Introduction to and State-of-the-Art in Earthquake Source Inversion

– Part 2 –

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Roadmap

Earthquake Source Inversion

(1) Introduction & Theory
- A brief overview
- Fundamentals
- From point-source to extended-fault modeling

(2) Applications & Implications
- Early developments & case studies
- What can we extract from them?
- What to learn from finite-fault source models?

(3) Challenges, Developments, Opportunities
- Imaging versus inversion, or combination of both?
- Alternative methods
- Uncertainty quantification

Applications & Implications

"Historic" developments and early landmark studies

- The 1979 Imperial Valley (M 6.6) earthquake was the first one studied in great detail, due to the existence of a cross-fault seismic array
- For the 1984 Morgan Hill (M6.2) earthquake a new method was developed
- The 1992 Landers (M 7.3) occurred on a geometrically complex fault, and triggered new ideas
- For the 1994 Northridge (M 6.7) lots of (new) data were available, and many groups worked on that earthquake
- 1995 Kobe (M 6.9), 1999 Izmit (M 7.6), 1999 Chi-Chi (M 7.6), 2004 Parkfield (M 6.0) are further landmark events that taught us numerous lessons …

Applications & Implications

The representation theorem

$$u_n(\mathbf{x},t) = \int_{-\infty}^{\infty} d\tau \int\int_{\Sigma} [s_i(\xi,\tau)] \cdot c_{ijpq} \cdot \nu_j \cdot \frac{\partial}{\partial \xi_q} G_{np}(\mathbf{x}, t-\tau; \xi, 0) d\Sigma$$

- $u_n(\mathbf{x},t)$: displacement seismogram at observer location **x**
- $[s_i(\xi,\tau)]$: time-dependent slip history on fault
- c_{ijpq}: elasticity tensor
- ν_j: fault-normal vector
- G_{np}: Green's tensor for geometry of interest

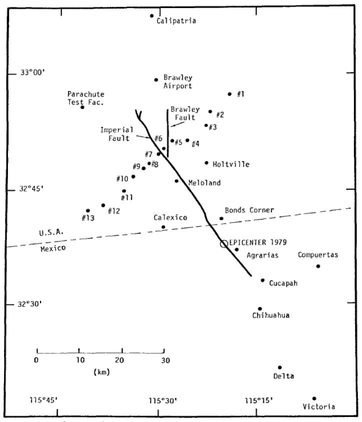
This equation is used to formulate the inverse problem:
*Given the seismic observations u_n(**x**,t), let us estimate the time-dependent slip history on the fault, assuming we know how to compute (or otherwise obtain) Green's functions and we have some knowledge about the fault geometry.*

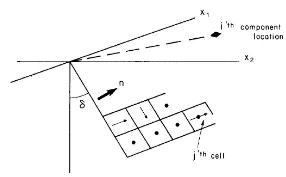
Applications & Implications

The “classic” finite-fault earthquake source inversion

- **1979 M 6.6 Imperial Valley earthquake** (e.g. Olson & Apsel, 1982; Hartzell & Heaton, 1983; Archuleta, 1984):
  - Olson & Apsel: linearized MTW inversion

$$U^i(y,t) = \sum_{j=1}^{J}\sum_{k=-K}^{K} s_{jk}\cdot g_j^i(y, t+k\delta t).$$





The damped least-squares solution

$$y_d = V\Lambda(\Lambda^2 + k_0^2 I)^{-1}U'f.$$

The least-squares solution to the augmented system is y_a, where

$$y_a = V\Lambda_a^{-2}(\Lambda U'f + k_0 HV'b).$$

Applications & Implications

“Standard” finite-fault earthquake source inversion

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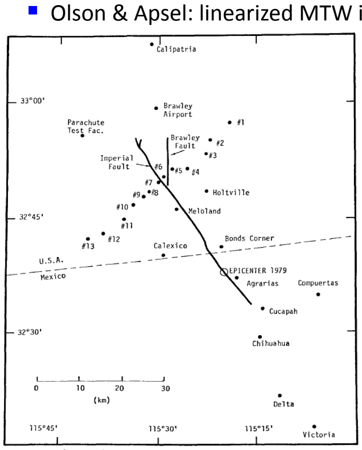

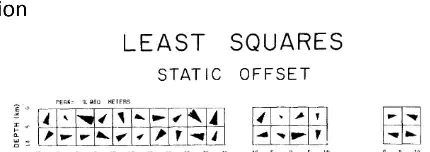

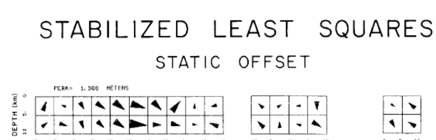

Applications & Implications

"Standard" finite-fault earthquake source inversion

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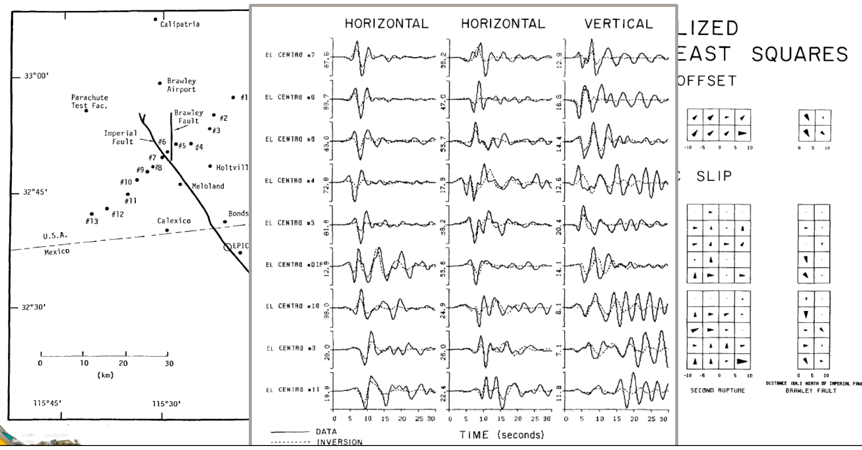
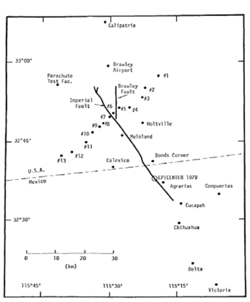
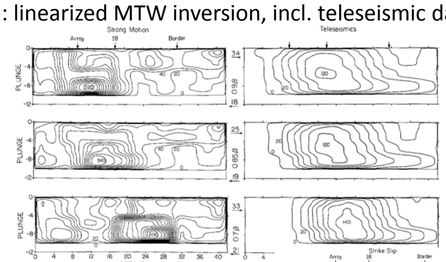


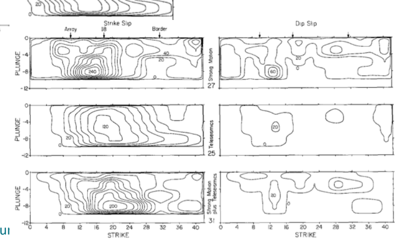
Applications & Implications

"Standard" finite-fault earthquake source inversion

- **1979 M 6.6 Imperial Valley earthquake** (e.g. Olson & Apsel, 1982; Hartzell & Heaton, 1983; Archuleta, 1984):
  - Hartzell & Heaton: linearized MTW inversion, incl. teleseismic data



