


Introduction to and State-of-the-Art in Earthquake Source Inversion

– Part 2 –

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Roadmap

Earthquake Source Inversion

- (1) Introduction & Theory**
 - A brief overview
 - Fundamentals
 - From point-source to extended-fault modeling
- (2) Applications & Implications**
 - Early developments & case studies
 - What can we extract from them?
 - What to learn from finite-fault source models?
- (3) Challenges, Developments, Opportunities**
 - Imaging versus inversion, or combination of both?
 - Alternative methods
 - Uncertainty quantification




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Applications & Implications

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“Historic” developments and early landmark studies

- ⊙ The 1979 Imperial Valley (M 6.6) earthquake was the first one studied in great detail, due to the existence of a cross-fault seismic array
- ⊙ For the 1984 Morgan Hill (M6.2) earthquake a new method was developed
- ⊙ The 1992 Landers (M 7.3) occurred on a geometrically complex fault, and triggered new ideas
- ⊙ For the 1994 Northridge (M 6.7) lots of (new) data were available, and many groups worked on that earthquake
- ⊙ 1995 Kobe (M 6.9), 1999 Izmit (M 7.6), 1999 Chi-Chi (M 7.6), 2004 Parkfield (M 6.0) are further landmark events that taught us numerous lessons ...



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Applications & Implications

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The representation theorem

$$u_n(\mathbf{x}, t) = \int_{-\infty}^{\infty} d\tau \int_{\Sigma} [s_i(\xi, \tau)] \cdot c_{ijpq} \cdot \nu_j \cdot \frac{\partial}{\partial \xi_q} G_{np}(\mathbf{x}, t - \tau; \xi, 0) d\Sigma$$

displacement seismogram
at observer location \mathbf{x}


time-dependent slip
history on fault

elasticity
tensor

fault-normal
vector

Green's tensor for
geometry of interest

This equation is used to formulate the inverse problem:
Given the seismic observations $u_n(\mathbf{x}, t)$, let us estimate the time-dependent slip history on the fault, assuming we know how to compute (or otherwise obtain) Green's functions and we have some knowledge about the fault geometry.



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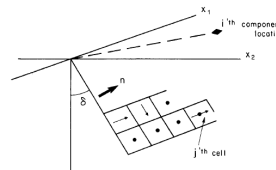
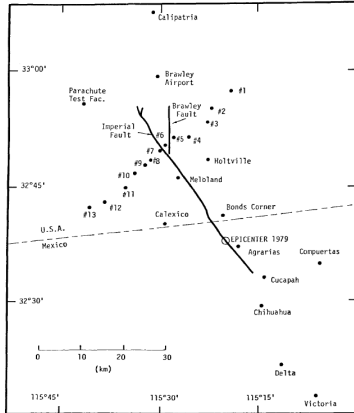
Applications & Implications

The “classic” finite-fault earthquake source inversion

- 1979 M 6.6 Imperial Valley earthquake (e.g. Olson & Apsel, 1982; Hartzell & Heaton, 1983; Archuleta, 1984):

- Olson & Apsel: linearized MTW inversion

$$U^i(\mathbf{y}, t) = \sum_{j=1}^J \sum_{k=-K}^K \mathbf{s}_{jk} \cdot \mathbf{g}_j^i(\mathbf{y}, t + k\delta t).$$



The damped least-squares solution

$$\mathbf{y}_d = \mathbf{V}\Lambda(\Lambda^2 + k_0^2\mathbf{I})^{-1}\mathbf{U}'\mathbf{f}.$$

The least-squares solution to the augmented system is \mathbf{y}_a , where

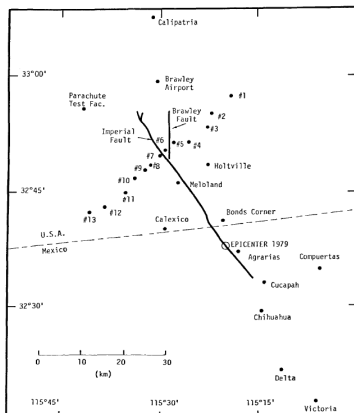
$$\mathbf{y}_a = \mathbf{V}\Lambda_a^{-2}(\Lambda\mathbf{U}'\mathbf{f} + k_0\mathbf{H}\mathbf{V}'\mathbf{b}).$$

Applications & Implications

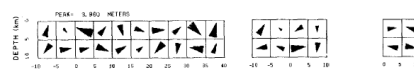
“Standard” finite-fault earthquake source inversion

- 1979 M 6.6 Imperial Valley earthquake (e.g. Olson & Apsel, 1982; Hartzell & Heaton, 1983; Archuleta, 1984):

