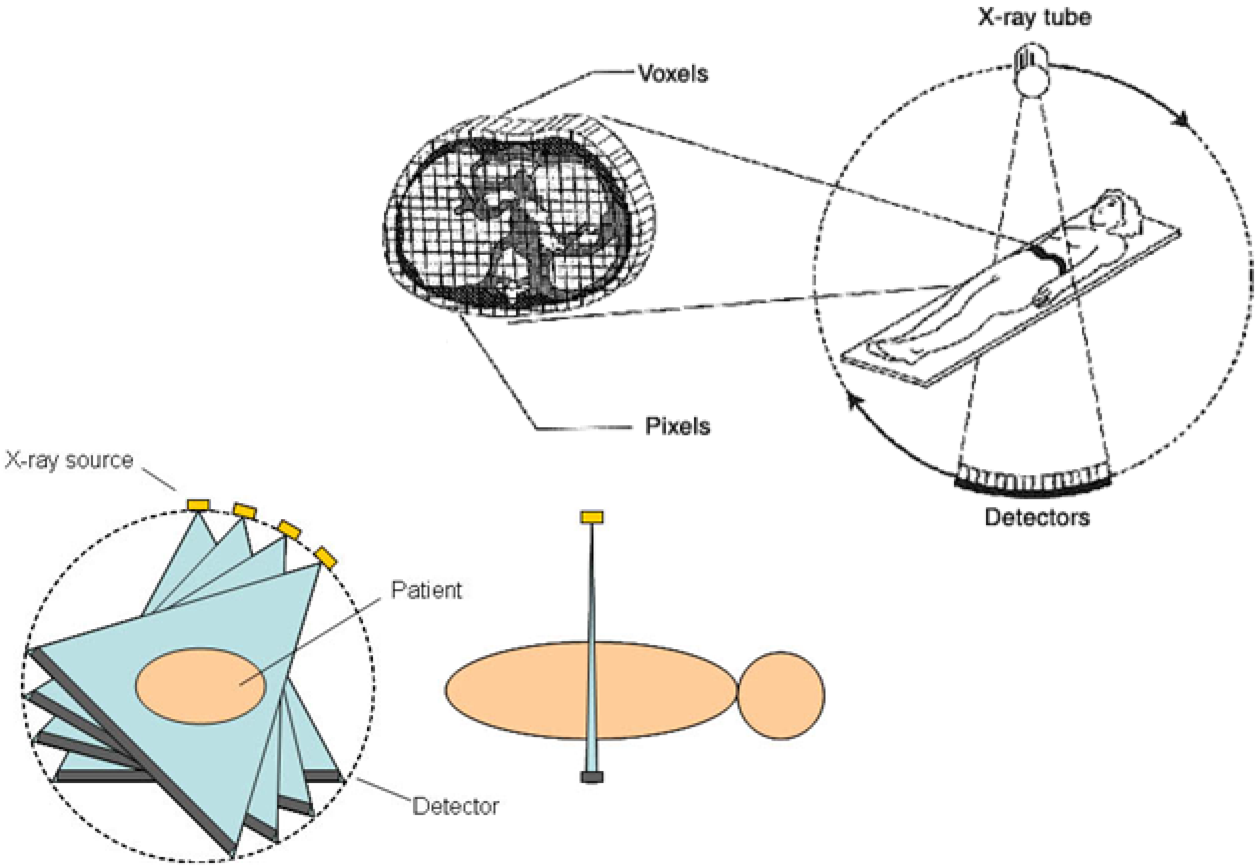
DATA: X-ray imaging in multiple plane -- by how much do x-rays get attenuated?

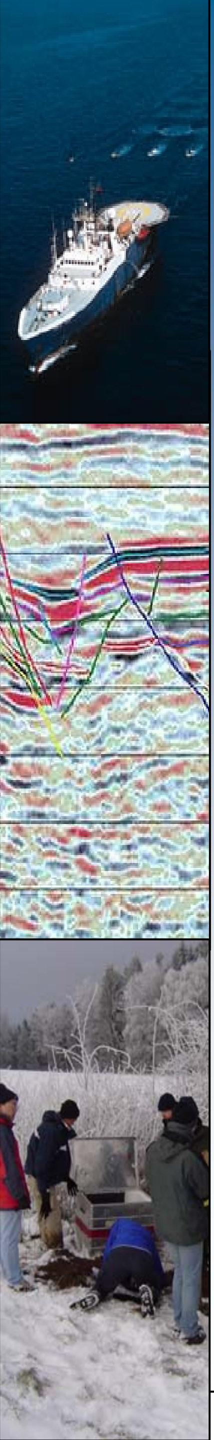
MAPPING_FCT: wave propagation and attenuation (optics, geometrical ray approximation)



Inverse problems are common (also outside geophysics)

Medical Imaging: Computed Tomography



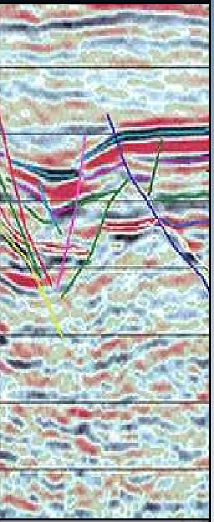


Inverse problems are common

Planetary Science:Composition of a Jupiter moon

EARTH_PROPERTY: density as function of x,y,z
DATA: gravity measurements: deflection of a satellite upon its fly-by
MAPPING_FCT: Newton's law of gravity

Common theme: measure interior properties from the outside…
…but not all inverse problems are like that…



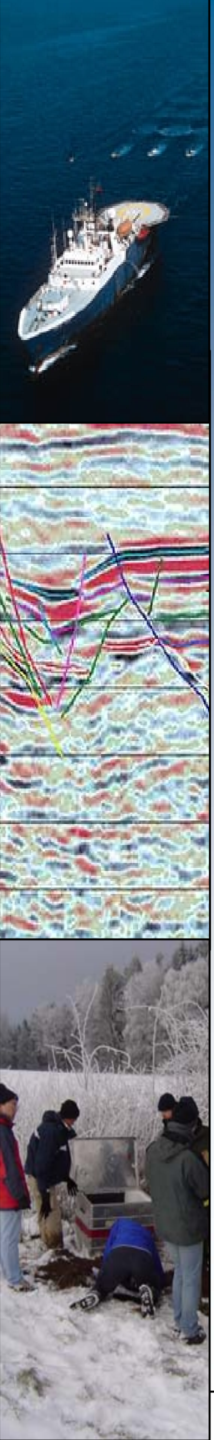
Inverse problems are common

Borehole seismics:

EARTH_PROPERTY: shallow earth properties (velocity, density, attenuation,…) as a function of depth

DATA: hydrophone recordings inside the borehoel

MAPPING_FCT: wave equation: reflections



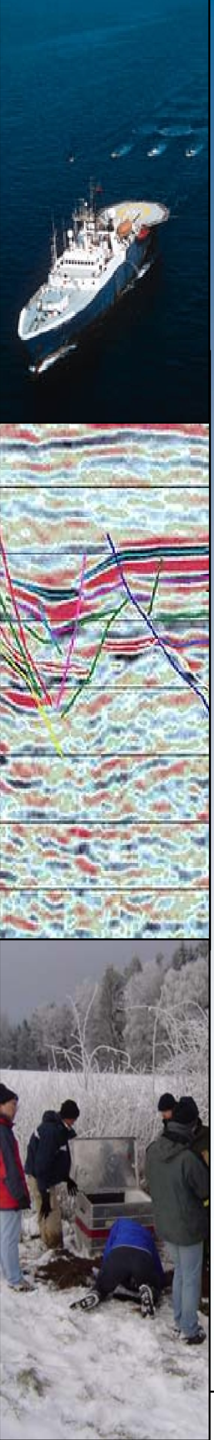
Inverse problems are common

Environmental remediation/hydrology:

EARTH_PROPERTY: source location(s) and quantity of contaminants

DATA: contaminant sensors in several deep holes around a chemical factory

MAPPING_FCT: diffusion equation/transport in porous media



Summary: What is an inverse problem?

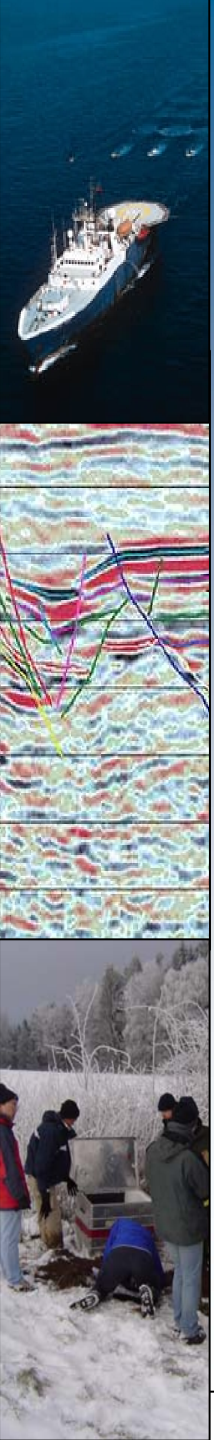
We are unable to directly measure an interesting EARTH_PROPERTY.

Instead we measure some other DATA, because we know how to derive/compute a physical relationship MAPPING_FCT so that:

DATA = MAPPING_FCT(EARTH_PROPERTY)

We try to find the „inverse“ MAPPING_FCT^{-1}, so that

EARTH_PROPERTY = MAPPING_FCT^{-1}(DATA)

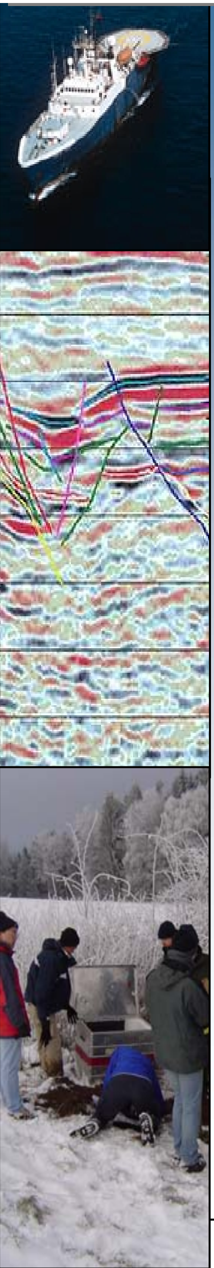


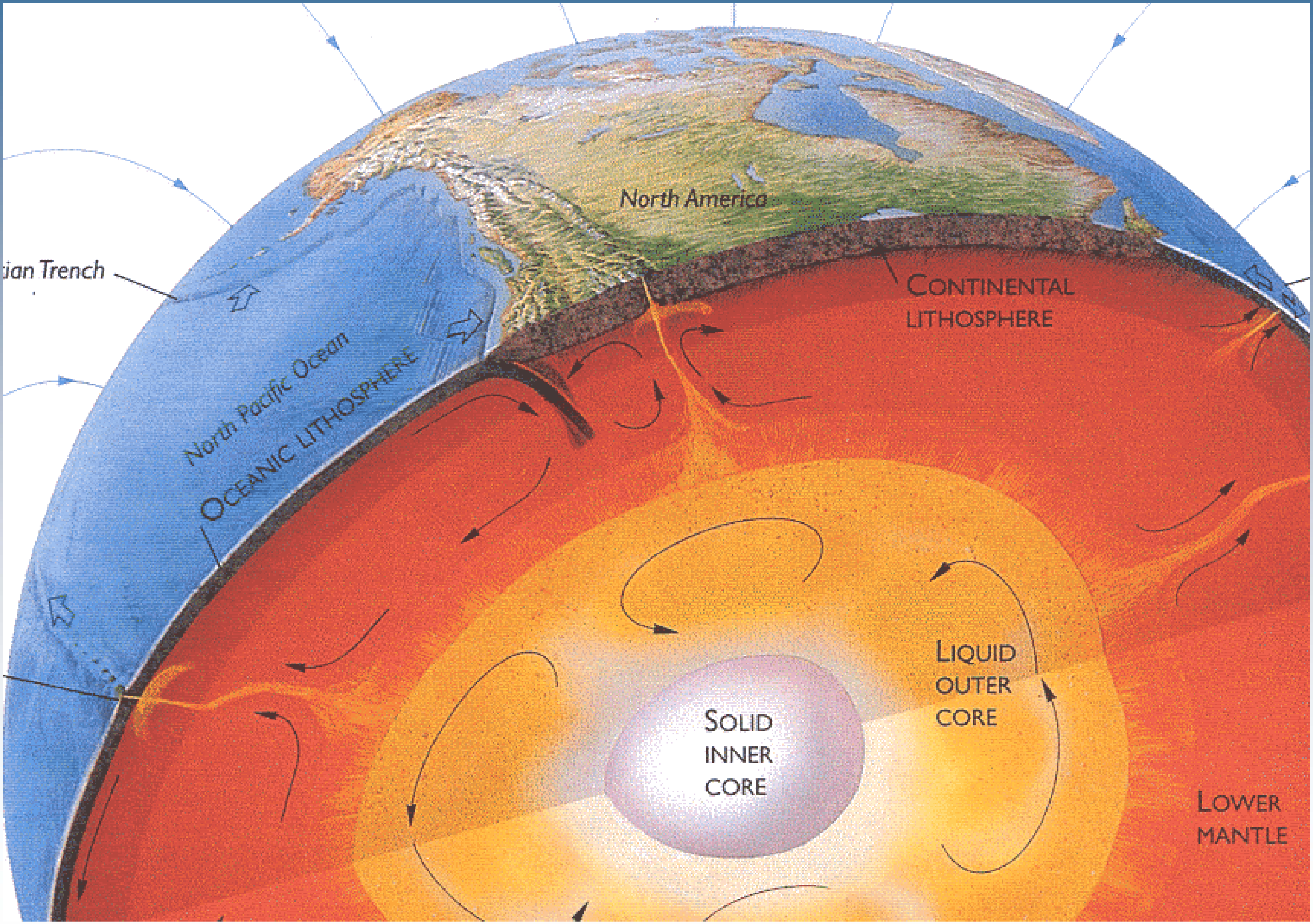
A realistic experiment: Seismic tomography of the Earth's mantle

Geophysicists' mission: Discover **new** things about the Earth.