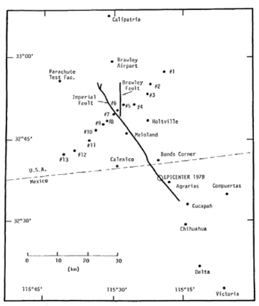


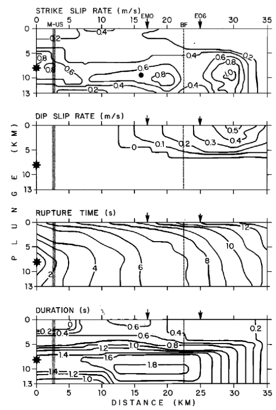


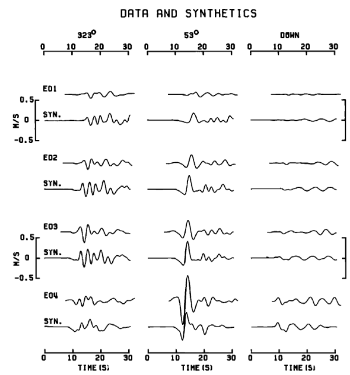
Applications & Implications

"Standard" finite-fault earthquake source inversion

- **1979 M 6.6 Imperial Valley earthquake** (e.g. Olson & Apsel, 1982; Hartzell & Heaton, 1983; Archuleta, 1984):
  - Archuleta: tedious "manual" forward modeling







Applications & Implications

"Standard" finite-fault earthquake source inversion

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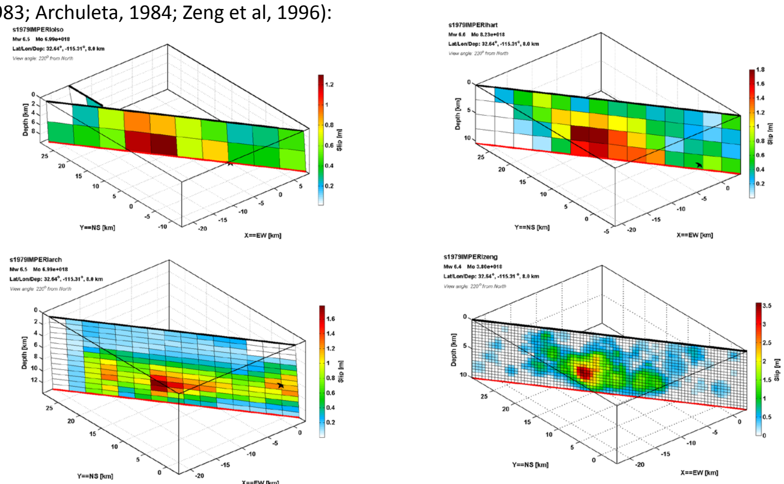
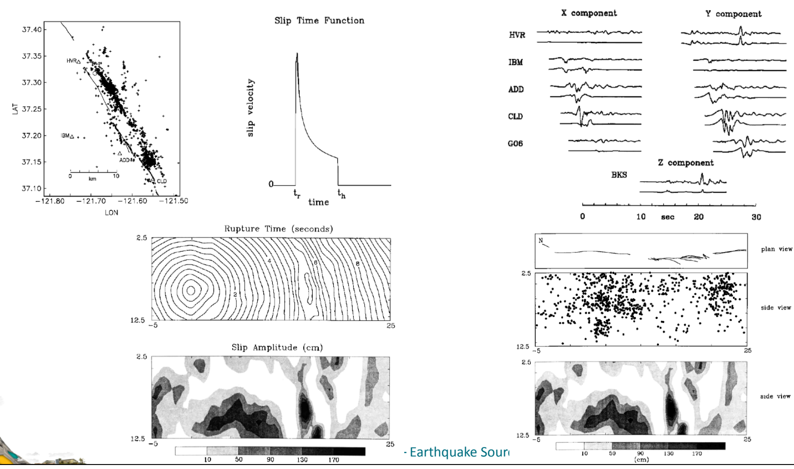


Applications & Implications

Further early developments

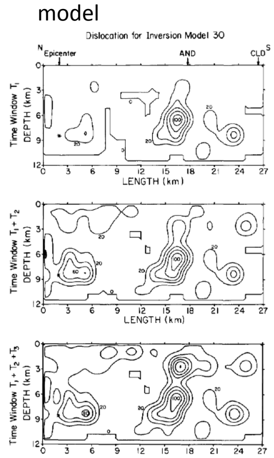
- **1984 M 6.2 Morgan Hill earthquake** (Heaton & Hartzell, 1986; Beroza & Spudich, 1988)
  - Fault from aftershock locations; rupture-dynamics inspired source-time function

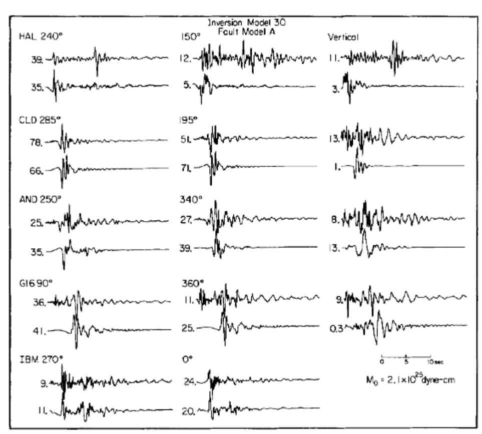


Applications & Implications

Further early developments

- **1984 M 6.2 Morgan Hill earthquake** (Heaton & Hartzell, 1986; Beroza & Spudich, 1988)
  - H&H ran numerous trial inversions, some are reported, one chosen a "best" model





Applications & Implications

Further early developments

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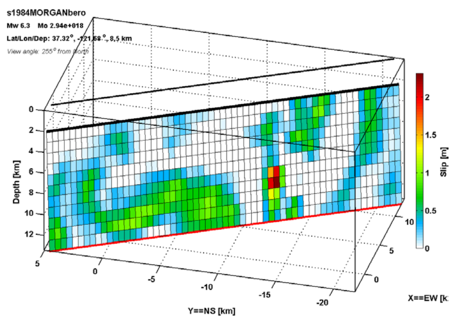
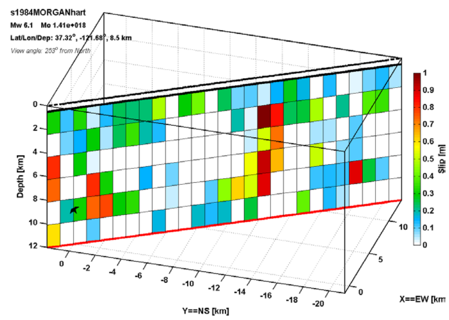
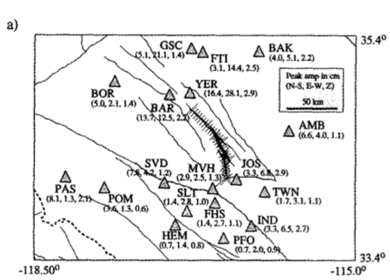




Applications & Implications

Rupture on a geometrically complex fault

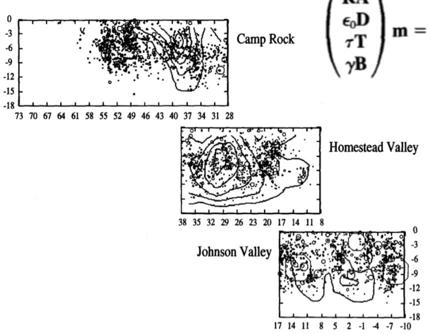
- **1992 M 7.3 Landers earthquake** (Cohee & Beroza, 1994; Wald et al., 1994, Cotton & Campillo, 1996):