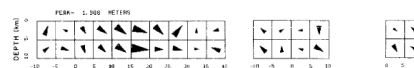
- Olson & Apsel: linearized MTW inversion



LEAST SQUARES
STATIC OFFSET



STABILIZED LEAST SQUARES
STATIC OFFSET



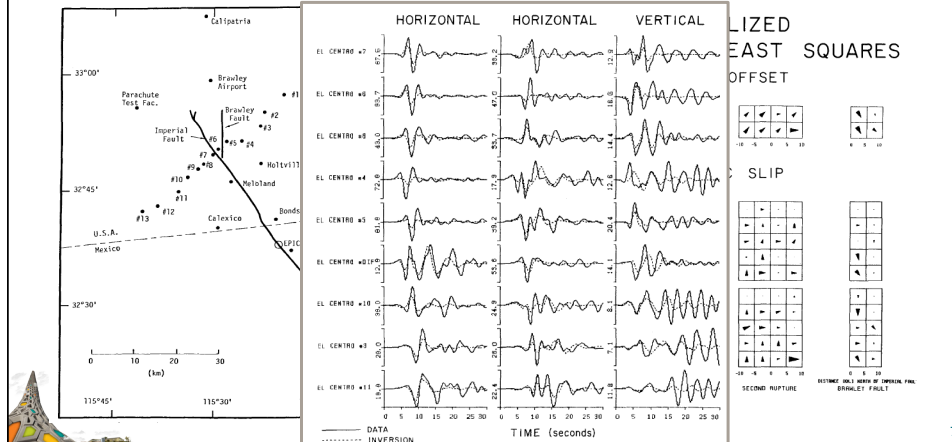
Applications & Implications



“Standard” finite-fault earthquake source inversion

1979 M 6.6 Imperial Valley earthquake (e.g. Olson & Apsel, 1982; Hartzell & Heaton, 1983; Archuleta, 1984):

Olson & Apsel: linearized MTW inversion



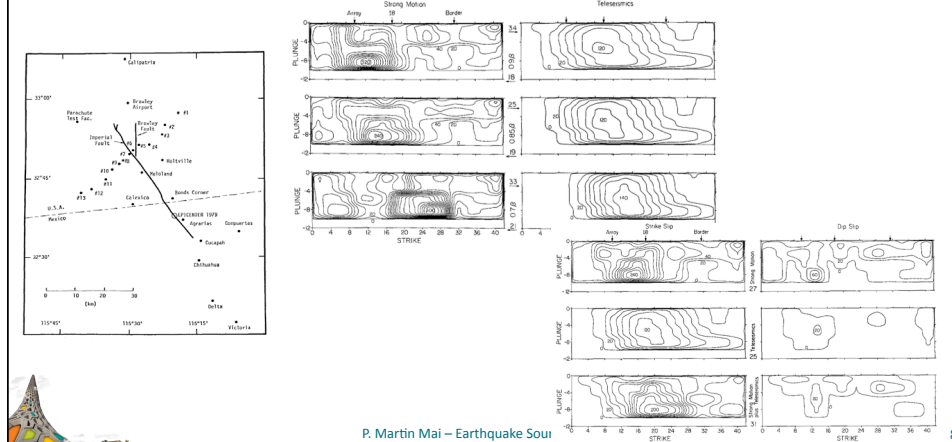
Applications & Implications



“Standard” finite-fault earthquake source inversion

1979 M 6.6 Imperial Valley earthquake (e.g. Olson & Apsel, 1982; Hartzell & Heaton, 1983; Archuleta, 1984):

Hartzell & Heaton: linearized MTW inversion, incl. teleseismic data



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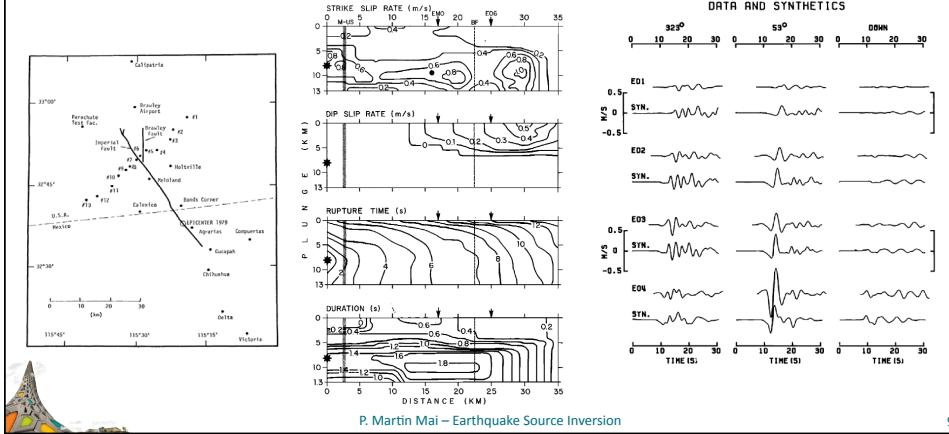
Applications & Implications