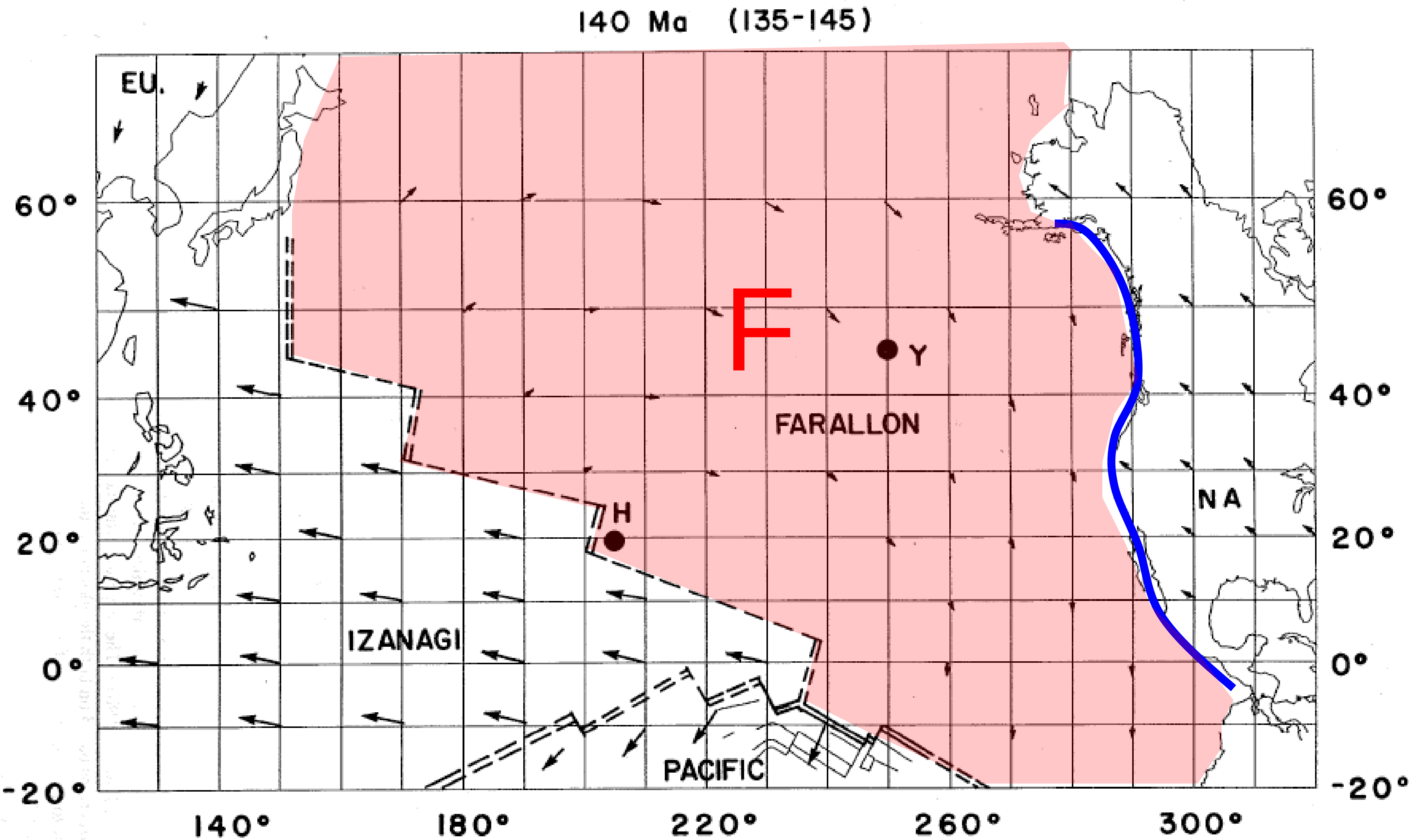
If it is a good problem then many other people (geologists, geodynamicists, economists, etc.) will be interested in the results.

Imaging the subducted Farallon plate under North America



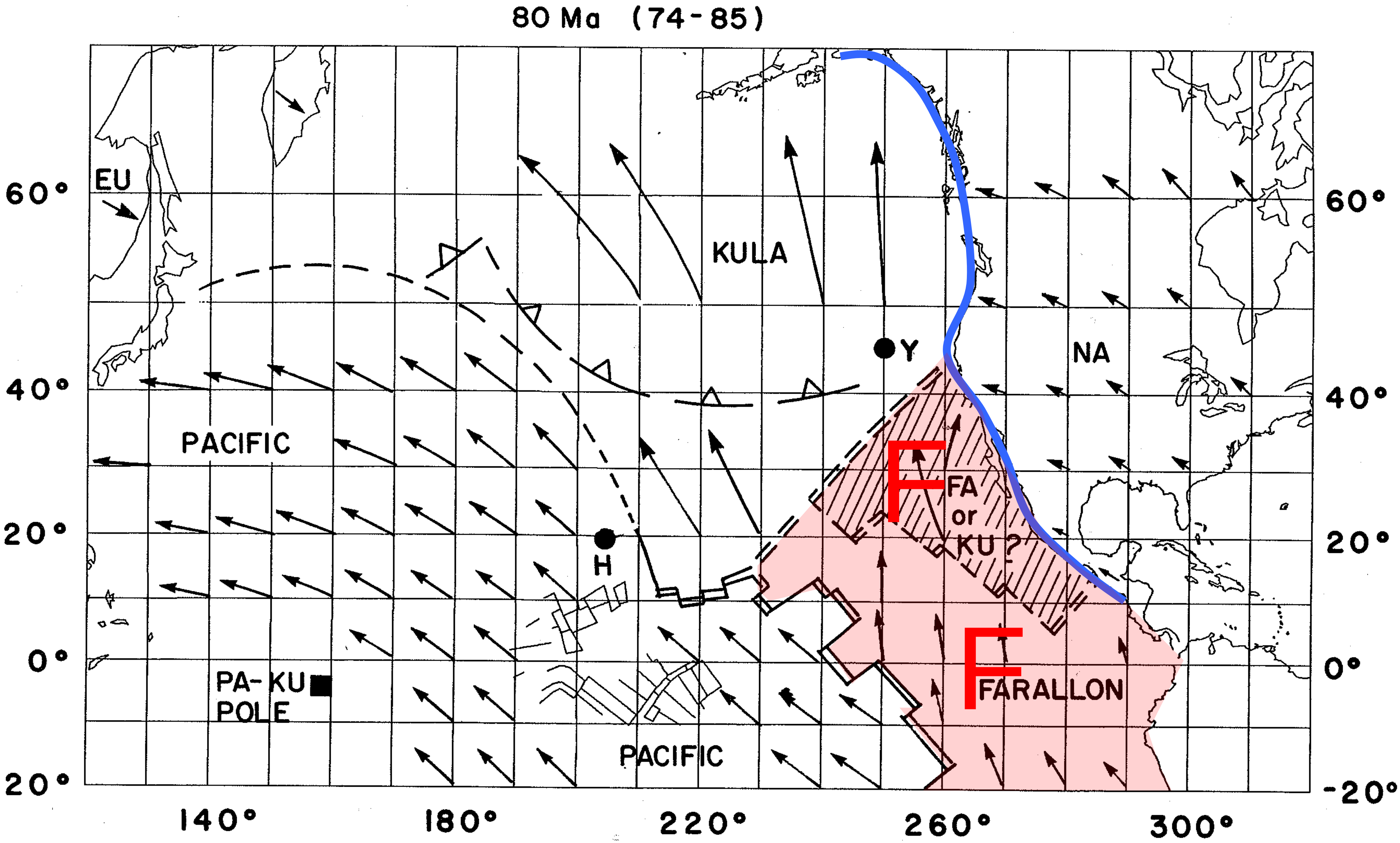


The Farallon plate 140 Myr ago…



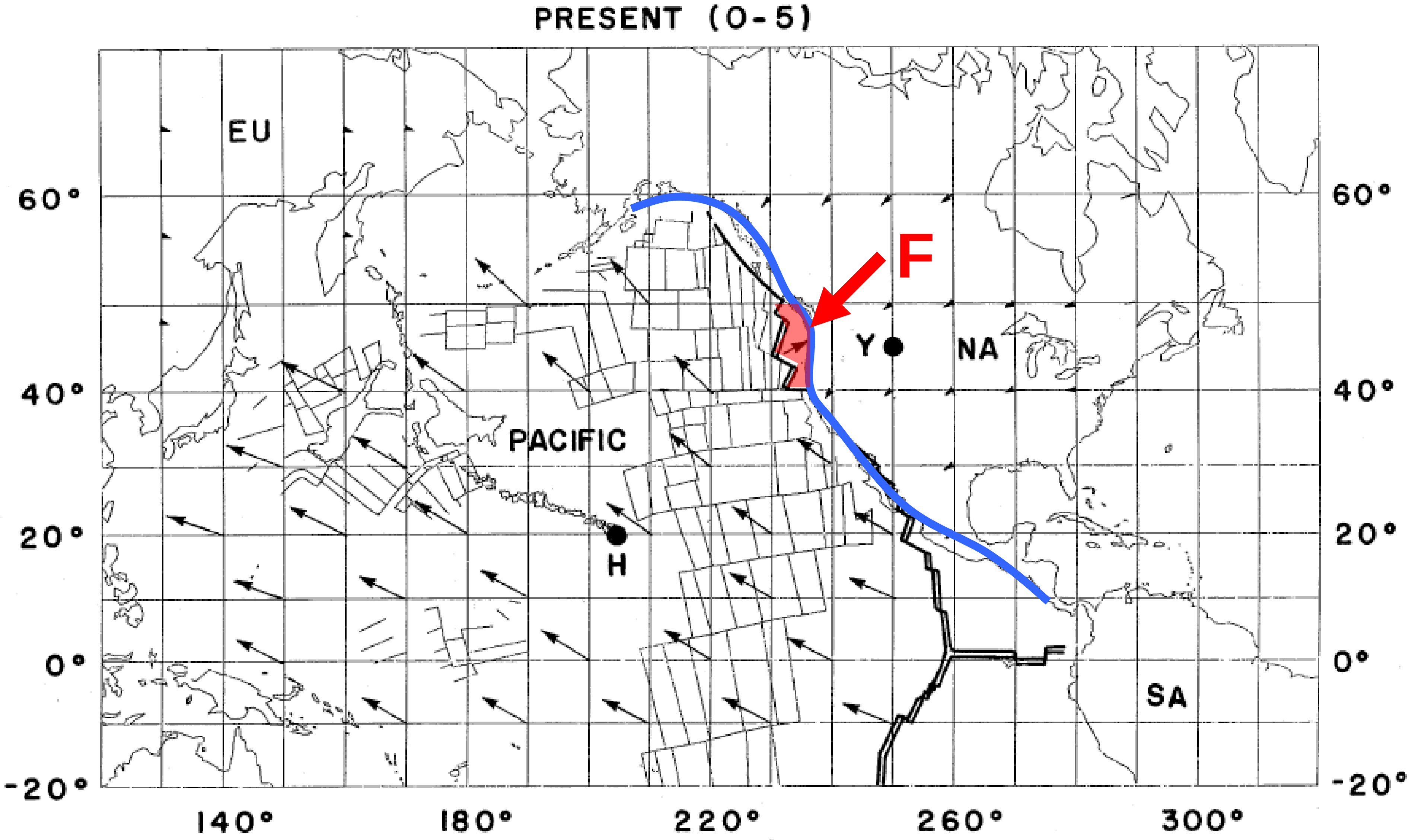
Engebretson et al. 1985

…and 80 Myr ago…



Engebretson et al. 1985

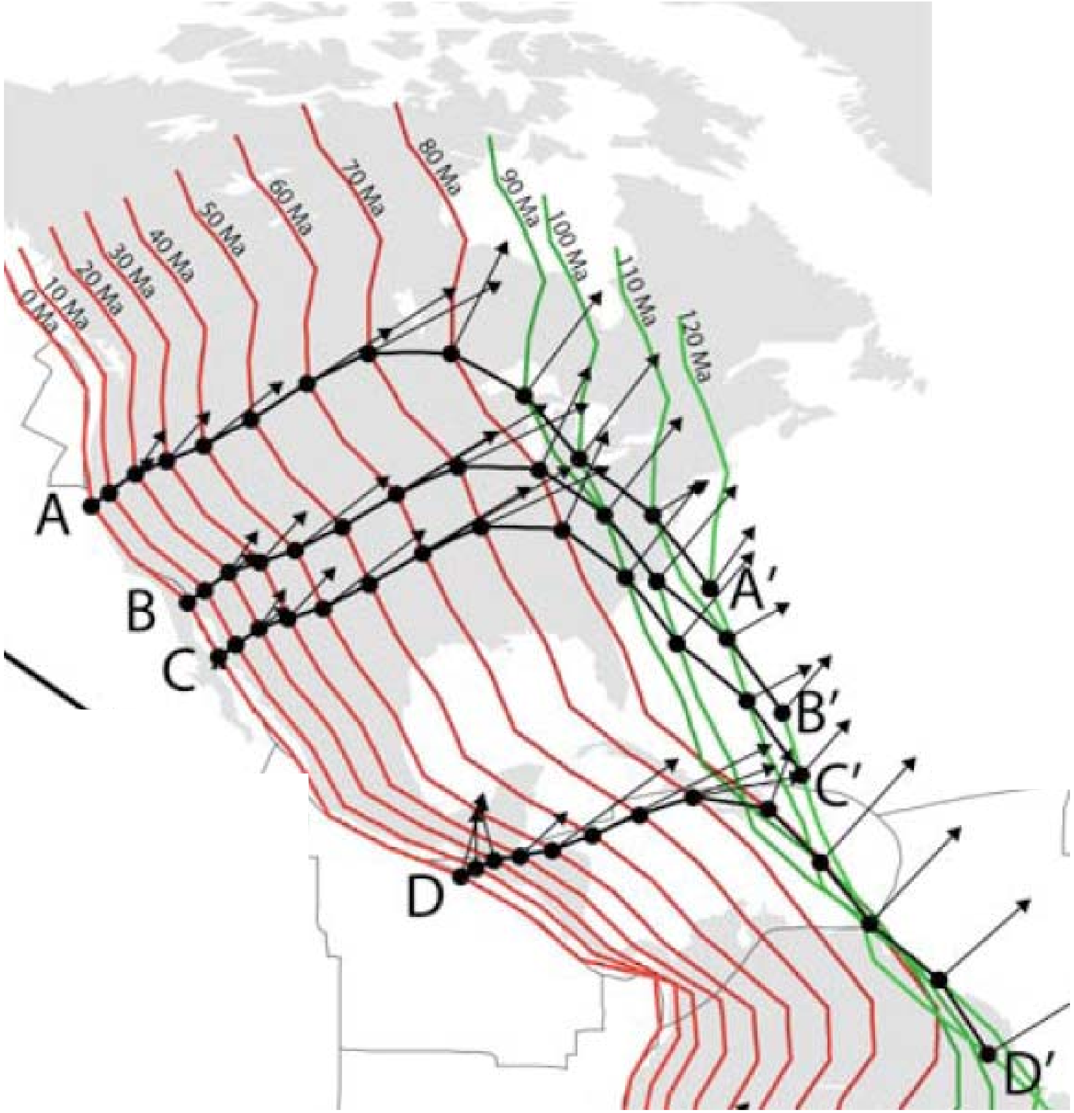
…and today.



Engebretson et al. 1985

150 million years of textbook-like subduction?

A single large plate has been subducting beneath the North American west coast for 150+ million years. No significant interference from other plates.

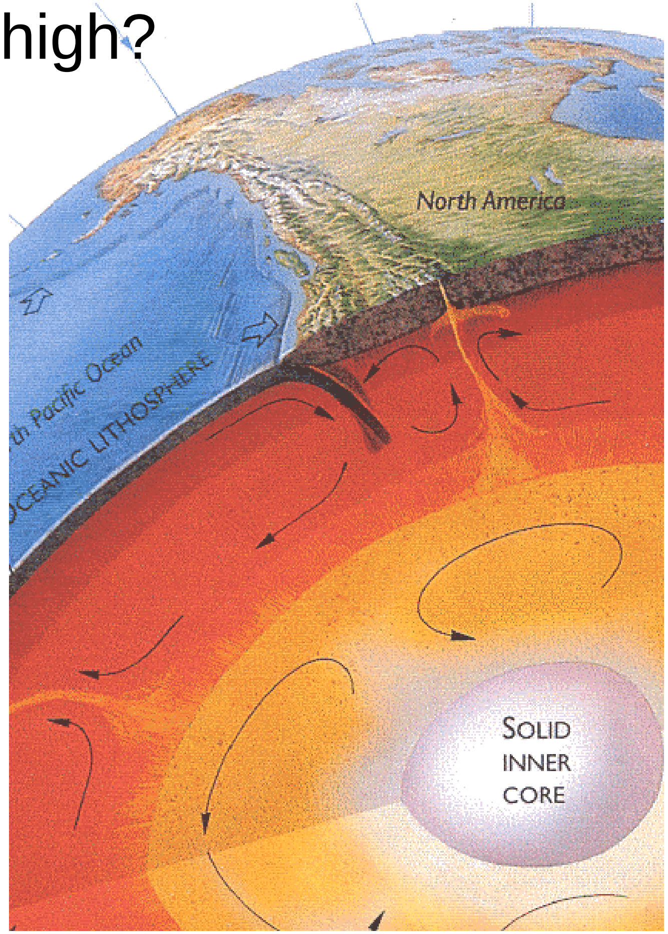


Ren et al 2007, after Engebretson 1985

A simple story? Yes, but.