Applications & Implications

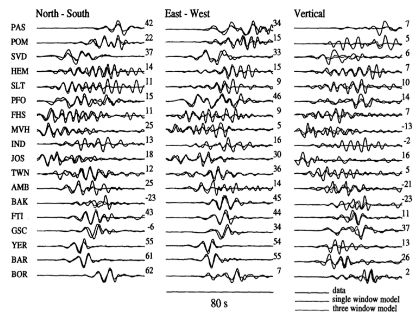
Rupture on a geometrically complex fault

- **1992 M 7.3 Landers earthquake** (Cohee & Beroza, 1994; Wald et al., 1994, Cotton & Campillo, 1996):
  - C & B applied smoothing, boundary conditions at the top and bottom, and defined the variance reduction as misfit norm



$$\begin{pmatrix} \mathbf{RA} \\ \epsilon_0\mathbf{D} \\ \tau\mathbf{T} \\ \gamma\mathbf{B} \end{pmatrix} \mathbf{m} = \begin{pmatrix} \mathbf{Rd} \\ 0 \\ \tau\mathbf{s} \\ 0 \end{pmatrix}.$$

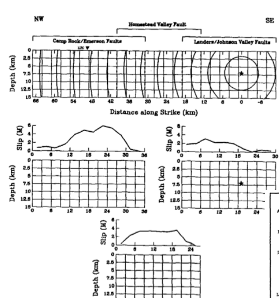
$$\Delta\sigma^2 = \left(1 - \frac{[\mathbf{d} - \mathbf{g}(\mathbf{m})]^{\mathrm{T}}\mathbf{C}_d^{-1}[\mathbf{d} - \mathbf{g}(\mathbf{m})]}{\mathbf{d}^{\mathrm{T}}\mathbf{C}_d^{-1}\mathbf{d}}\right) \times 100\%,$$



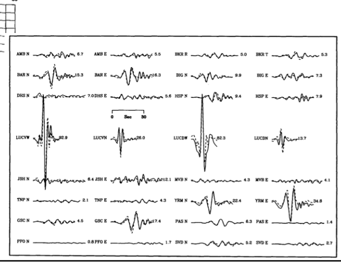
Applications & Implications

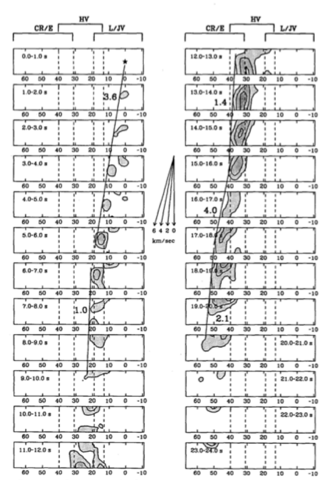
Rupture on a geometrically complex fault

- **1992 M 7.3 Landers earthquake** (Cohee & Beroza, 1994; Wald et al., 1994, Cotton & Campillo, 1996):
  - Wald et al applied the "standard" MTW approach and multiple data sets



$$\begin{pmatrix} \mathbf{C}_d^{-1}\mathbf{A} \\ \lambda_1\mathbf{S} \\ \lambda_2\mathbf{M} \\ \lambda_3\mathbf{F} \end{pmatrix} \mathbf{x} \cong \begin{pmatrix} \mathbf{C}_d^{-1}\mathbf{b} \\ 0 \\ 0 \\ \mathbf{D} \end{pmatrix}.$$

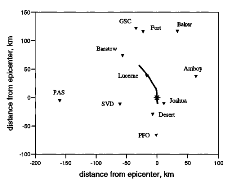


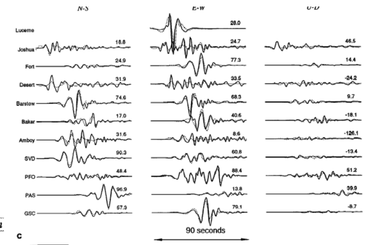


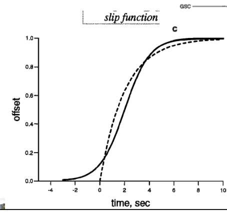
Applications & Implications

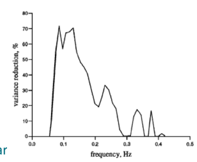
Rupture on a geometrically complex fault

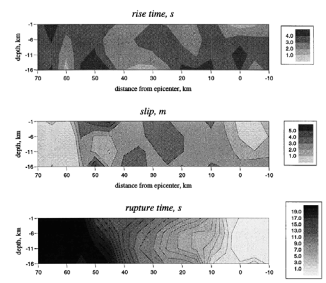
- **1992 M 7.3 Landers earthquake** (Cohee & Beroza, 1994; Wald et al., 1994, Cotton & Campillo, 1996):
  - C &C developed and applied the inversion in the <u>frequency domain</u>, used a different slip function, and made clear that they cannot fit the higher frequencies







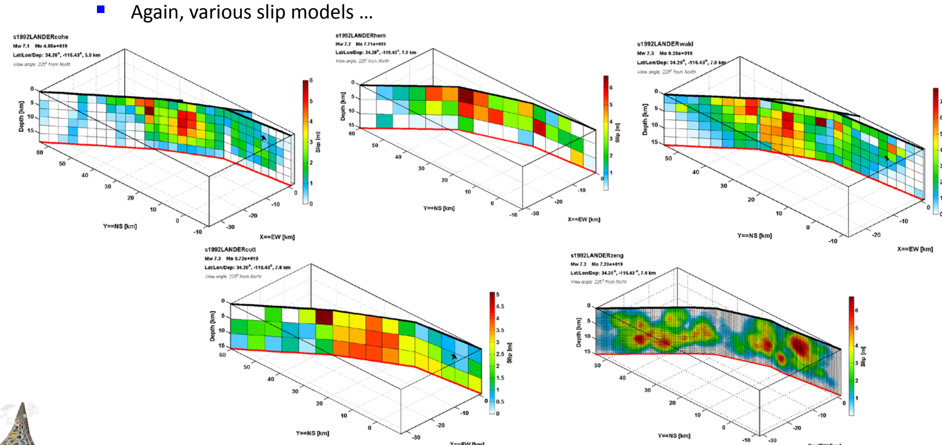




Applications & Implications

Rupture on a geometrically complex fault

- **1992 M 7.3 Landers earthquake** (Cohee & Beroza, 1994; Wald et al., 1994, Cotton & Campillo, 1996):
  - Again, various slip models ...