“Standard” finite-fault earthquake source inversion

1979 M 6.6 Imperial Valley earthquake (e.g. Olson & Apsel, 1982; Hartzell & Heaton, 1983; Archuleta, 1984):

- Archuleta: tedious “manual” forward modeling

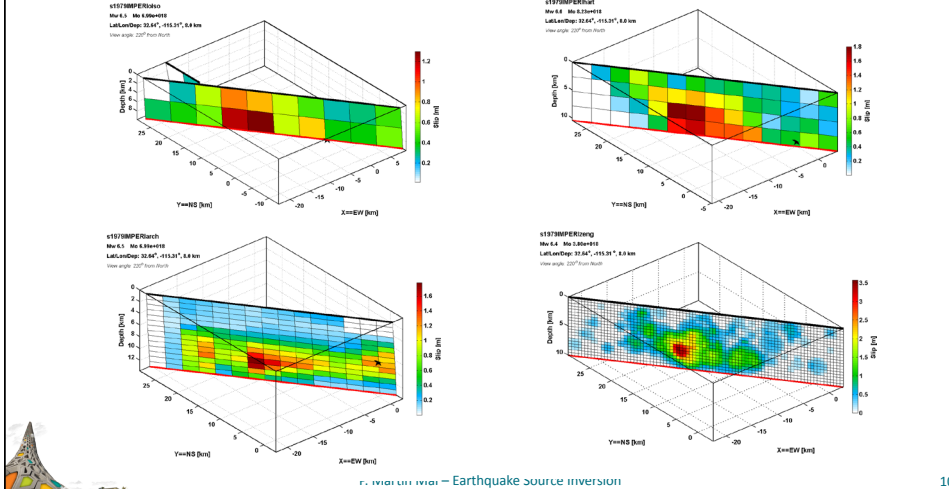


Applications & Implications



“Standard” finite-fault earthquake source inversion

1979 M 6.6 Imperial Valley earthquake (e.g. Olson & Apsel, 1982; Hartzell & Heaton, 1983; Archuleta, 1984; Zeng et al, 1996):

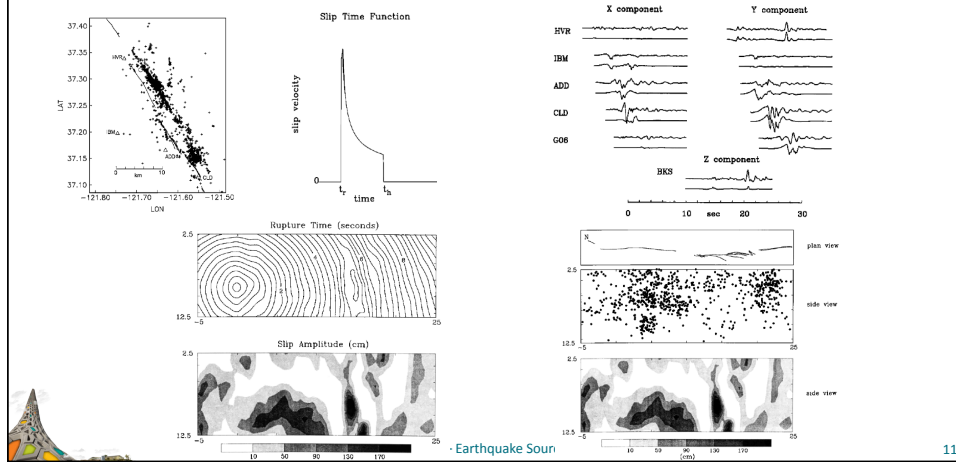


Applications & Implications



Further early developments

- ⦿ **1984 M 6.2 Morgan Hill earthquake** (Heaton & Hartzell, 1986; Beroza & Spudich, 1988)
 - Fault from aftershock locations; rupture-dynamics inspired source-time function