Extensive mountain building and volcanism far inland (since ~70 Myr). Not a “conventional” volcanic arc.

Why is the North American Cordillera so wide and stands so high?



The “Laramide orogeny”: Rapid uplift, far inland at ~70 Myr ago

75 Myr ago

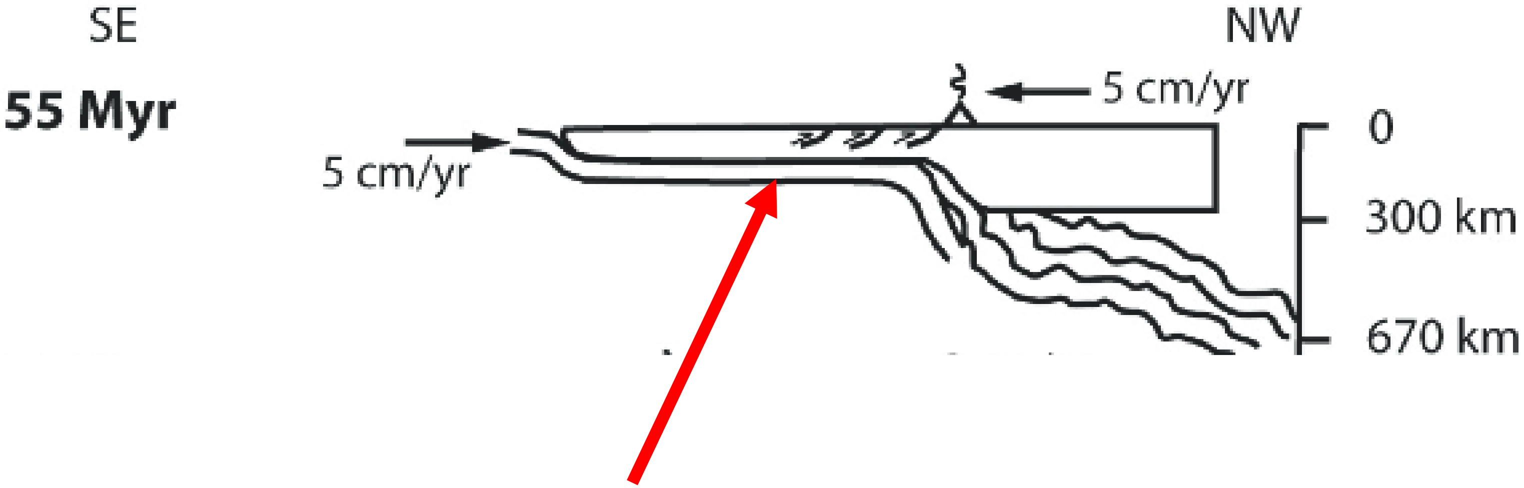
A shallow inland sea covers the Rocky Mountain area

65 Myr ago

Laramide orogeny (70-50 Myr): basement uplift by thrust faulting, volcanic arc along trench has shut off.

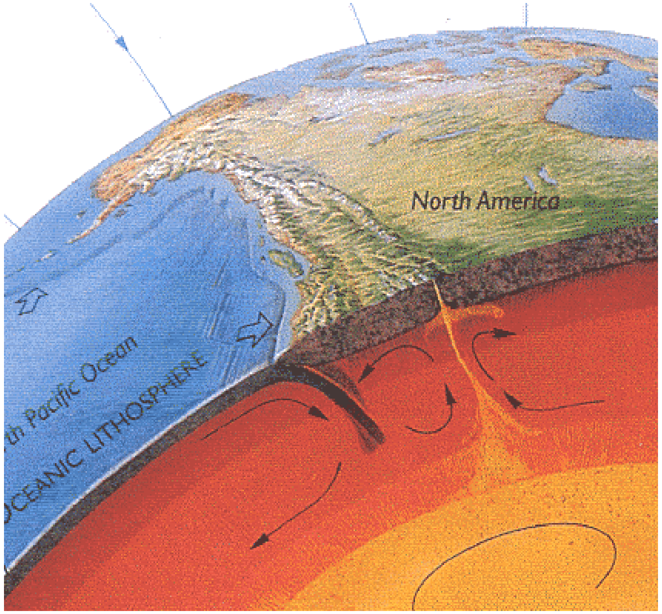
graphics: Blakely (online)

Geologists' explanation: Laramide thrust faulting was caused by anomalously flat subduction



Extremely flat slab scrapes along bottom of continental crust

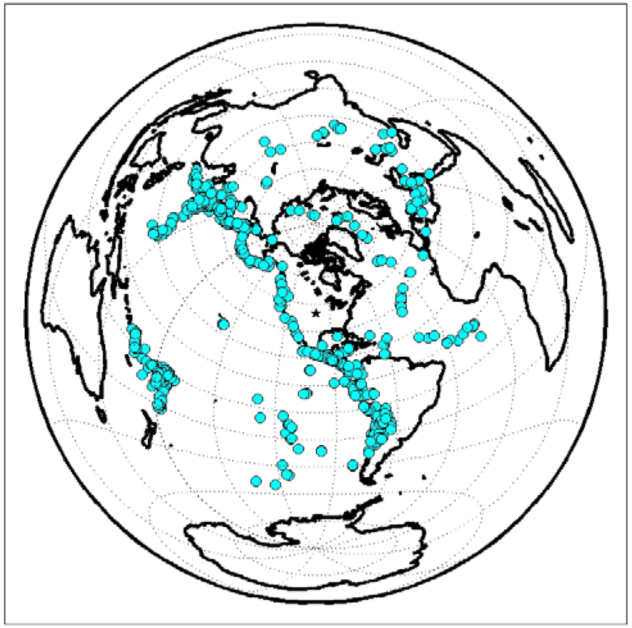
…but Western North America has stood high ever since.



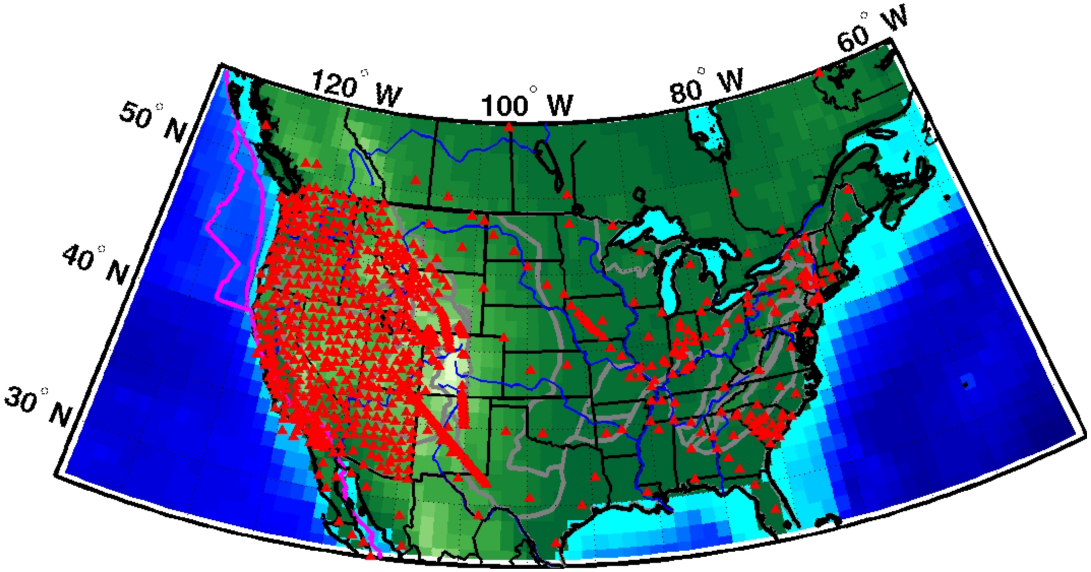
NASA satellite photo of Western U.S.; mountains are from Laramide times

Our tomographic experiment

637 earthquake sources



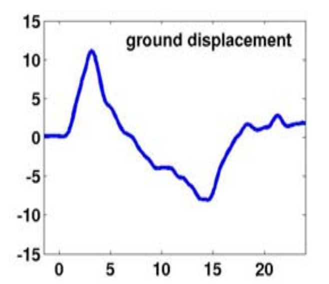
1125 broadband receivers (seismometers)



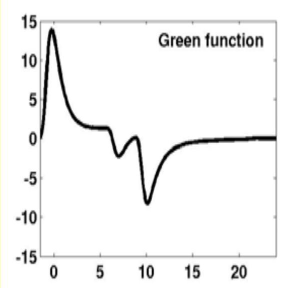
- We use teleseismic P-wave seismograms from large earthquakes (magnitude >= 5.8, 1990-2007)
- Many new USArray stations in Western U.S since 2005

Tomography step by step

observed seismogram

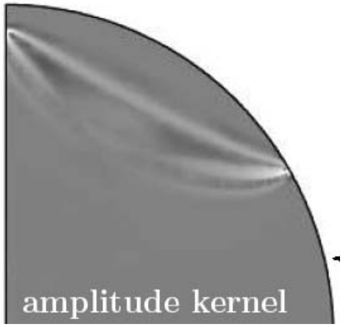


predicted seismogram

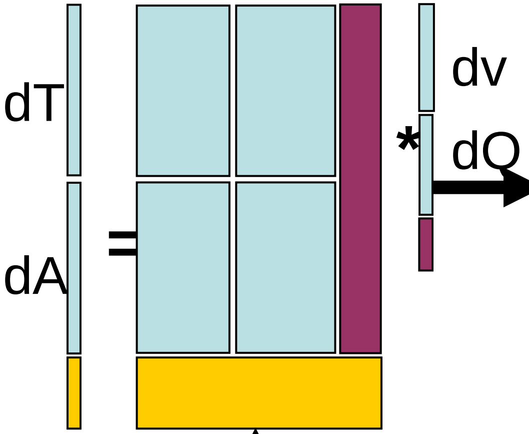


- deconvolve source time function
- extract scalar observables dT(f), dA(f)

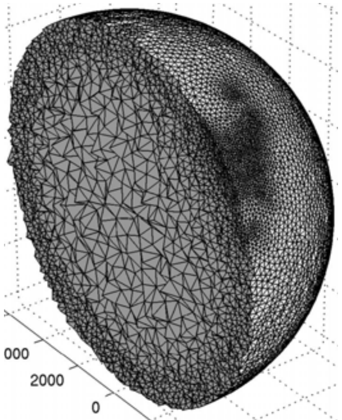
compute sensitivity kernels



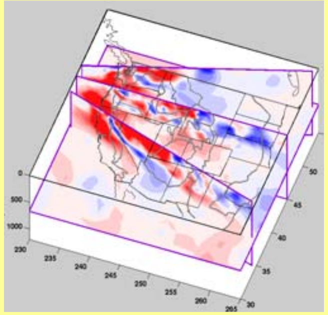
solve



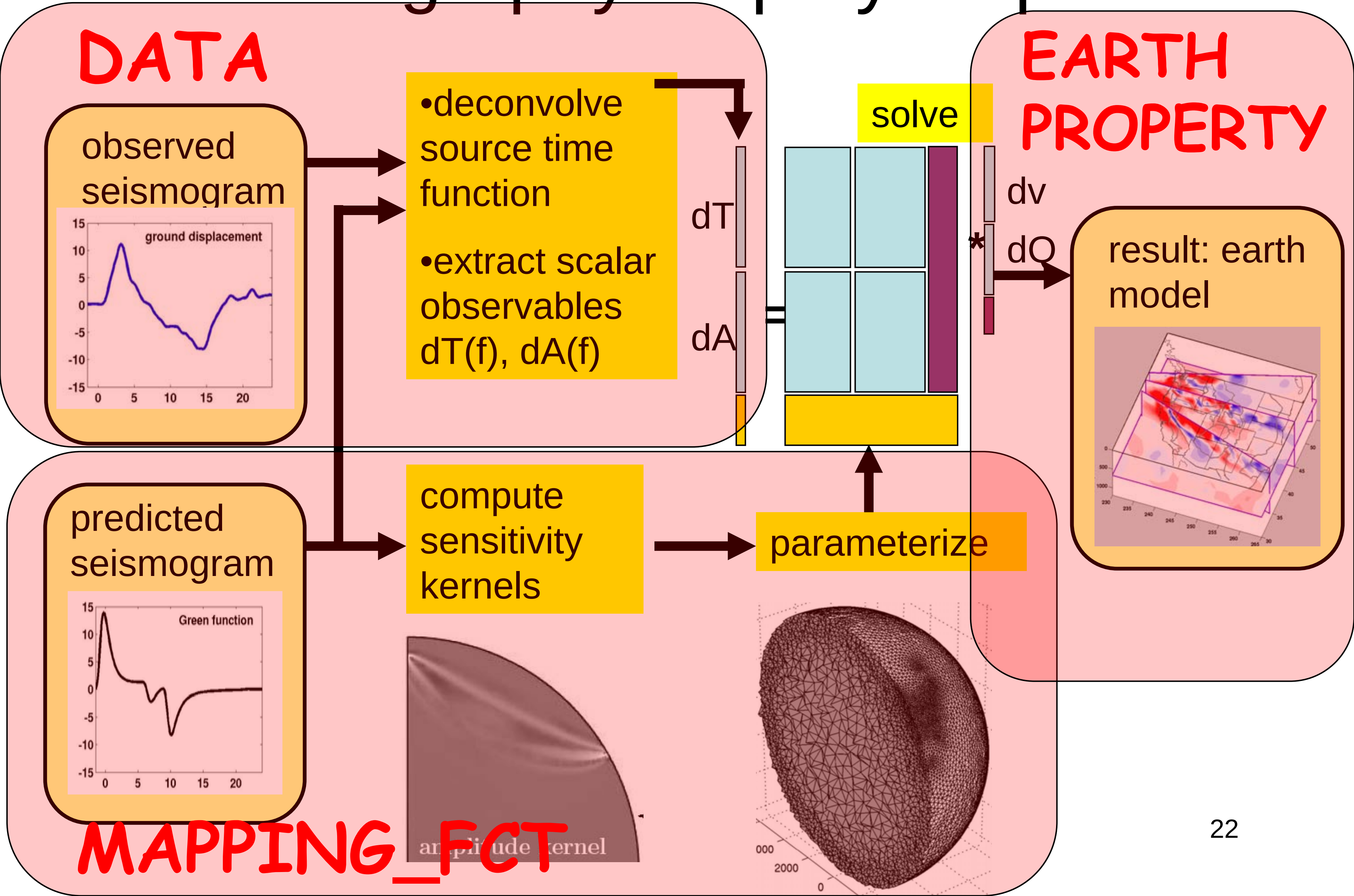
parameterize



result: earth model



Tomography step by step



DATA

observed seismogram

- deconvolve source time function
- extract scalar observables dT(f), dA(f)