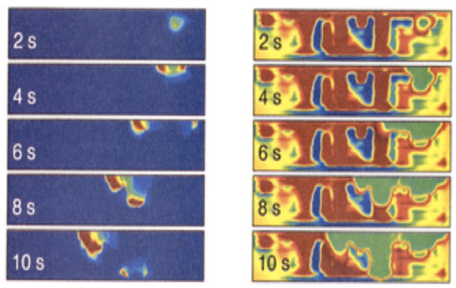


Source

Applications & Implications

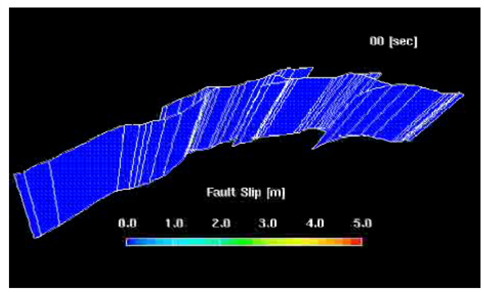
Dynamics on simple and complex faults

- 1992 M 7.3 Landers earthquake



Peyrat et al, 2001
Heterogeneous pre-stress

Aochi & Madariaga, 2002
Complex fault geometry



Applications & Implications

Dynamics on simple and complex faults

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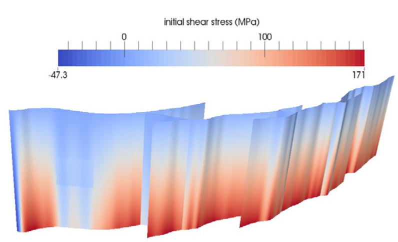
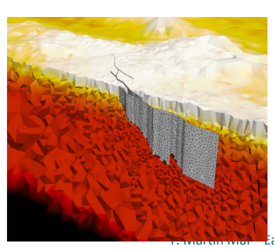
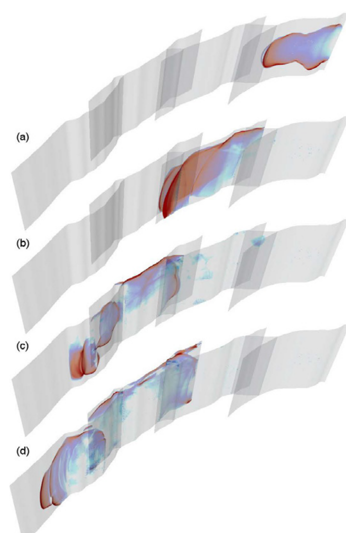




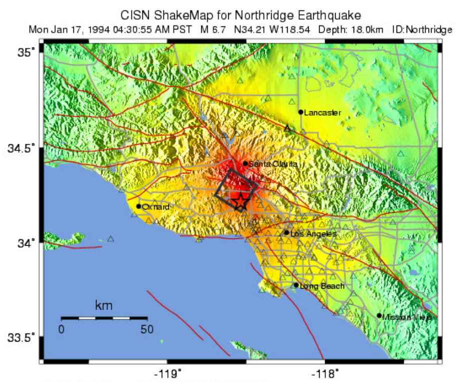


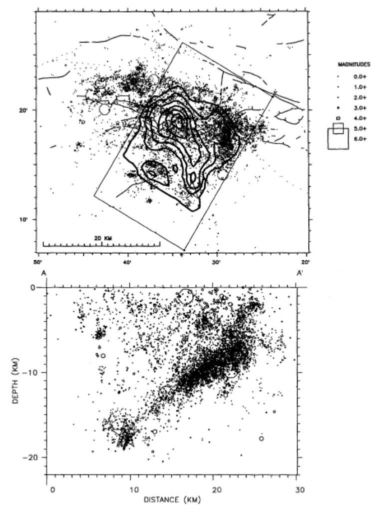
Figures courtesy Alice Gabriel

Applications & Implications

Further early developments

- **1994 M 6.7 Northridge earthquake** (numerous studies …) -- **a "hidden" blind-thrust earthquake !**





Applications & Implications

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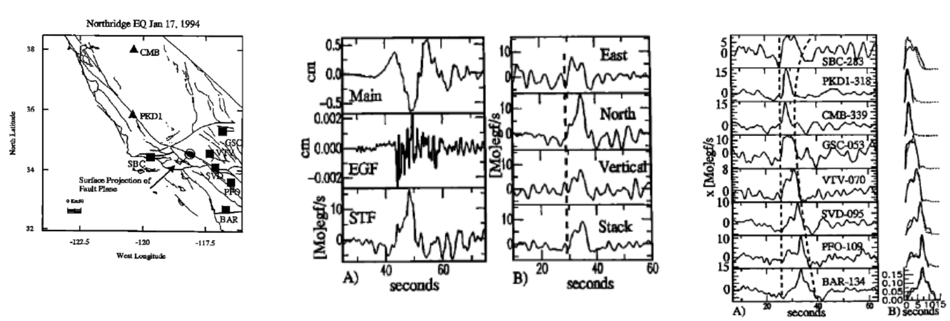
  - Dreger (1994) used empirical Green's functions; deconvolved from the recorded seismograms to obtain local moment-rate functions for inversion



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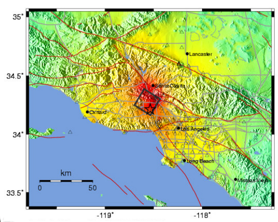
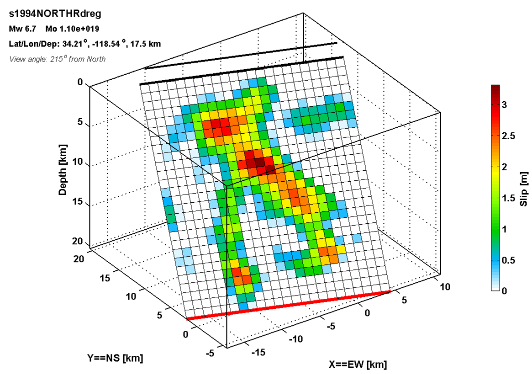
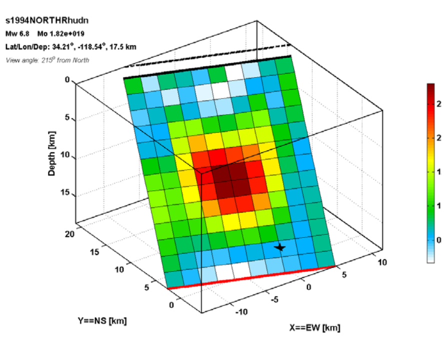


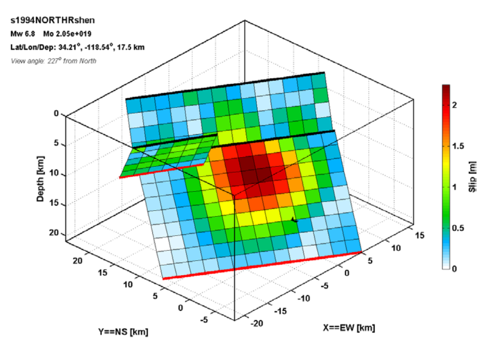


Applications & Implications

Further early developments

- **1994 M 6.7 Northridge earthquake** (numerous studies …)
  - Hudnut et al (1996) and Shen et al (1996) used geodetic data





Applications & Implications

Further early developments

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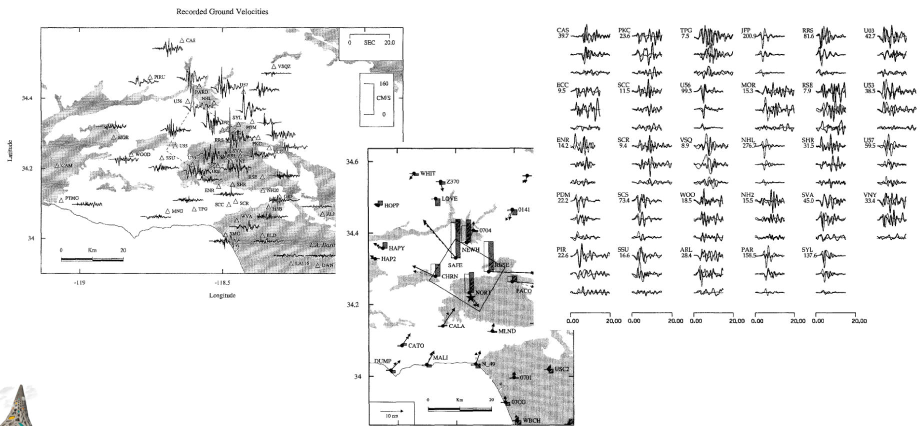
  - Wald et al (1996) use all available data, in the "standard" MTW approach