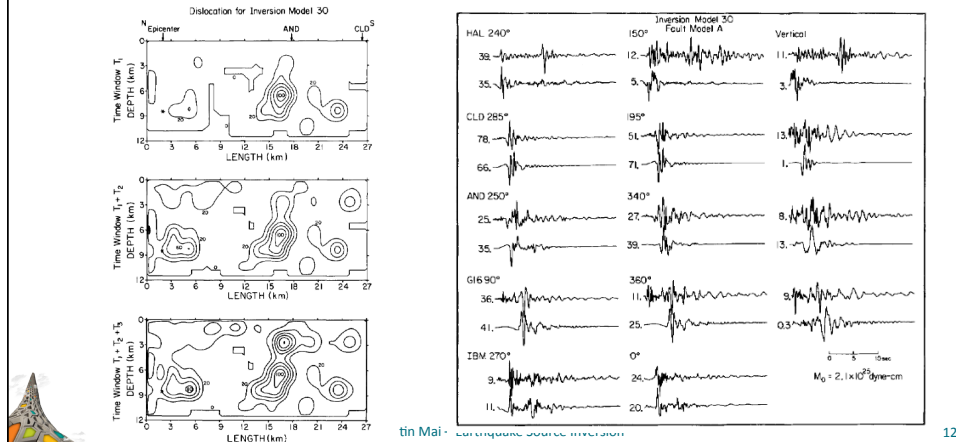
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Applications & Implications



Further early developments

- ⦿ **1984 M 6.2 Morgan Hill earthquake** (Heaton & Hartzell, 1986; Beroza & Spudich, 1988)
 - H&H ran numerous trial inversions, some are reported, one chosen a “best” model

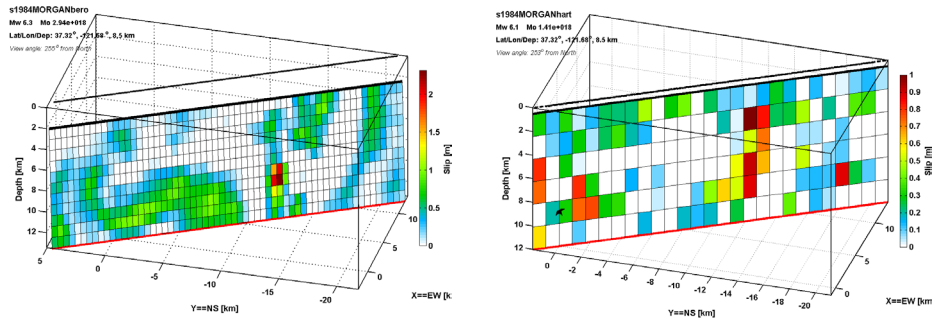


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Applications & Implications

Further early developments

- 1984 M 6.2 Morgan Hill earthquake (Heaton & Hartzell, 1986; Beroza & Spudich, 1988):

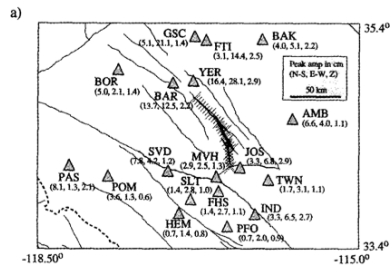


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Applications & Implications

Rupture on a geometrically complex fault

- 1992 M 7.3 Landers earthquake (Cohee & Beroza, 1994; Wald et al., 1994, Cotton & Campillo, 1996):



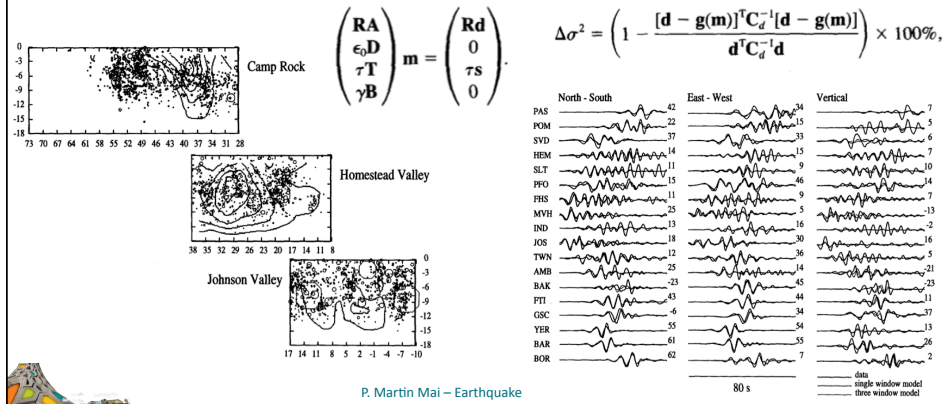
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Applications & Implications

Rupture on a geometrically complex fault

1992 M 7.3 Landers earthquake (Cohee & Beroza, 1994; Wald et al., 1994, Cotton & Campillo, 1996):

- C & B applied smoothing, boundary conditions at the top and bottom, and defined the variance reduction as misfit norm