dT

dA

=

solve

*

EARTH PROPERTY

dv

dQ

result: earth model

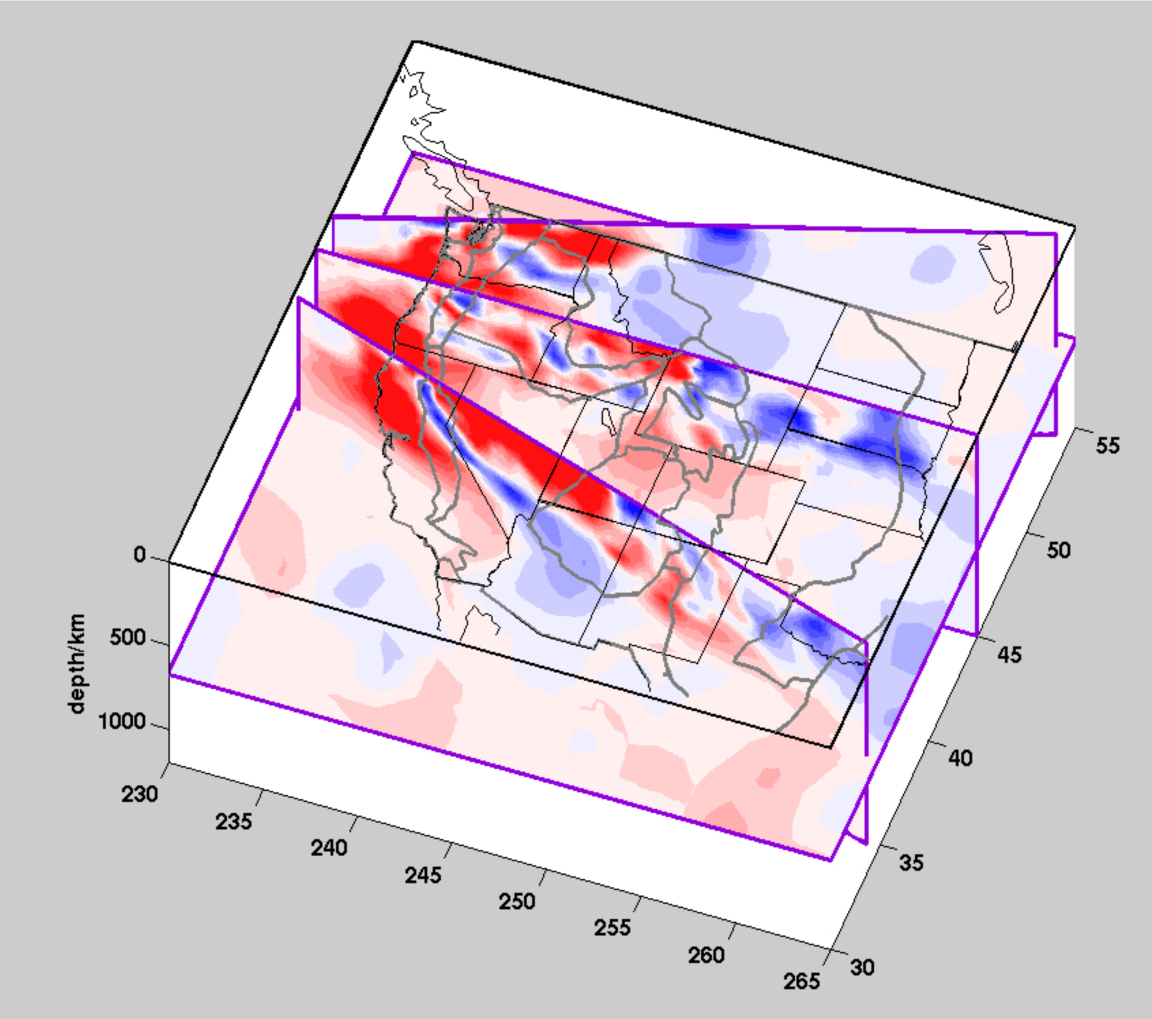
predicted seismogram

compute sensitivity kernels

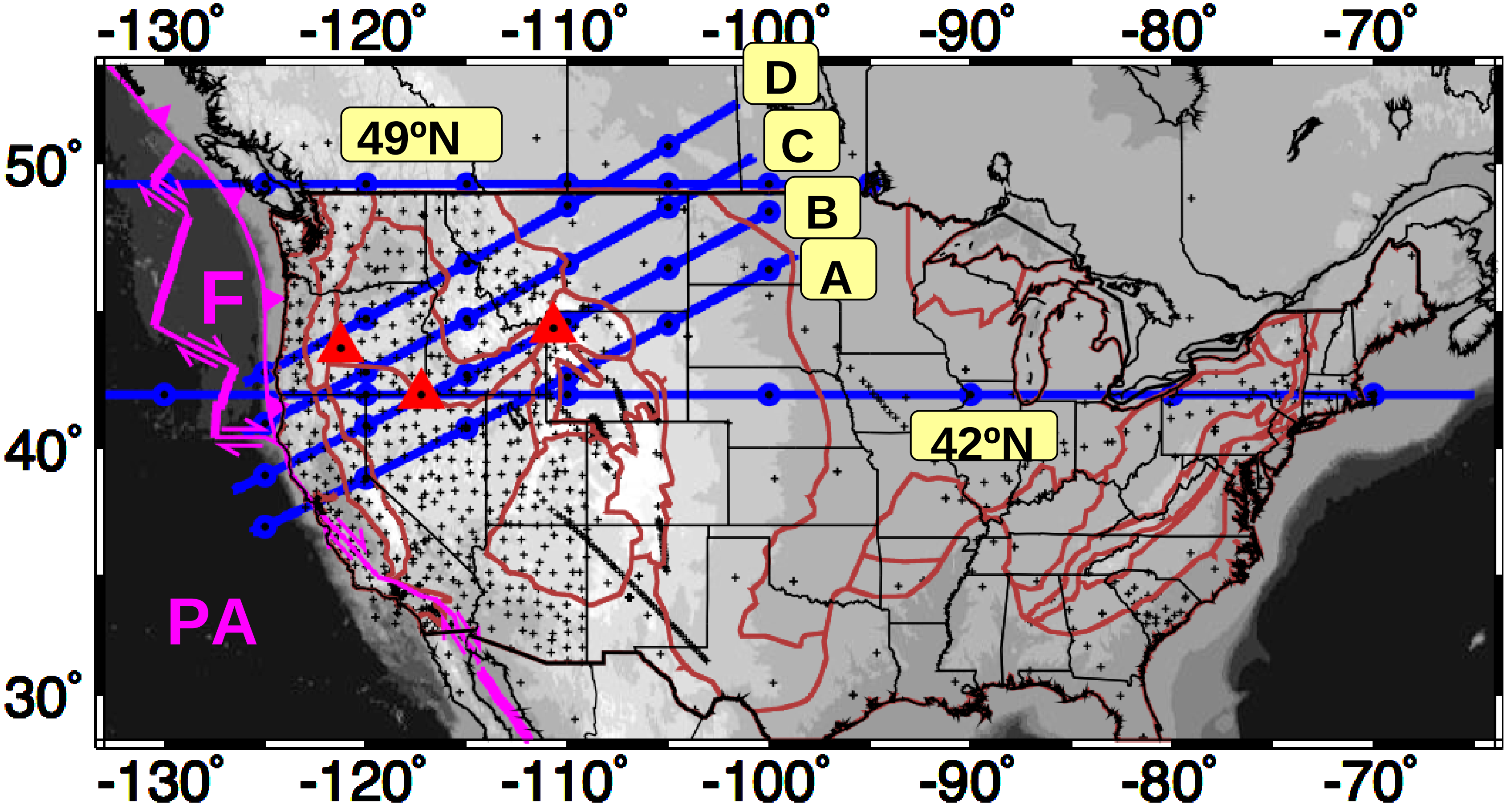
parameterize

MAPPING_FCT

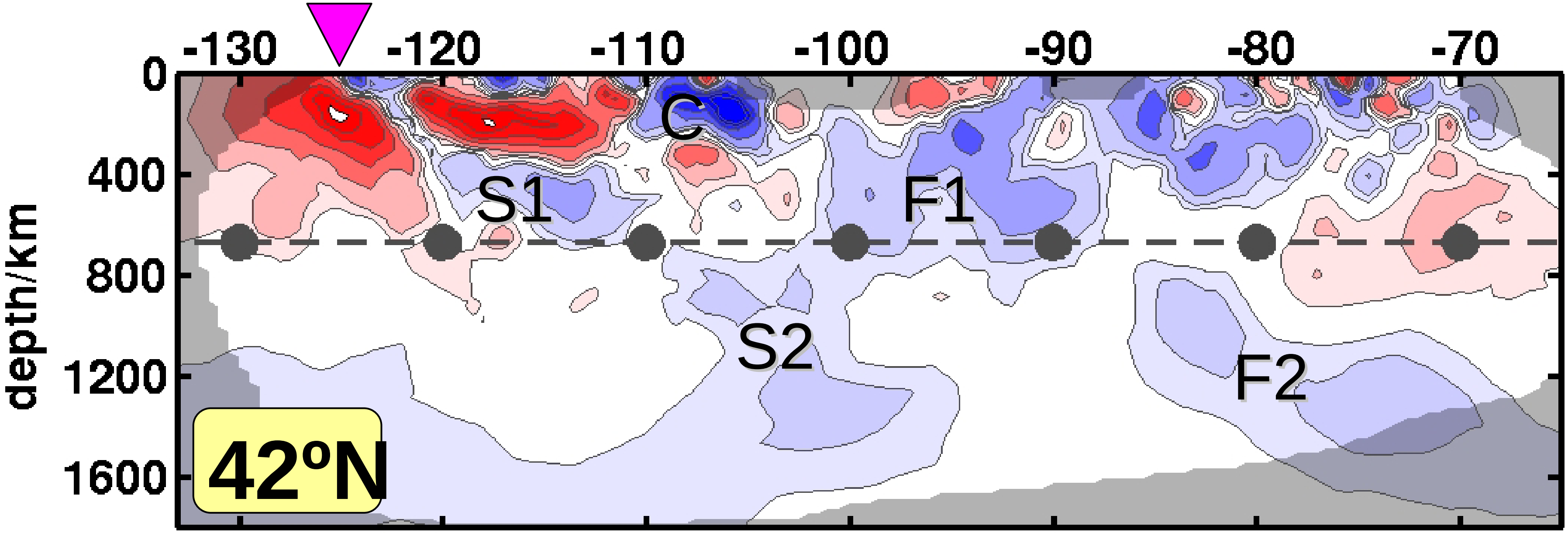
Result: 3-D model of P-wave velocities under North America

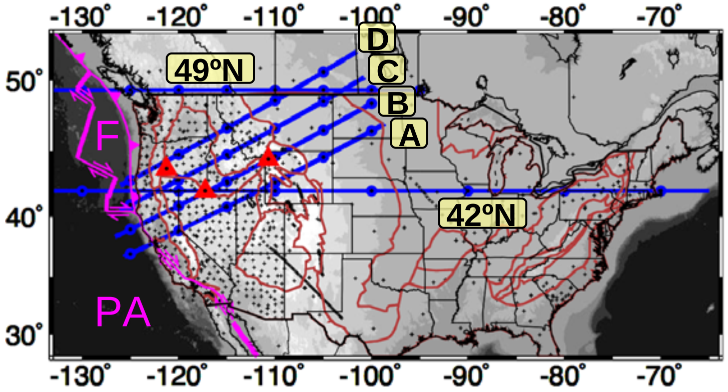


Locations of interesting cross-sections



The big picture: Not one, but two episodes of whole mantle subduction





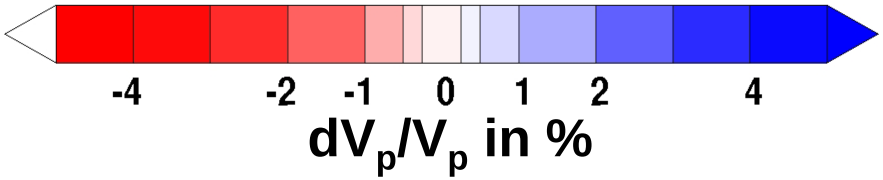
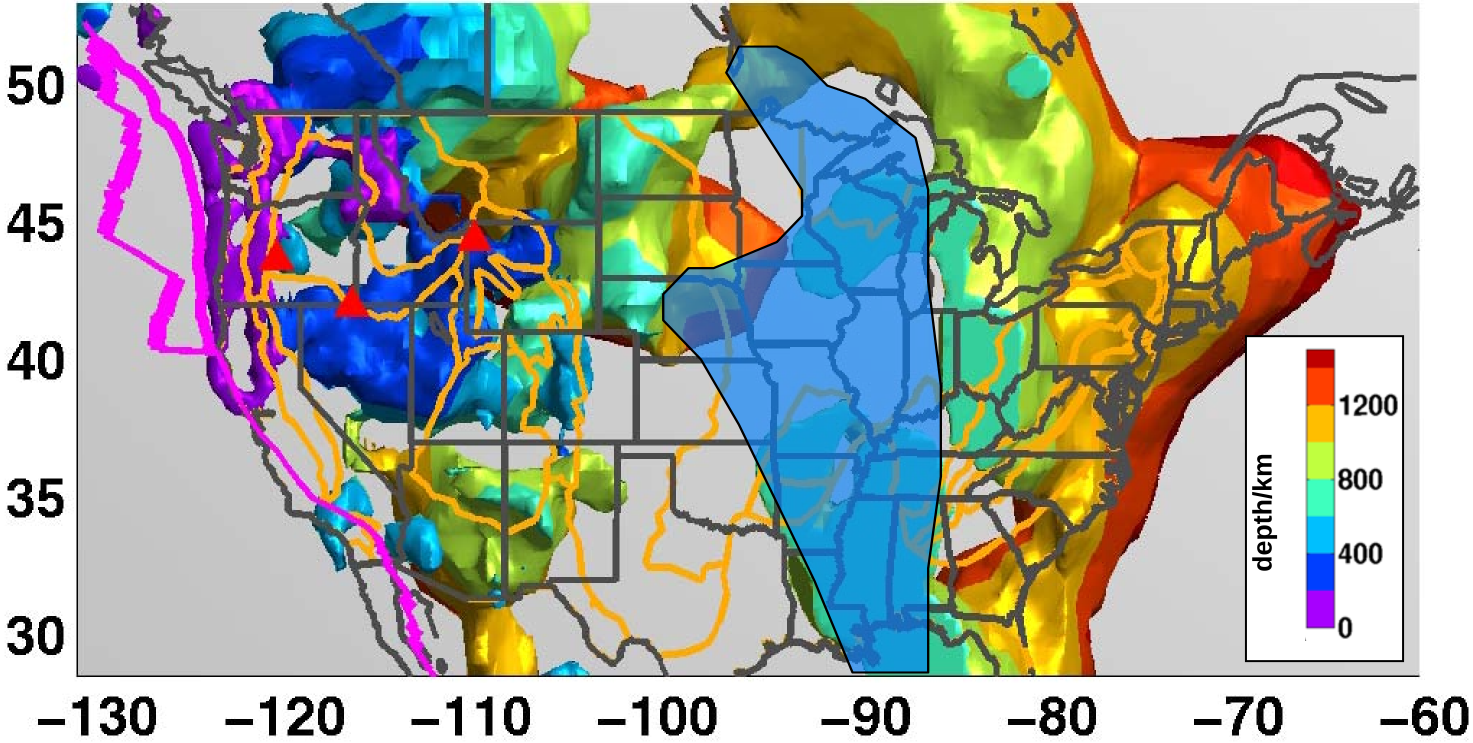
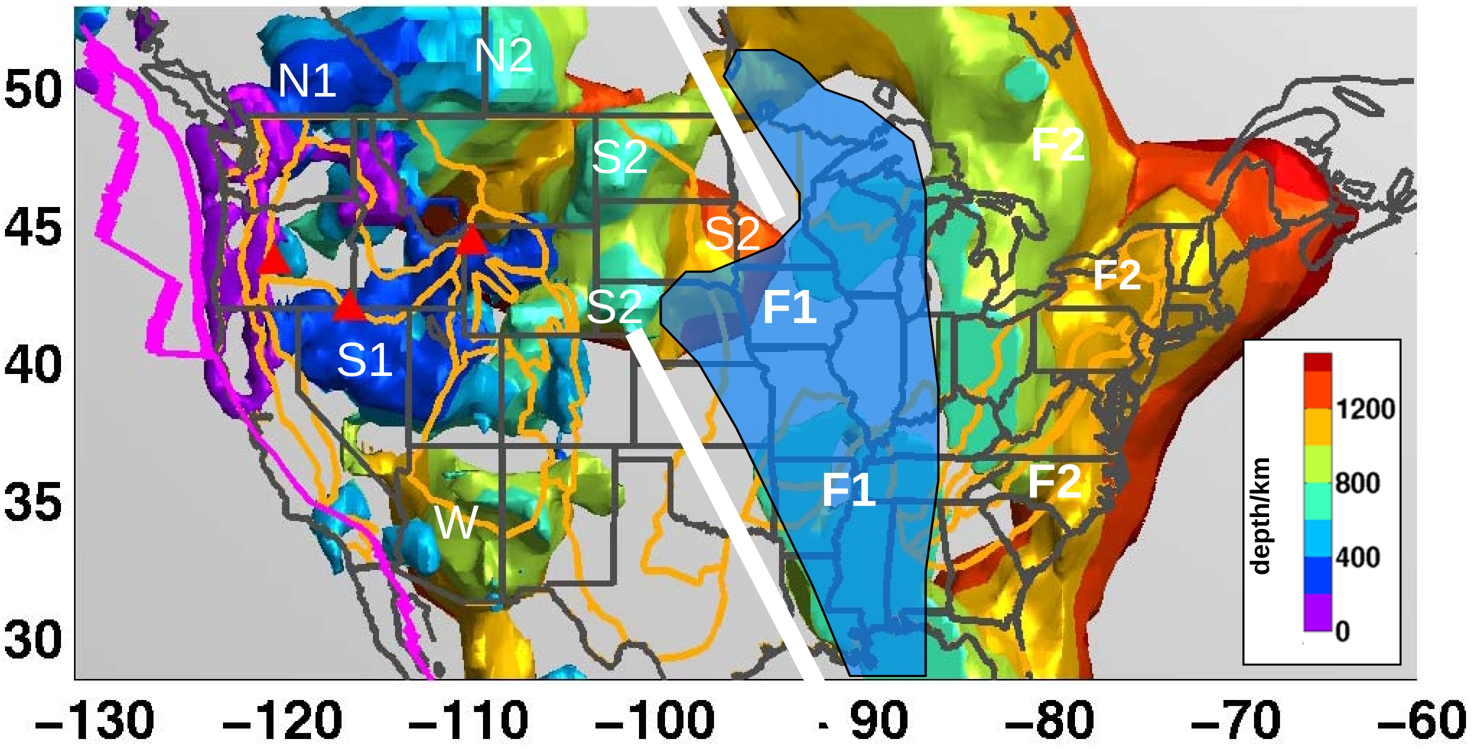


Image of the subducted Farallon slab in the mantle



- Seismically fast material is contoured (fast means cold).
- Color signifies depth. We can confidently image ~1500 km deep.
- Crust and lithosphere not rendered.