Applications & Implications

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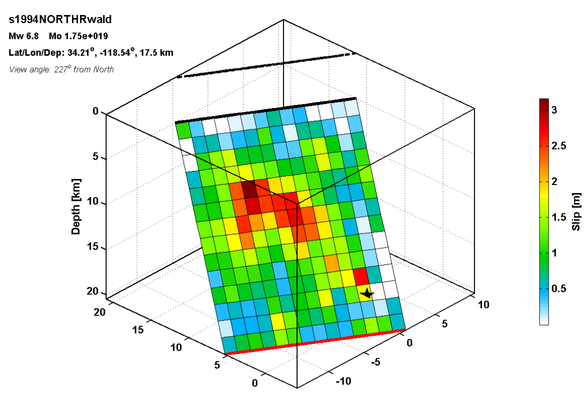
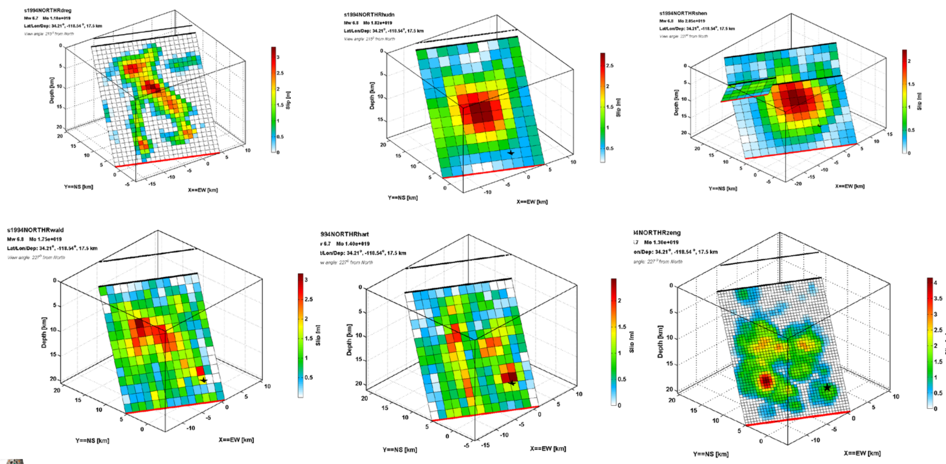


Applications & Implications

Further early developments

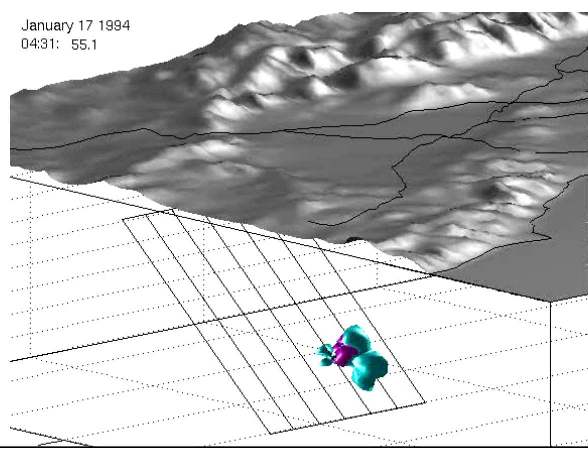
- 1994 M 6.7 Northridge earthquake: a visual comparison



Applications & Implications

Using the inversion results

- 1994 M 6.7 Northridge earthquake: The story does not end here …
  - We still don't fully understand extreme shaking levels during the event, certain site, path, topography effects, or the precise dynamics of the rupture



Courtesy K. Olsen

Applications & Implications

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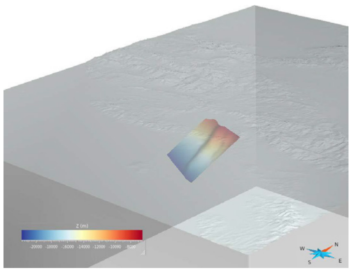

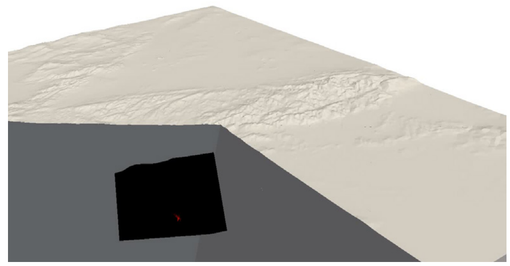

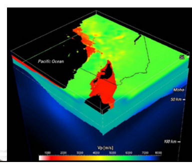

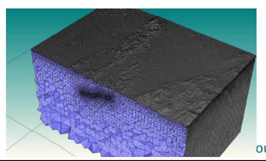

Courtesy A. Gabriel

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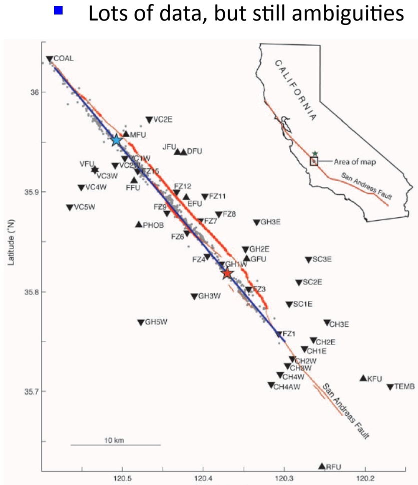
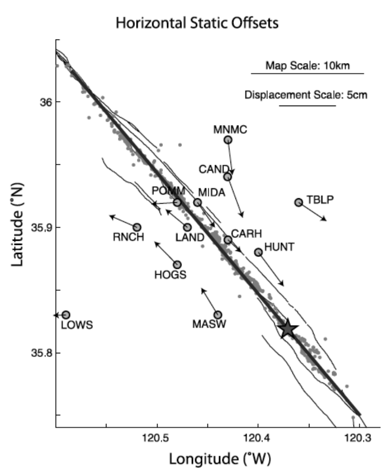
Applications & Implications

Another important case

- **2004 M 6.0 Parkfield earthquake** (numerous studies …)
  - Lots of data, but still ambiguities



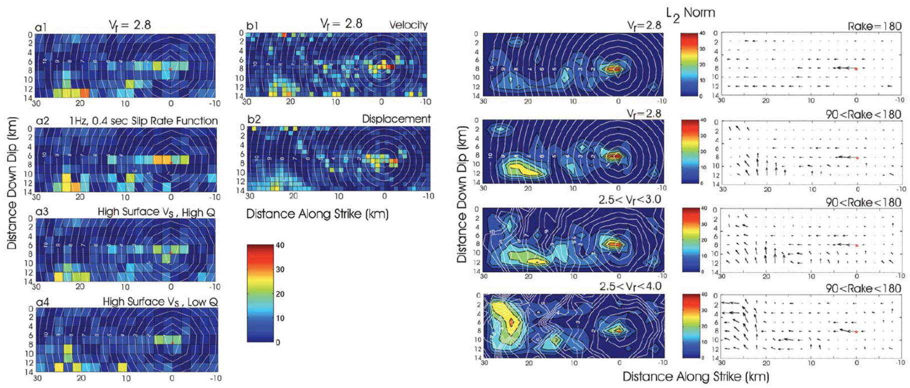


quake 30

Applications & Implications

Another important case

- **2004 M 6.0 Parkfield earthquake** (numerous studies …)
  - Lots of data, but still ambiguities
  - Hartzell et al (2007) tested various misfit norms & parameterizations

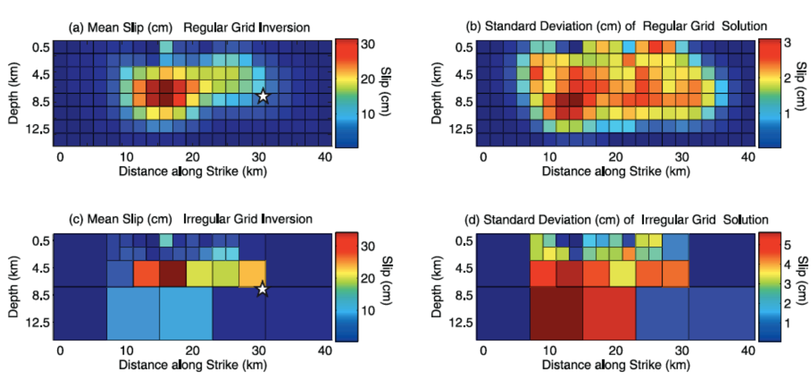


P. Martin Mai – Earthquake Source Inversion 31

Applications & Implications

Another important case

- **2004 M 6.0 Parkfield earthquake** (numerous studies …)
  - Lots of data, but still ambiguities
  - Page et al. (2009) used GPS data and various discretization

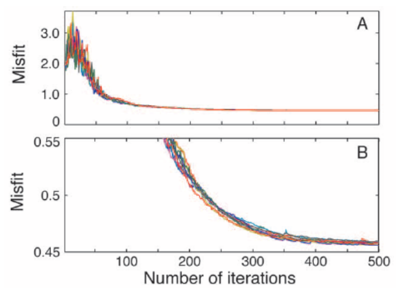


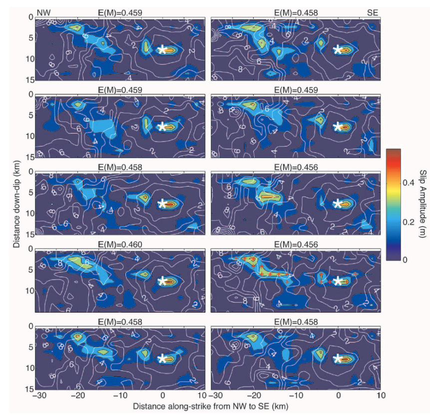
P. Martin Mai – Earthquake Source Inversion 32

Applications & Implications

Another important case

- **2004 M 6.0 Parkfield earthquake** (numerous studies …)
  - Lots of data, but still ambiguities
  - Lui et al. (2006) conducted a non-linear inversion and did statistics on the 10 "best" models

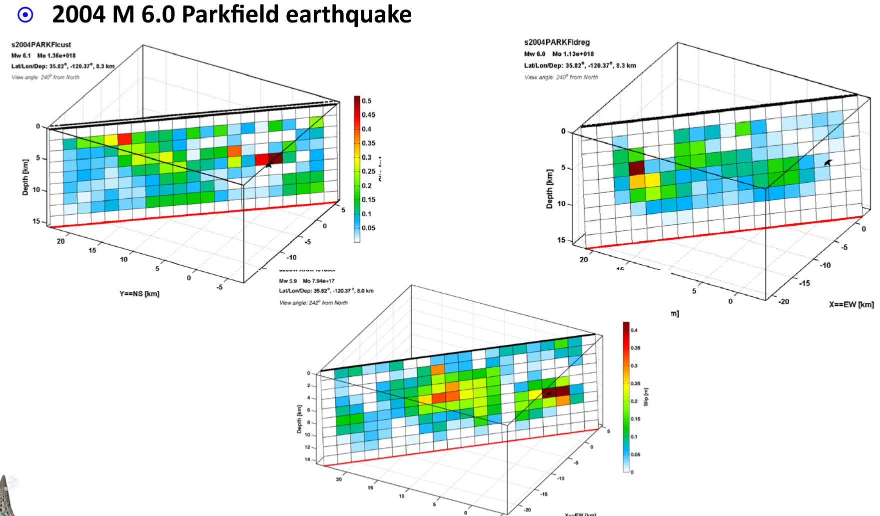




Applications & Implications

Another important case

- **2004 M 6.0 Parkfield earthquake**

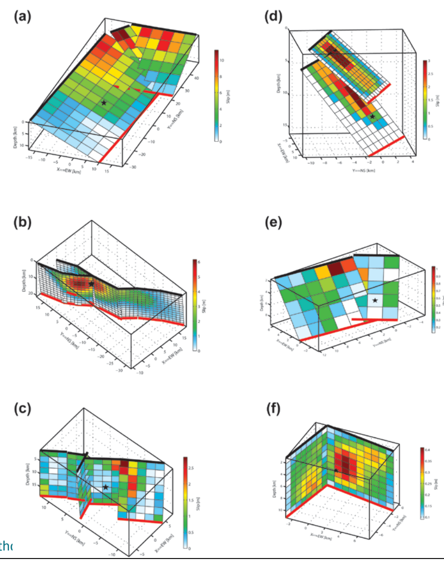


Applications & Implications

There are many more finite-fault inversions

- From large to small earthquakes

a) A model for the 1999 Chi-Chi (Taiwan) earthquake, M_w = 7.6
b) The 1999 M_w = 7.1 Hector Mine (CA) earthquake
c) Five segments for the M_w = 6.6 2000 Tottori (Japan) earthquake
d) Two overlapping planes in the 1971 M_w = 6.6 San Fernando (CA) event
e) The 2003 Miyagi-hokubo (Japan) earthquake (M_w = 6.1)
f) The 1997 Kagoshima (Japan) earthquake, M_w = 6.0