Interpretation: a frontal plate break ended the Laramide era at ~50 Myr



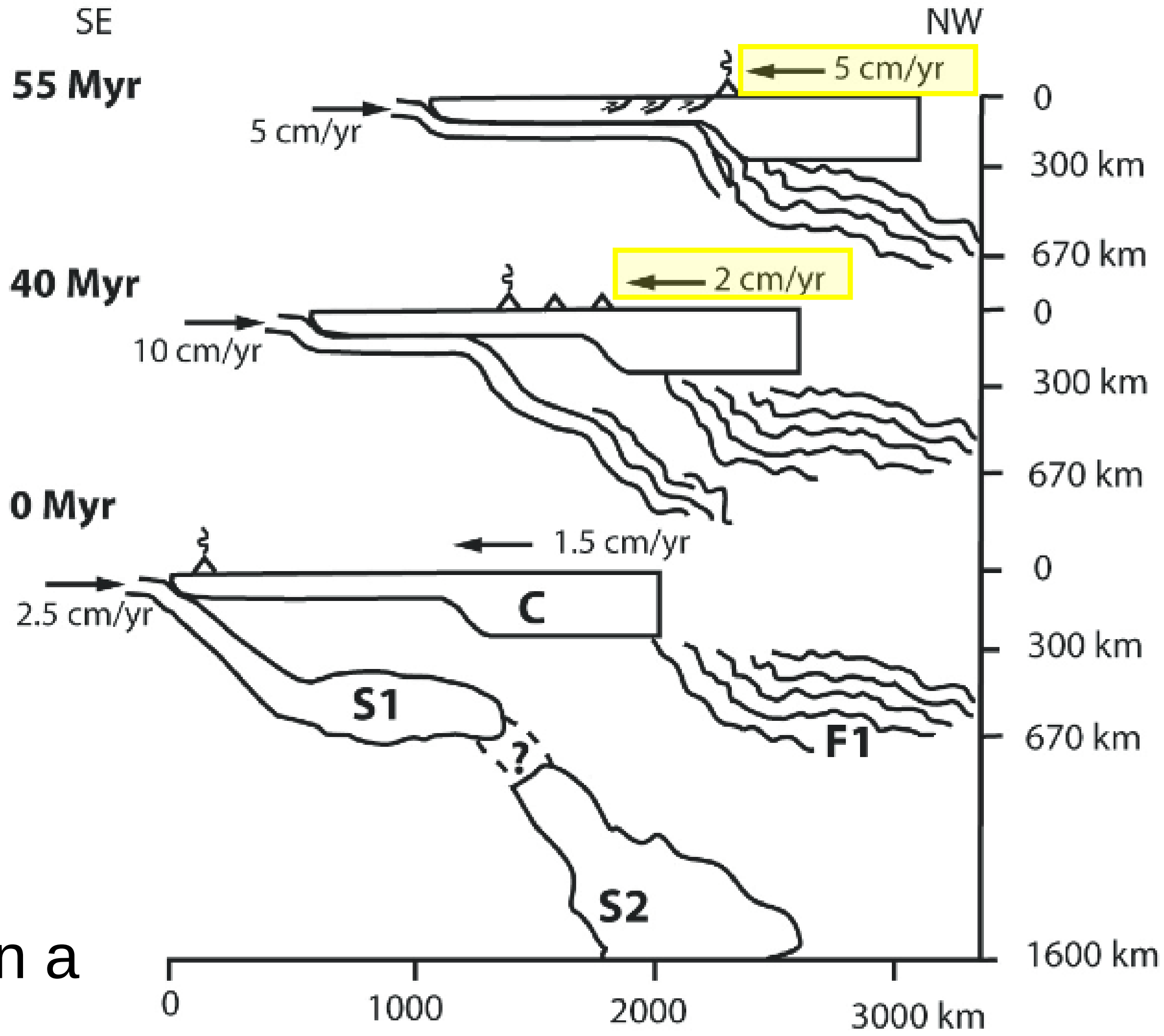
- F1 must have been part of the Laramide flat slab. It still fills the transition zone.
- Lower end of S2/N2 subducted ~55 Myr ago.

Interpretation: a frontal plate break ended the Laramide era at ~50 Myr

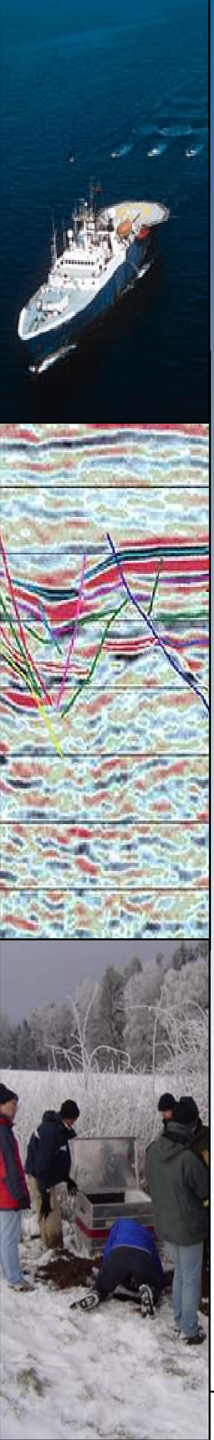
55 Myr ago

40 Myr ago

Today

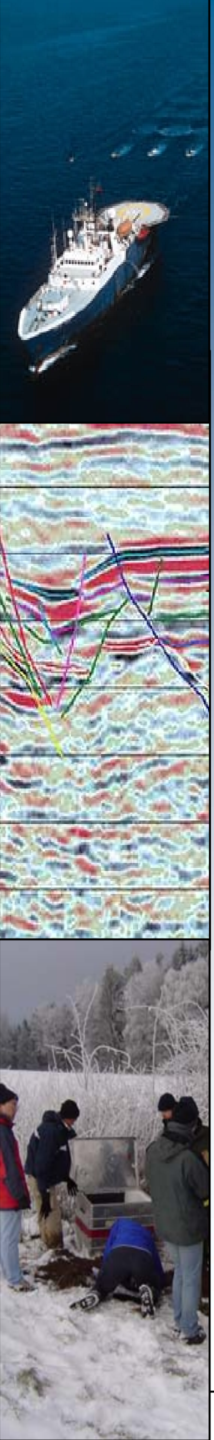


All velocities in a hotspot frame.



How does this work?

Some intuitive examples…



Tomography: Intuitive example 1

Surface waves (of a certain frequency) have sampled the shallow mantle of North America, along the shown raypaths.

Your prior guess is that traveltime ~ length of ray, meaning $v(x,y)=v_0$ = const. everywhere.

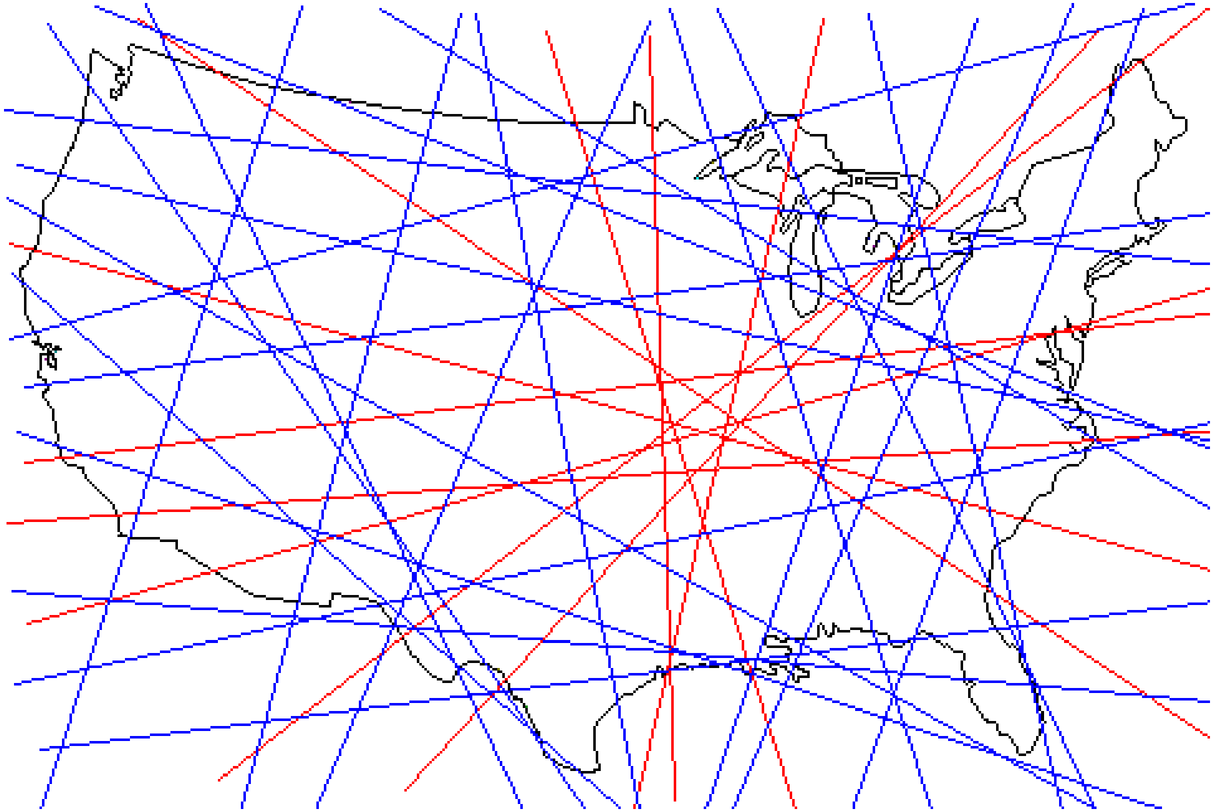
In reality you observe anomalies in the traveltime DATA:

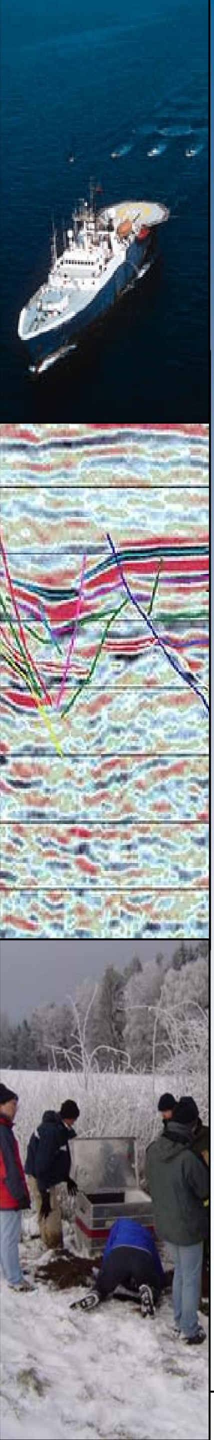
Red ray means traveltime was longer than expected.

Blue ray means traveltime was as expected.

Red rays must have traversed some slow material. Where exactly is is located?

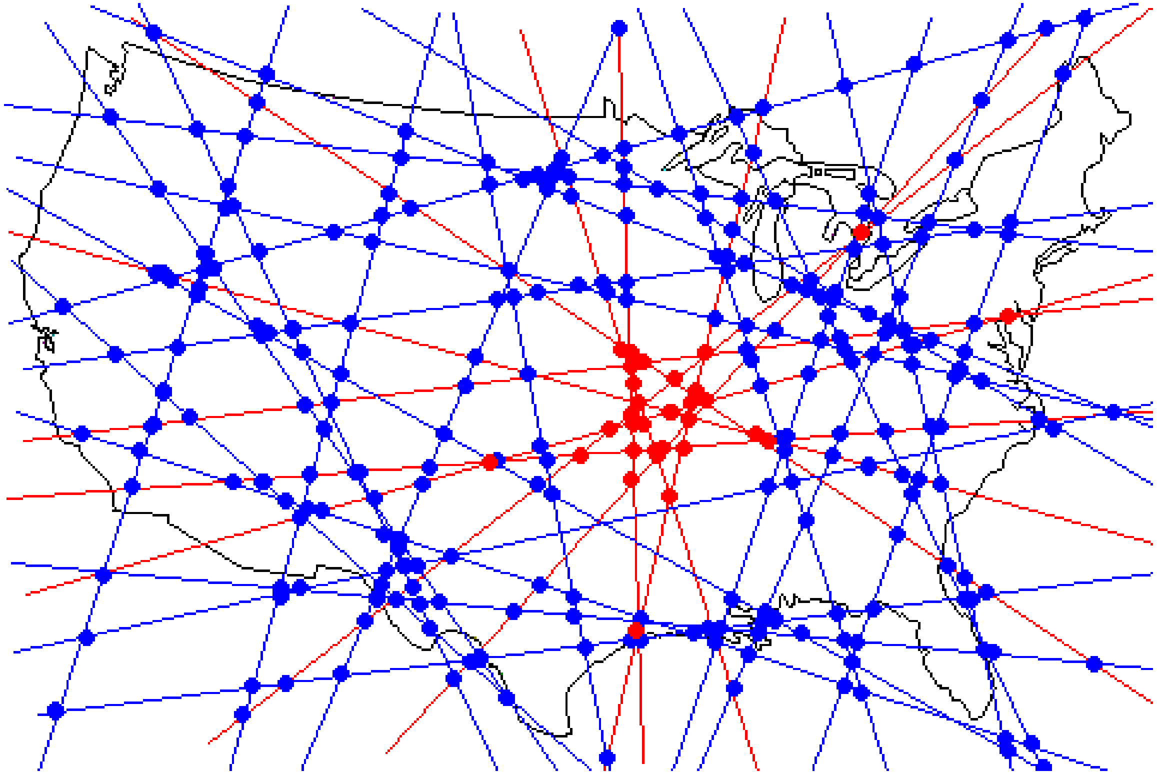
Tomography: Intuitive example 1

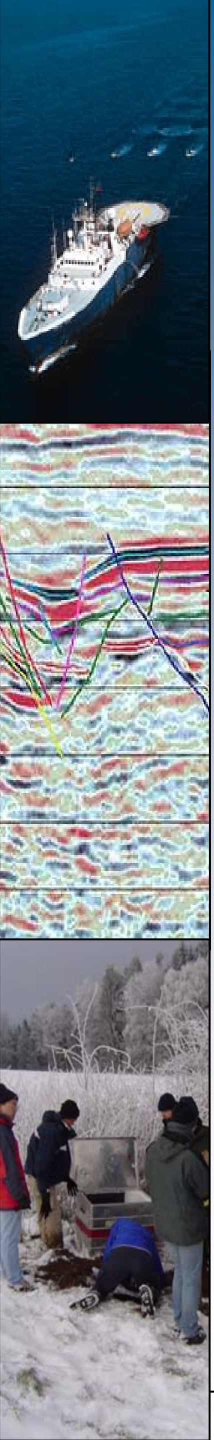




Tomography: Intuitive example 1

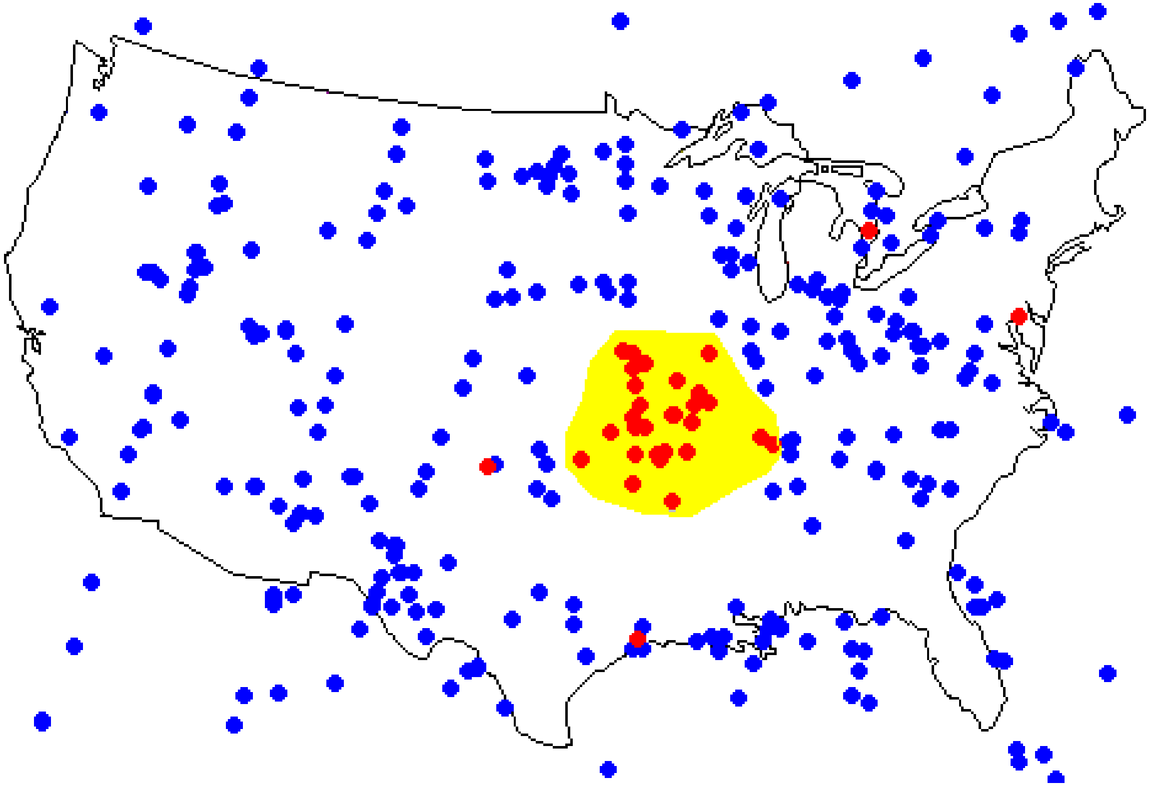
Idea: Consider ray crossings

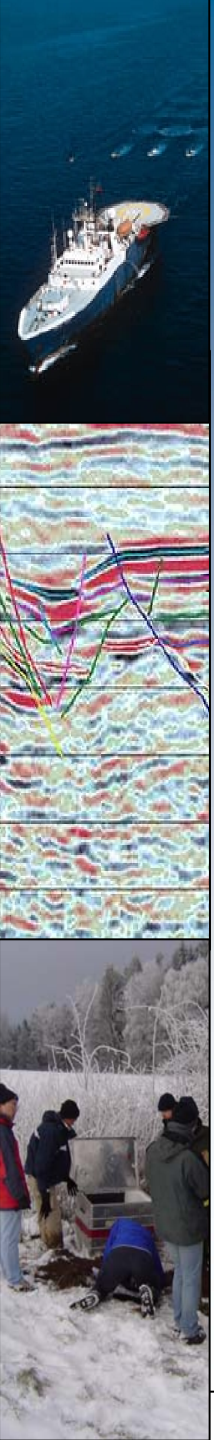




Tomography: Intuitive example 1

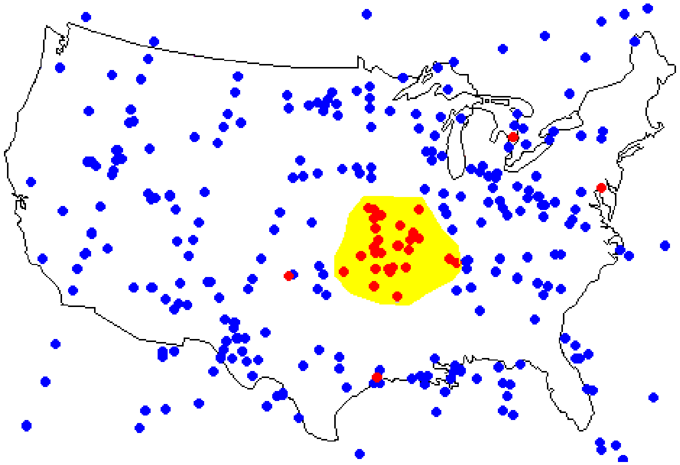
Idea: Consider ray crossings



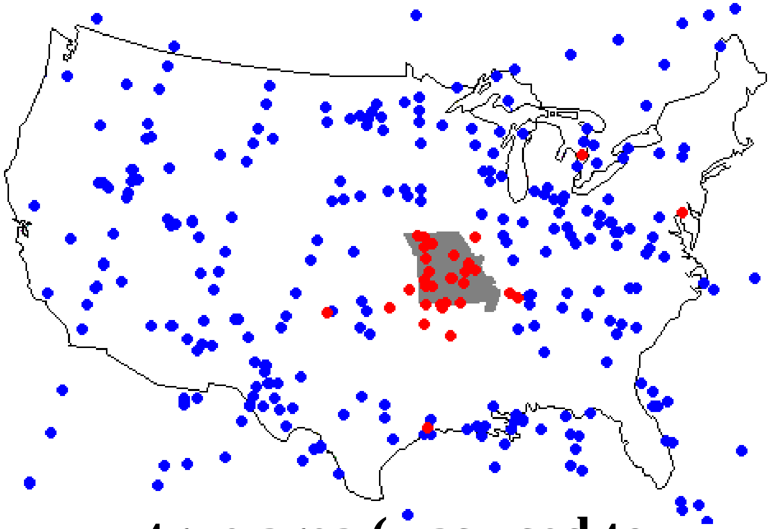


Tomography: Intuitive example 1

It worked pretty well.



recovered area

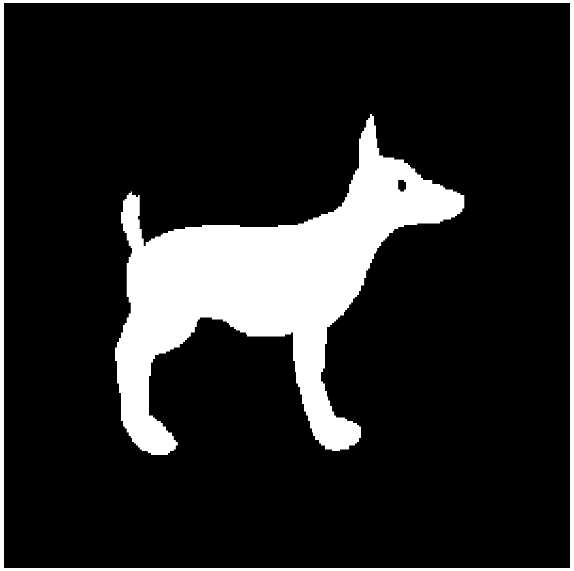


true area (was used to generate the colored rays)

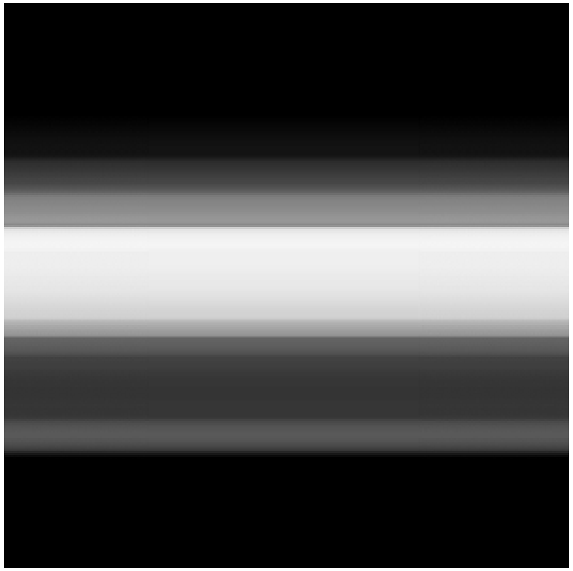
Why is the reconstruction not perfect?

Tomography: Intuitive example 2

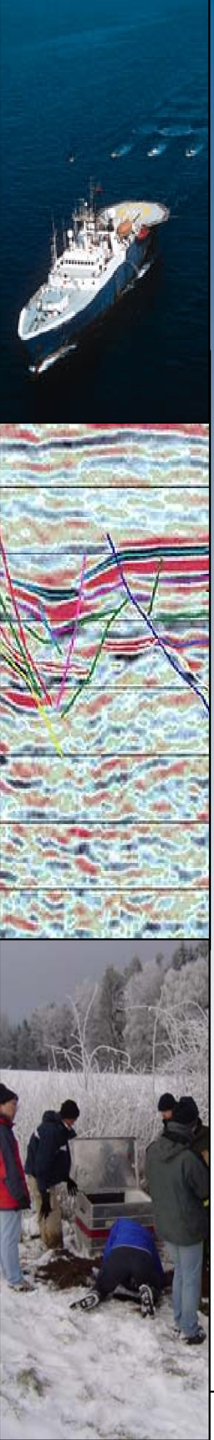
Image reconstruction



original image

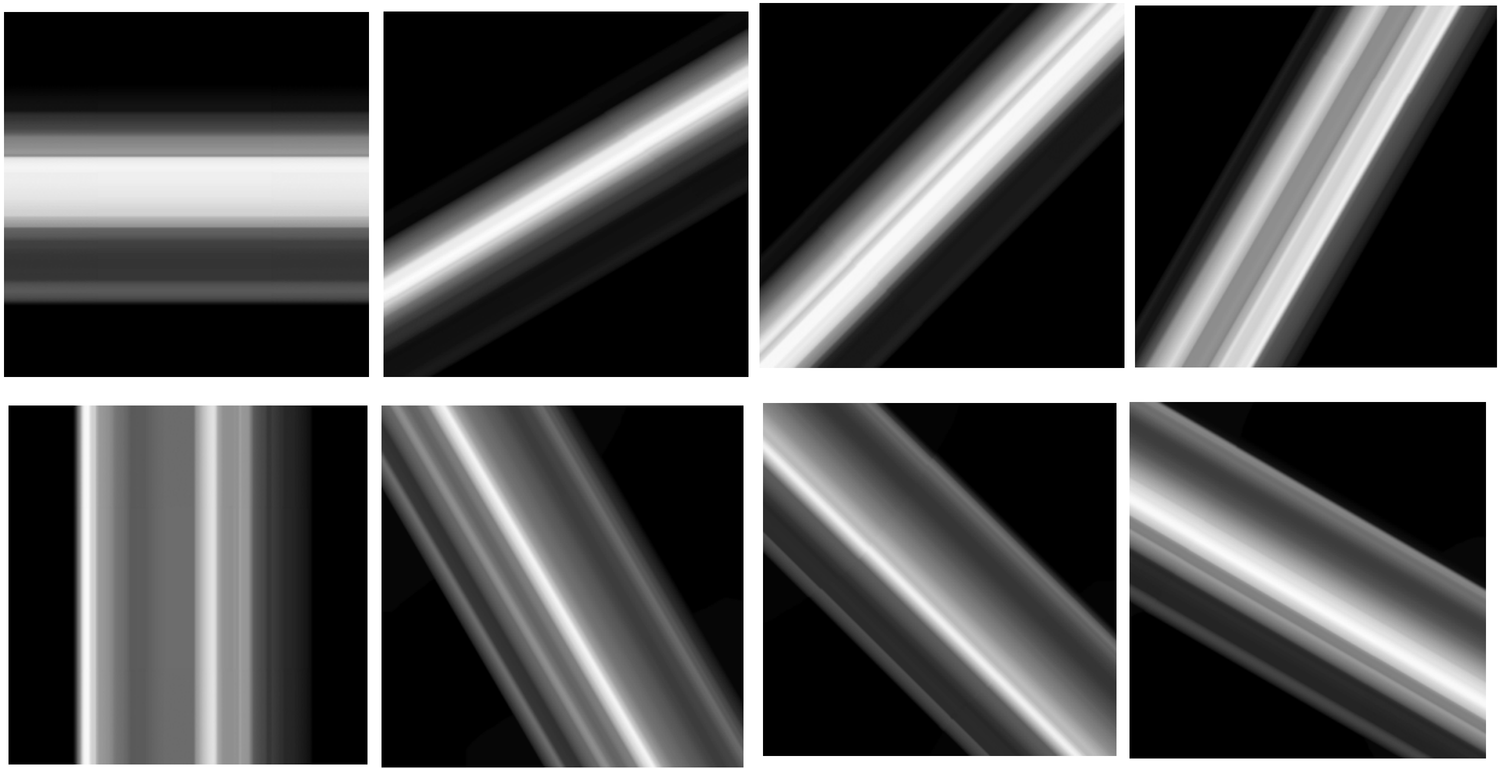


We smear the image in horizontal direction (like an x-ray integrates over different body tissues along its path)

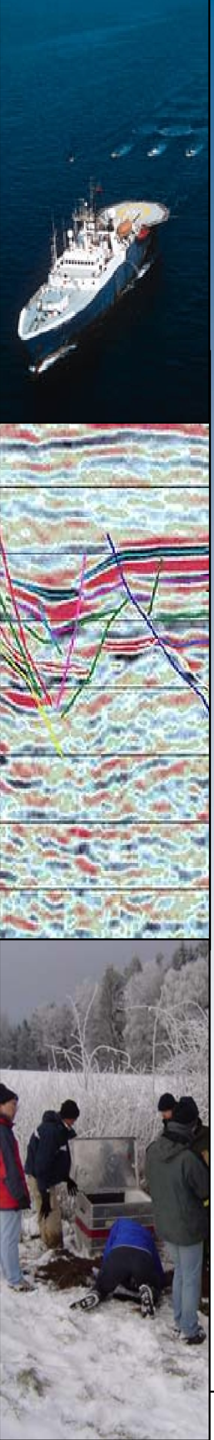


Tomography: Intuitive example 2

Image reconstruction: Generating „DATA"