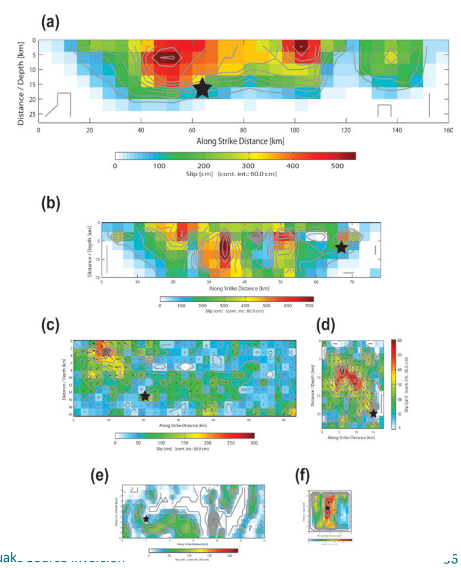


Applications & Implications

There are many more finite-fault inversions

- From large to small earthquakes

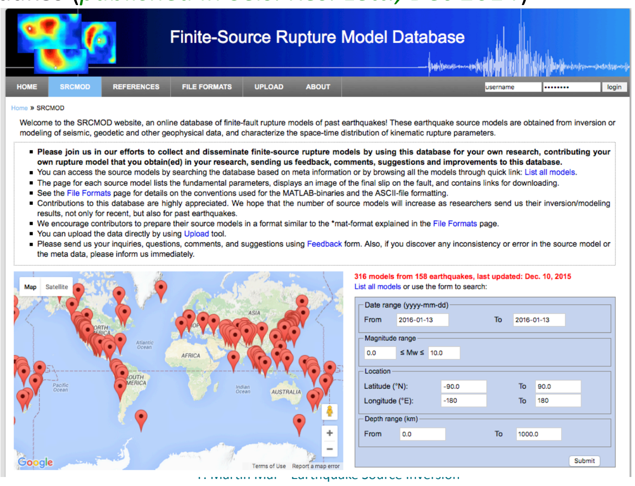
a) A model for the 1999 Izmit (Turkey) earthquake, M_w = 7.6, L = 160 km
b) The 1992 M_w = 7.3 Landers (CA) earthquake (L = 80 km)
c) The M_w = 6.9 1995 Kobe (Japan) earthquake (L = 60 km, arrows indicated slip direction = rake angle)
d) The 1994 M_w = 6.7 Northridge (CA) earthquake, L = 20 km, W = 24 km.
e) The 1984 Morgan Hill earthquake (M_w = 6.1), L = 30 km, W = 10 km
f) An event of the 1998 Hida Mountain sequence (Japan) M_w = 4.5, L = W = 4 km



Applications & Implications

A compilation: http://equake-rc.info/srcmod

- Database of finite-fault rupture models: > 300 rupture models for >150 earthquakes (*published in Seis. Res. Lett., Dec 2014*)



Applications & Implications

So, what can we extract from all this?

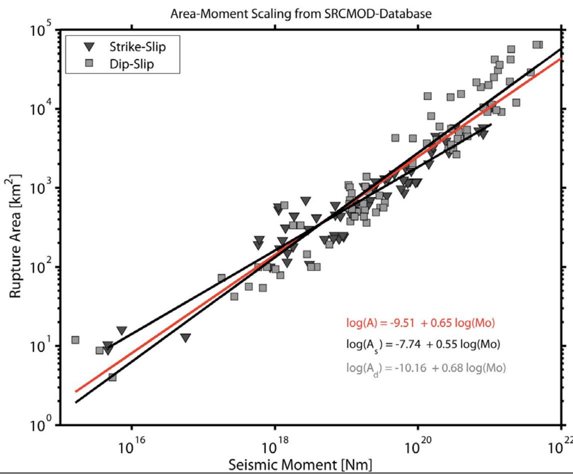
- Earthquake source inversion is a tough problem, with non-unique solutions that depend on many aspects:
  - available data, data distribution, data selection, data processing
  - inversion methodology (linearized, non-linear)
  - inversion parameterization (e.g. gridding, smoothing) and added constraints

- Despite the variability in the solutions, we can extract some common features
  - Source-scaling relations
  - Slip heterogeneity (in fact: rupture complexity in general)

Applications & Implications

Source-scaling relations

- The overall scaling follows the classic log(M_0) ≈ 2/3 log(area) ….
  …. but important differences exist for large strike-slip earthquakes …



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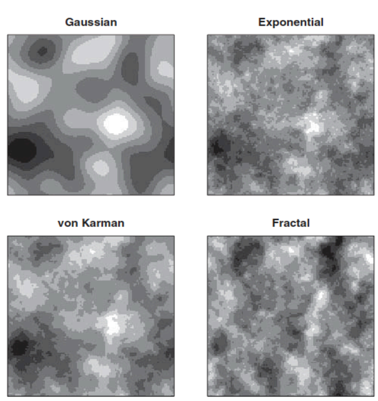
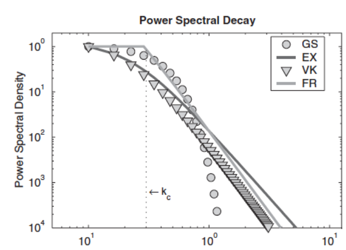
Applications & Implications

Slip is heterogeneous, and shows characteristic behavior

- **Slip complexity as a spatial random field** (Mai & Beroza, 2002; Lavallee and Archuleta, 2003, 2004)





self-affine fractal

$$P(k) \propto \frac{1}{k^{\beta+1}} \propto \frac{1}{(k_x^2+k_z^2)^{4-D}}$$

$$D = E + 1 - H.$$

auto-correlation

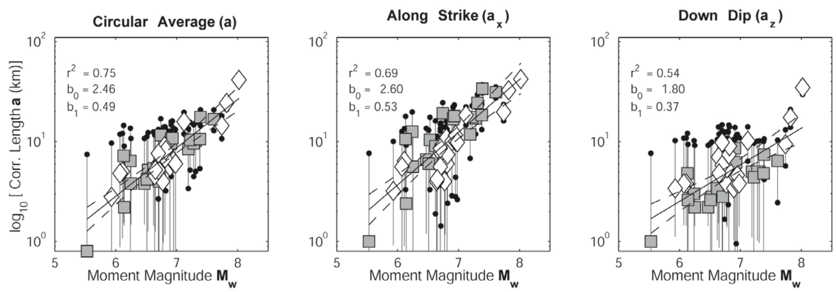
	$C(r)$	$P(k)$
GS	e^{-r^2}	$\frac{a_x a_z}{2} e^{-\frac{1}{4}k^2}$
EX	e^{-r}	$\frac{a_x a_z}{(1+k^2)^{\frac{3}{2}}}$
VK	$\frac{G_H(r)}{G_H(0)}$	$\frac{a_x a_z}{(1+k^2)^{H+1}}$.

Mai and Beroza (2002)

Applications & Implications

Slip is heterogeneous, and shows characteristic behavior

- **Slip complexity as a spatial random field** (Mai & Beroza, 2002; Lavallee and Archuleta, 2003, 2004)
- **a van Karman ACF with magnitude-dependent correlation lengths best fits the PSD of the slip functions**



$$P(k) = \frac{4\pi H}{K_0(0)} \cdot \frac{a_x a_z}{\left(1+k^2\right)^{H+1}}; \quad k = \sqrt{a_x^2 k_x^2 + a_z^2 k_z^2}$$

$$a_x \approx 2.0 + \tfrac{1}{3} L_{eff}; \quad \log(a_x) \approx -2.5 + \tfrac{1}{2} M_w$$

$$a_z \approx 1.0 + \tfrac{1}{3} W_{eff}; \quad \log(a_z) \approx -1.5 + \tfrac{1}{3} M_w$$

Mai and Beroza (2002)

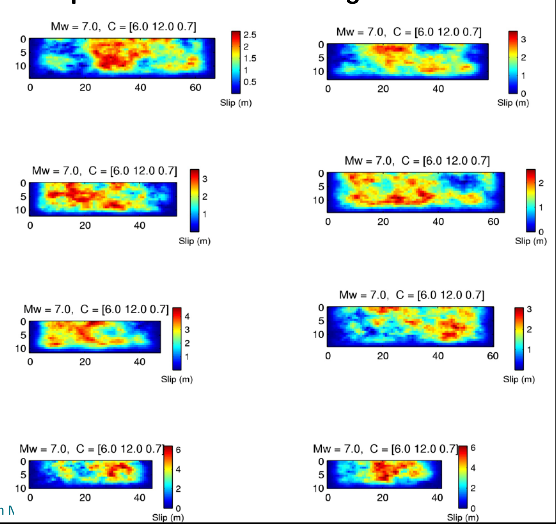
Applications & Implications

Slip is heterogeneous, and shows characteristic behavior

- **This allows to generate "realistic" slip distributions for shaking simulations**

Realizations for an M 7 strike-slip earthquake:

- Slip as a "constrained" stochastic process (*Mai and Beroza, 2002*)
- correlation lengths are computed from scaling with seismic moment
- Fault dimension based on source-scaling relations (*Wells & Coppersmith, 1994; Mai and Beroza, 2000*).



Applications & Implications

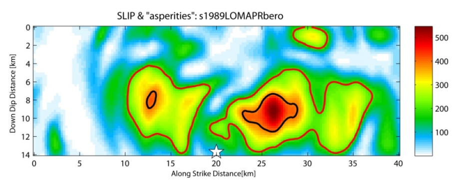
Slip is heterogeneous → stress drop even more so

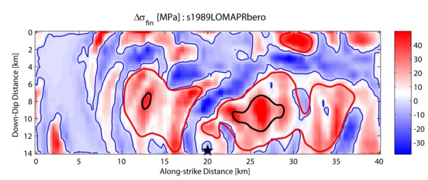
- **Slip inversion reveal how variable stress-drop is on the fault**

$$\epsilon_{xx} = \frac{\partial u_x}{\partial x} \approx \frac{\overline{D}}{L_c} \qquad \Delta\sigma = \mu\epsilon_{xx} = \mu\frac{\overline{D}}{L_c}$$

$$\Delta\sigma_{\parallel}(\mathbf{k}) = K_{\parallel}(\mathbf{k})D(\mathbf{k}) \qquad K_{\parallel}(\mathbf{k}) = -\frac{1}{2}\frac{\mu}{\sqrt{k_{\parallel}^2 + k_{\perp}^2}}\left[\frac{2(\lambda+\mu)}{\lambda+2\mu}k_{\parallel}^2 + k_{\perp}^2\right]$$

$$\Delta\sigma_{\perp}(\mathbf{k}) = K_{\perp}(\mathbf{k})D(\mathbf{k}), \qquad K_{\perp}(\mathbf{k}) = -\frac{1}{2}\frac{\mu}{\sqrt{k_{\parallel}^2 + k_{\perp}^2}}\left[\frac{2(\lambda+\mu)}{\lambda+2\mu} - 1\right]k_{\parallel}k_{\perp}$$



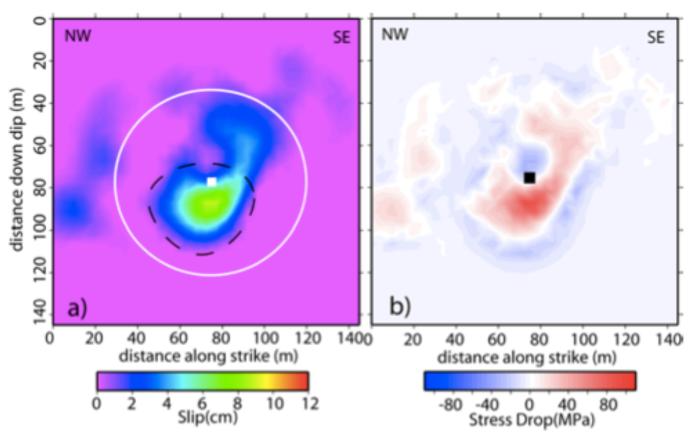


Ripperger & Mai (2004)

Applications & Implications

Slip is heterogeneous → stress drop even more so

- **Slip inversion reveal how variable stress-drop is on the fault, and shows much larger amplitudes than the "typical" value Δσ ~ 3MPa**

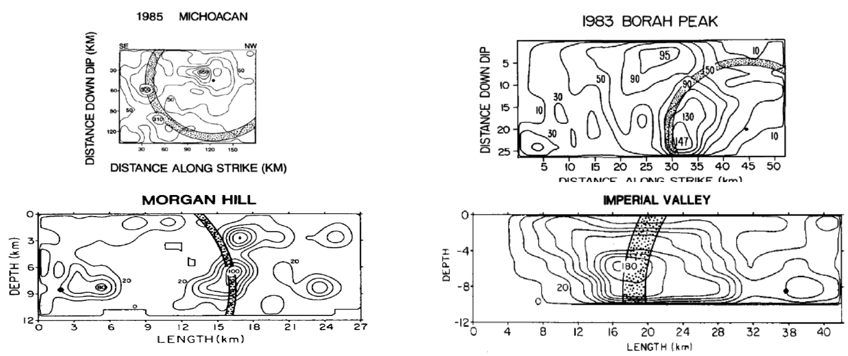


Dreger et al. (2007)

Applications & Implications

So, what can we learn from finite-fault source inversions

- The rupture process is highly complicated in space and time
  - These result informed early studies of rupture dynamics, which in fact were based on constant stress drop models ….
  - The "Heaton" slip-pulse (Heaton, 1990) used source inversions



Applications & Implications

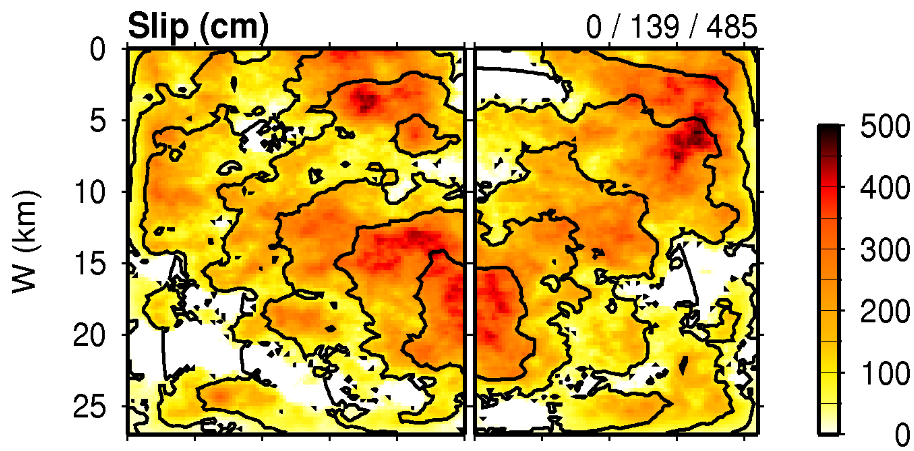
So, what can we learn from finite-fault source inversions

- The rupture process is highly complicated in space and time
  - These result informed early studies of rupture dynamics, which in fact were based on constant stress drop models ….
  - The "Heaton" slip-pulse (Heaton, 1990) used source inversions
  - Dynamic rupture simulations now use randomized stress on the fault almost routinely

- Slip-heterogeneity characterizations are now common-practice ("state-of-the-art) in ground-motion simulations, using so-called "rupture-generators (Guatteri et, 2002, 2003; Graves & Pitarka, 2004, 2010; and others)

Applications & Implications

So, what can we learn from finite-fault source inversions



Scenario-rupture for a M 7.2 on Puente Hills Fault, L.A. area (by Rob Graves)

Introduction & Theory

Next ?

(3) New developments, new challenges, new opportunities

- Imaging versus inversion; combination of both?
- Alternative methods
- Uncertainty quantification