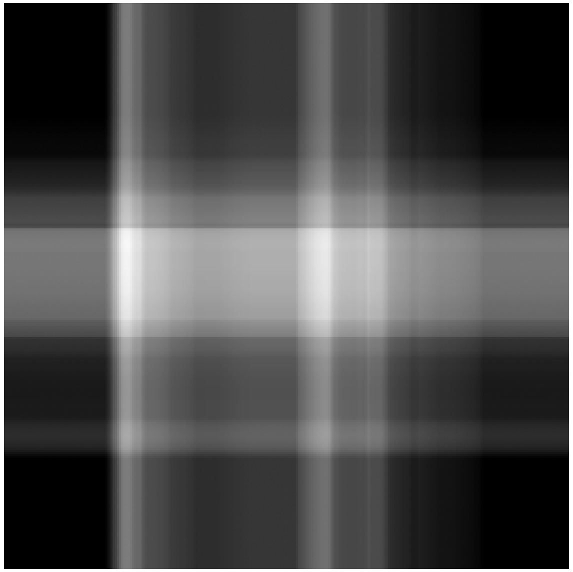


We smear over more directions to simulate more x-ray "data": what rays "see" from all these different angles

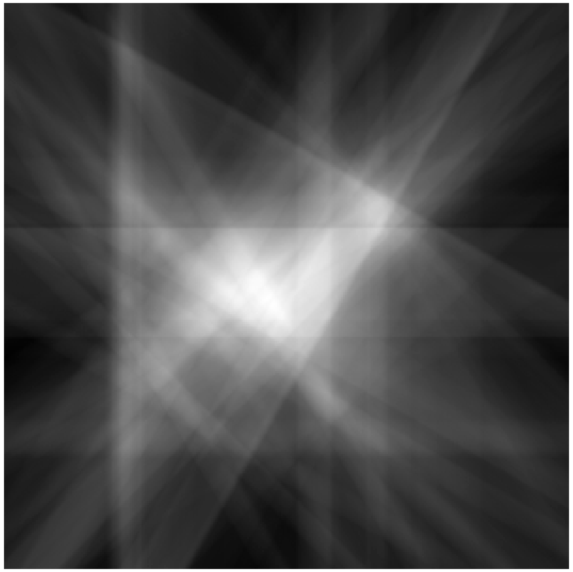


Tomography: Intuitive example 2

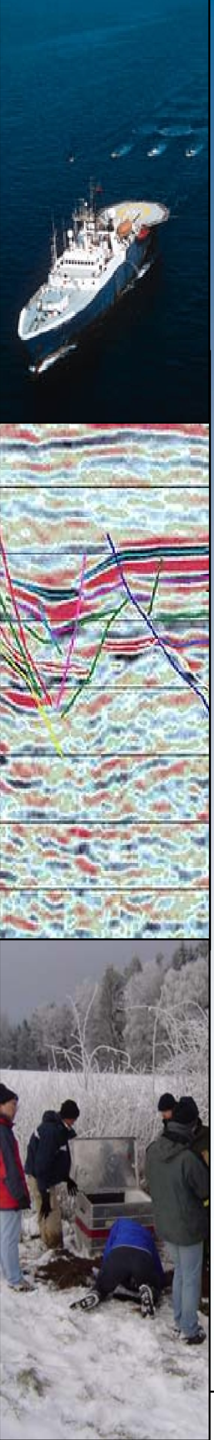
Now we try to reconstruct the image (principle of destructive/constructive interference):



Addition of two directions of the “data”

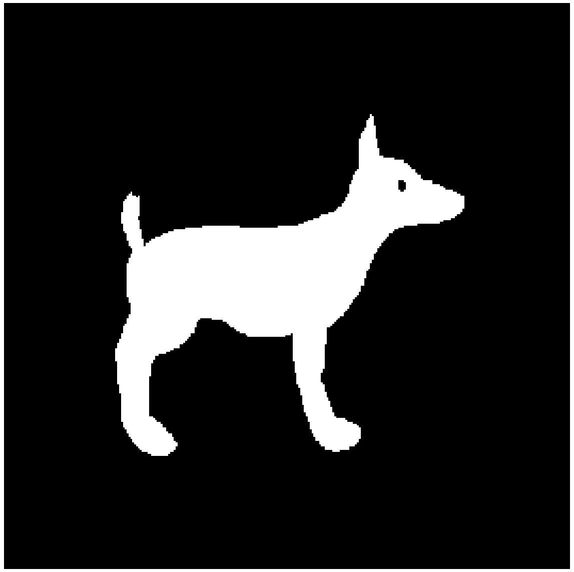


Addition of all 8 directions of the “data”

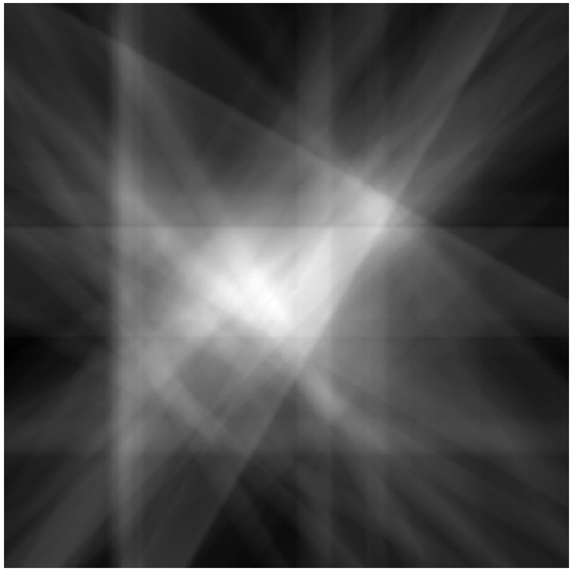


Tomography: Intuitive example 2

Reconstruction result

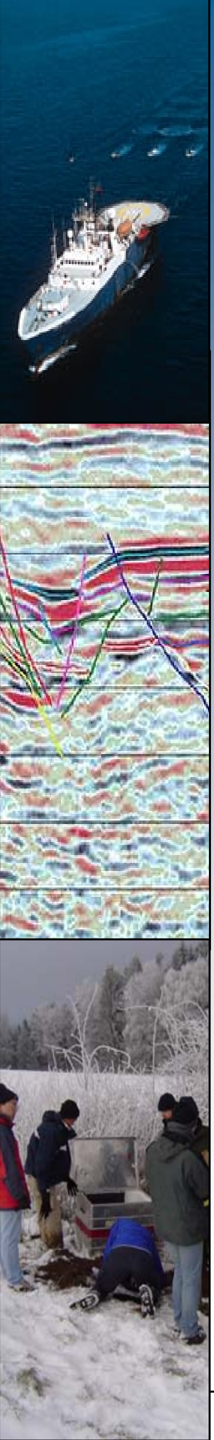


Original



Reconstruction

How could we improve on this?

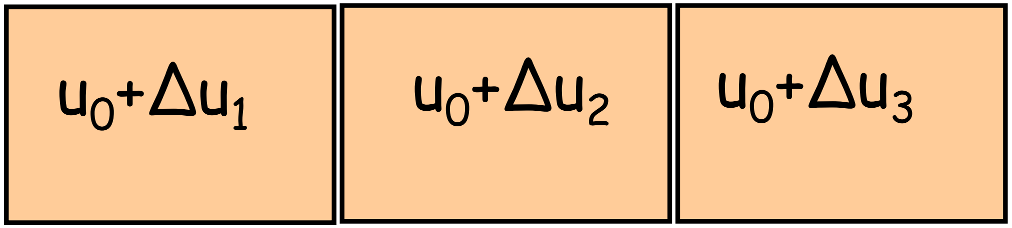


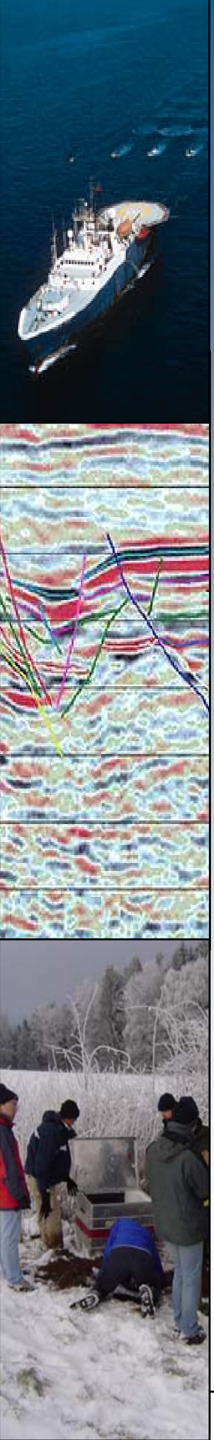
Basic modeling

Acoustic tomography:

A few bricks are standing next to each other. To first order they all have the same, known P-velocity v_0 (or slowness $u_0 = 1/v_0$), except for small variations: $u_i = u_0 + \Delta u_i$, where $\Delta u_i \ll u_0$

We want to estimate the small anomalies Δu_i





Basic modeling

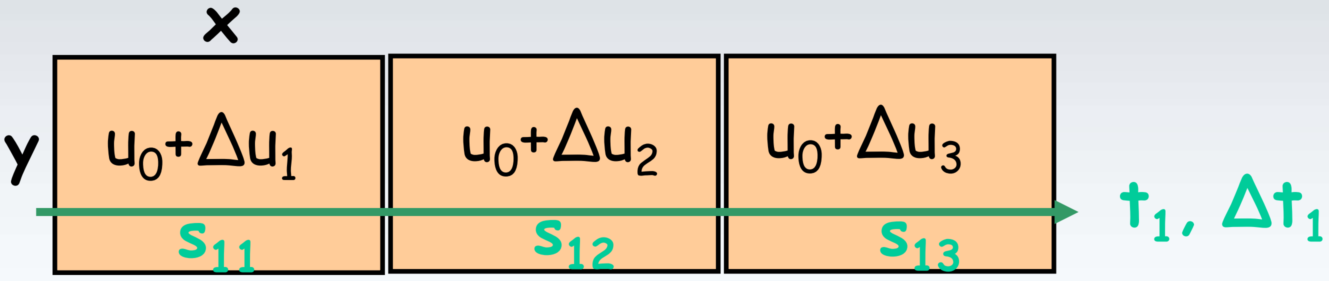
Acoustic tomography:

Blocks are x wide and y high.

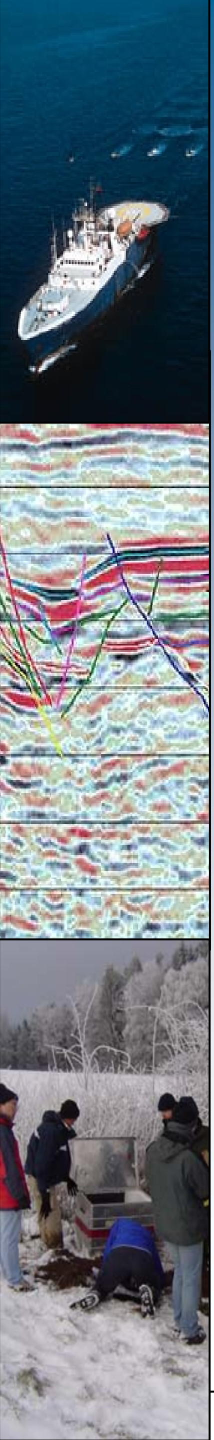
Traveltime: $t_1=u_1s_{11}+u_2s_{12}+u_3s_{13}$

Traveltime anomaly: $\Delta t_1=\Delta u_1s_{11}+\Delta u_2s_{12}+\Delta u_3s_{13}$

The s_{ij} can be computed from the given geometry (general case = Snell's law!)

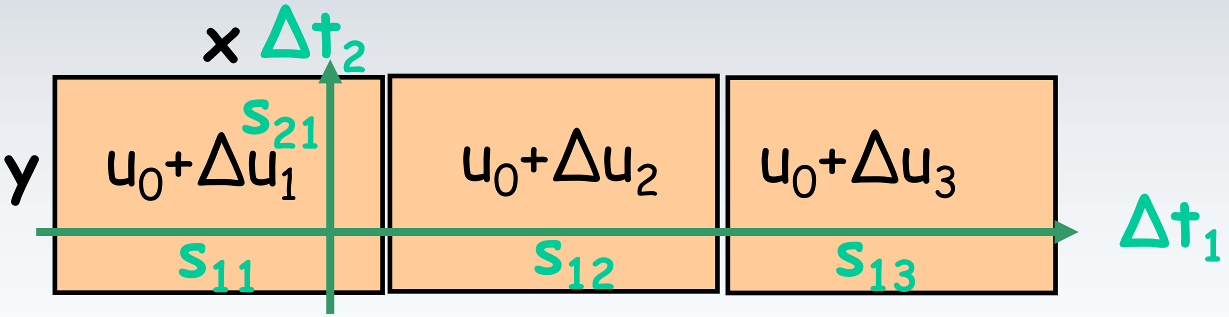


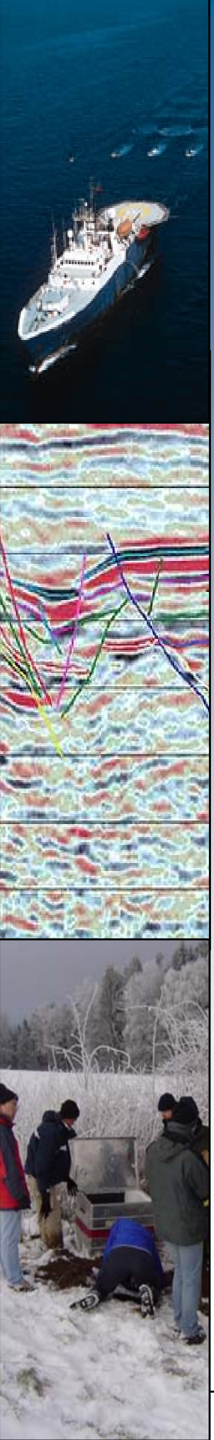
Basic modeling



Acoustic tomography:

Linear system: two equations, three unknowns
Matrix notation:

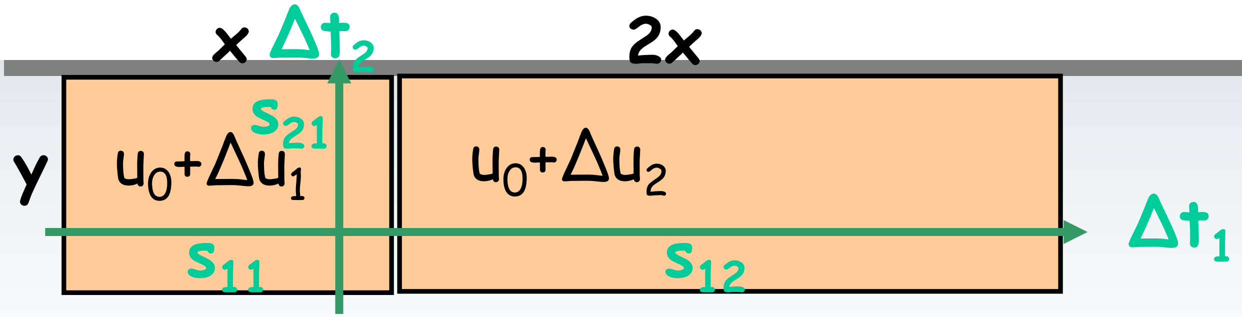


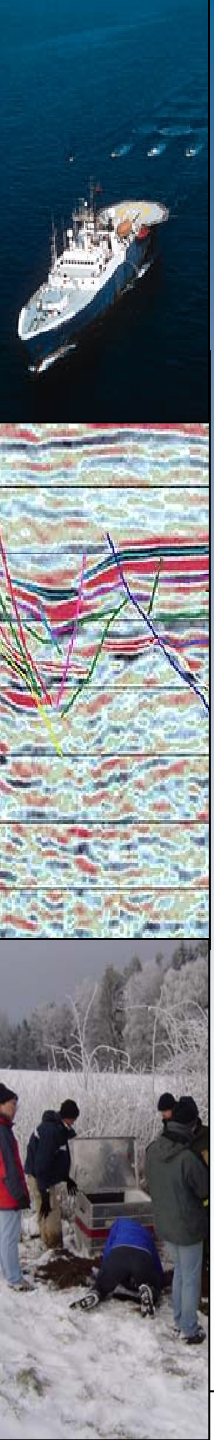


Basic modeling

Acoustic tomography:

For full rank: two equations, two unknowns.
Full rank means:
Matrix notation (and its inverse):



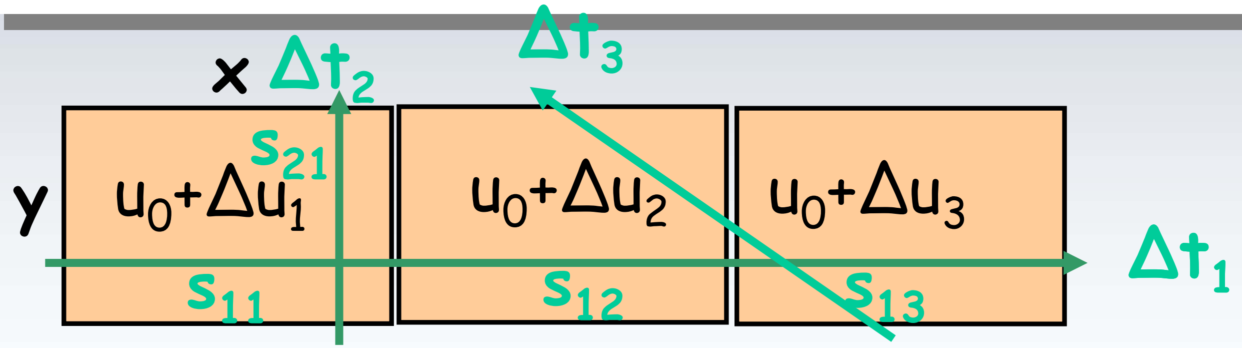


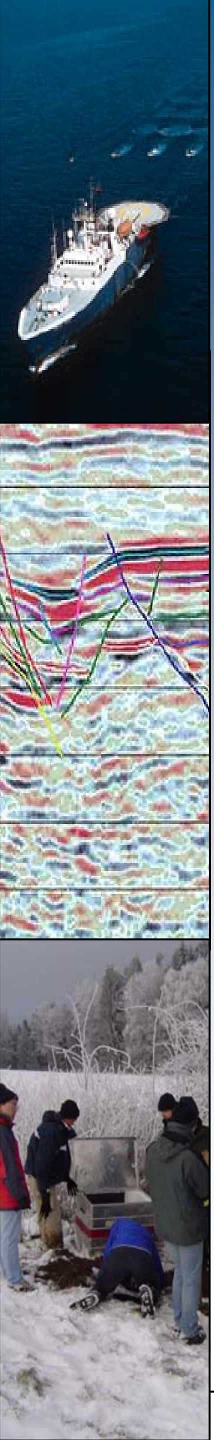
Basic modeling

Acoustic tomography:

Does this system have full rank?

How many measurements M need to be made for the matrix to have an inverse?

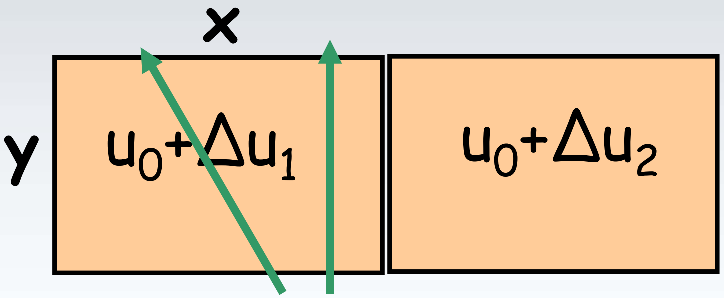


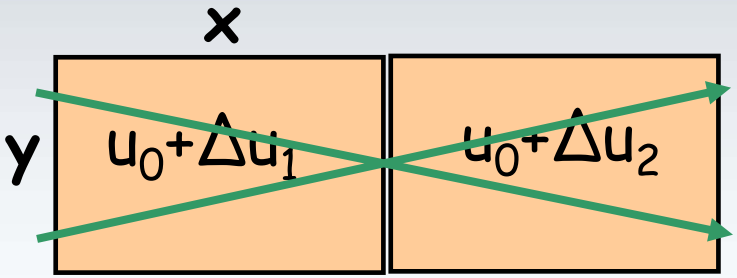


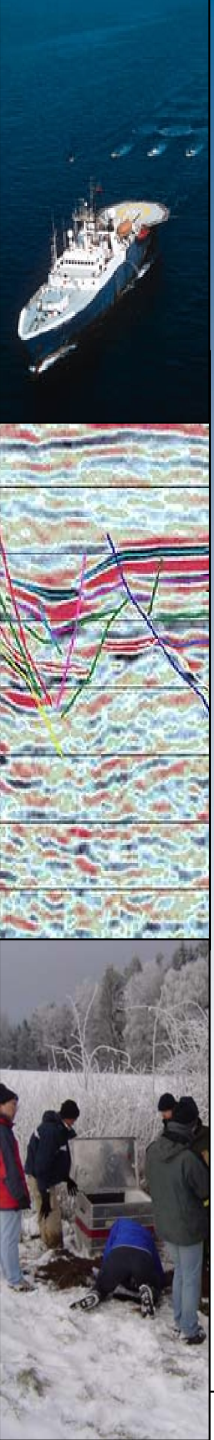
Basic modeling

Acoustic tomography: How about these geometries?

Ideally, each measurement should contribute as much new information as possible („independent" measurements --> experiment design)

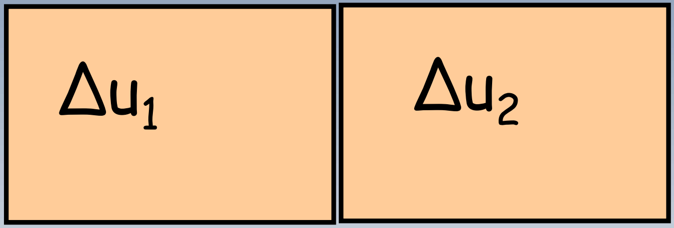






Real-world experiments

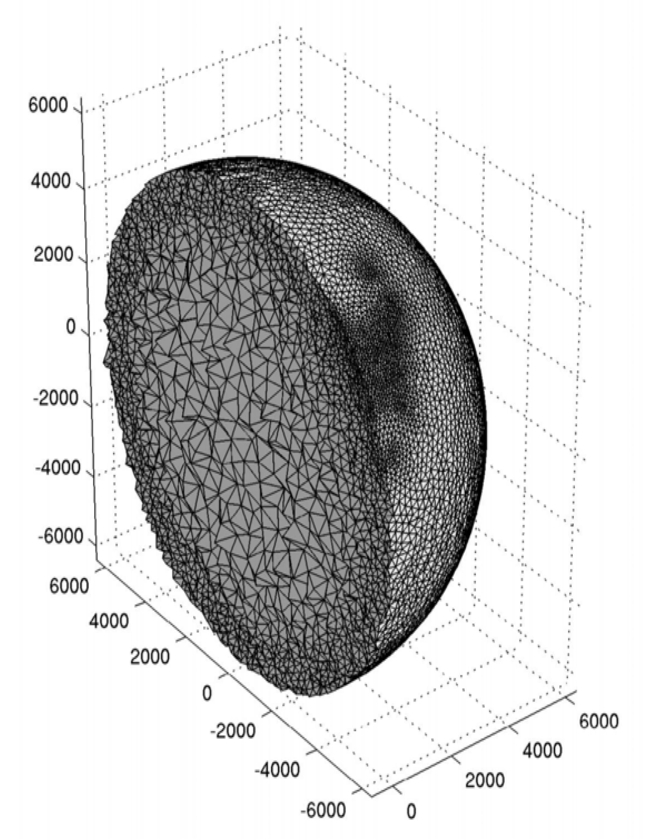
Parameterizations of the unknowns (grid)

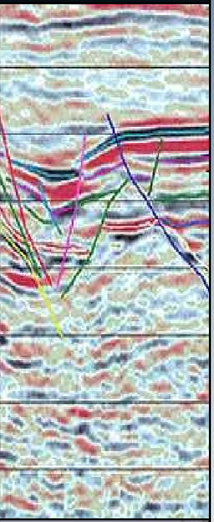


Coarse parameterization in blocks; few unknowns

vs.

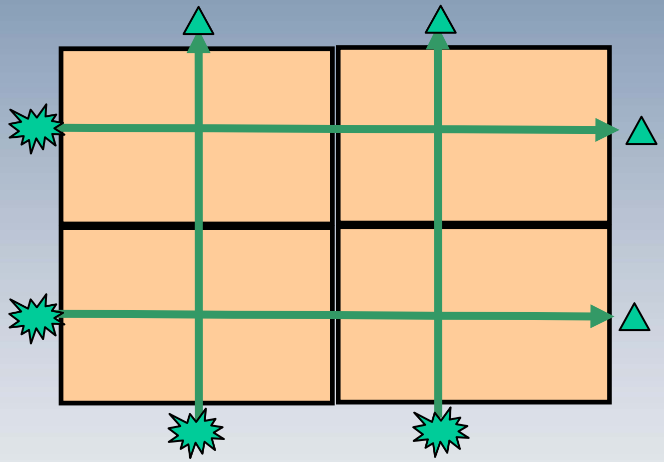
Complex parameterization (irregular tetrahedra, 10^5 unknowns

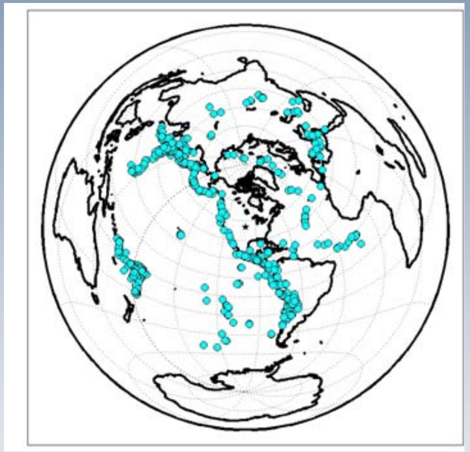




Real-world experiments

Source and receiver geometry

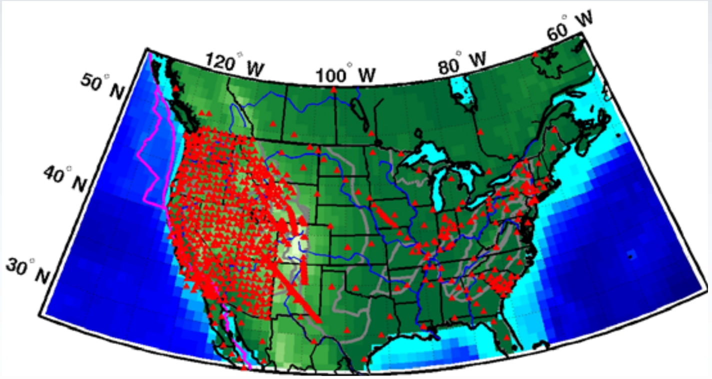


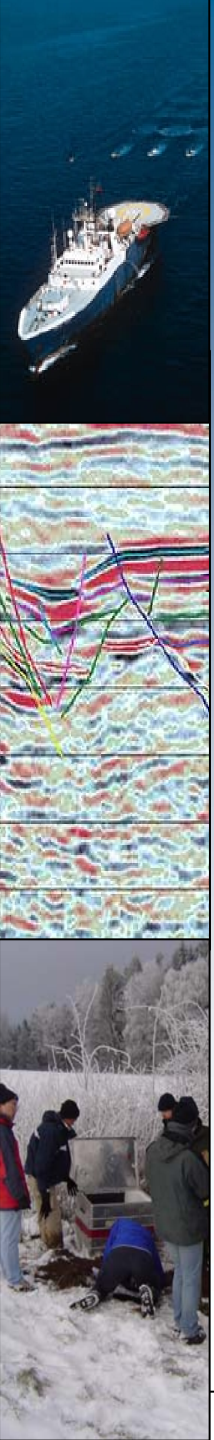


Optimally designed source-receiver geometry

vs.

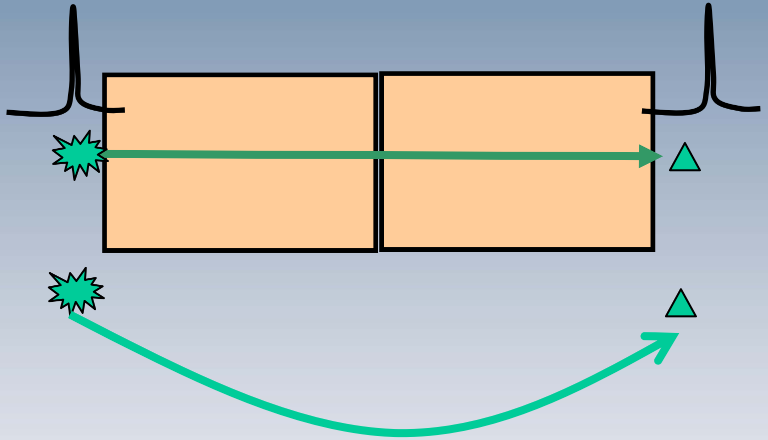
“Take what you can get” -->

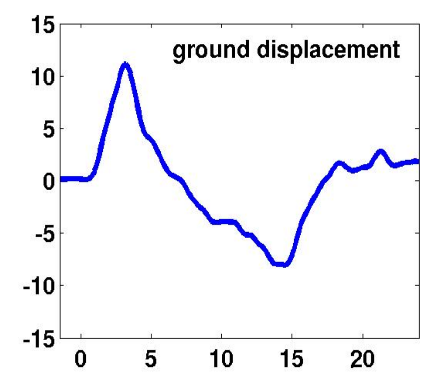




Real-world experiments

Signals and wave propagation modeling

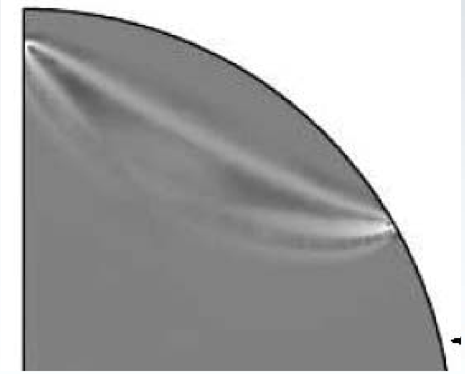


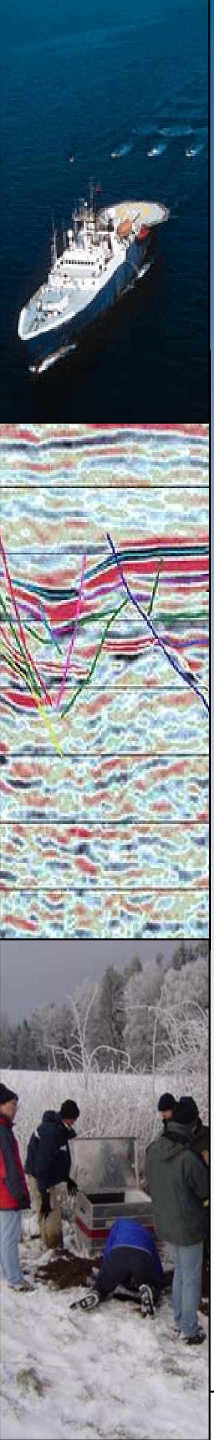


Sharp pulses modelled as optical rays

vs.

Realistic wavelets with broad Fresnel zones -->





Real-world experiments

System to solve

Small sytem, well conditioned, exactly determined

vs.

Huge system, ill conditioned, both underdetermined and overdetermined

ART OF TOMOGRAPHY: Finding smart ways to solve this anyway.