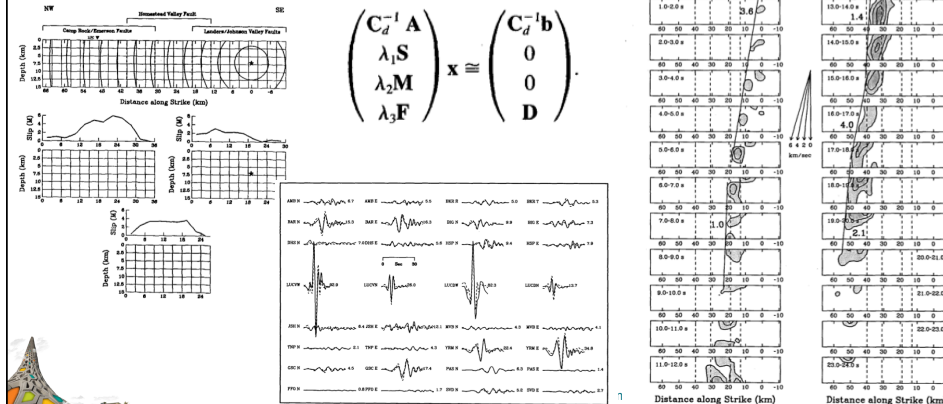


Applications & Implications

Rupture on a geometrically complex fault

1992 M 7.3 Landers earthquake (Cohee & Beroza, 1994; Wald et al., 1994, Cotton & Campillo, 1996):

- Wald et al applied the “standard” MTW approach and multiple data sets



Applications & Implications

Rupture on a geometrically complex fault

○ **1992 M 7.3 Landers earthquake** (Cohee & Beroza, 1994; Wald et al., 1994, Cotton & Campillo, 1996):

- C & C developed and applied the inversion in the frequency domain, used a different slip function, and made clear that they cannot fit the higher frequencies

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Applications & Implications

Rupture on a geometrically complex fault


○ **1992 M 7.3 Landers earthquake** (Cohee & Beroza, 1994; Wald et al., 1994, Cotton & Campillo, 1996):

- Again, various slip models ...

Source

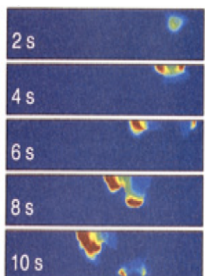
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Applications & Implications



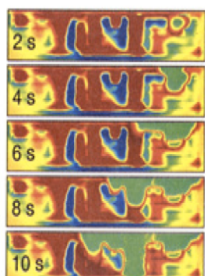
Dynamics on simple and complex faults

- 1992 M 7.3 Landers earthquake



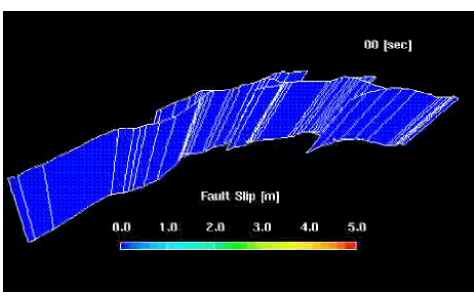
2 s
4 s
6 s
8 s
10 s

Peyrat et al, 2001
Heterogeneous pre-stress



2 s
4 s
6 s
8 s
10 s


Aochi & Madariaga, 2002
Complex fault geometry



00 [sec]
Fault Slip [m]
0.0 1.0 2.0 3.0 4.0 5.0

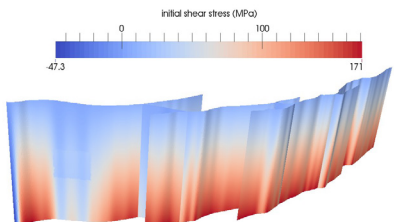
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Applications & Implications

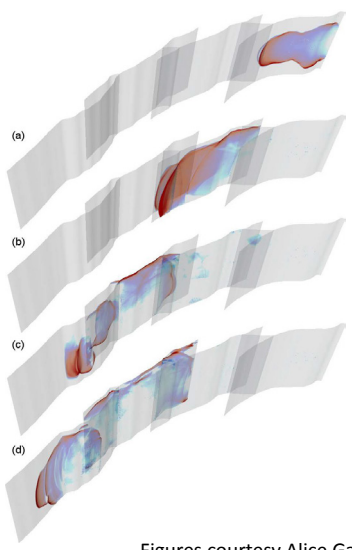


Dynamics on simple and complex faults

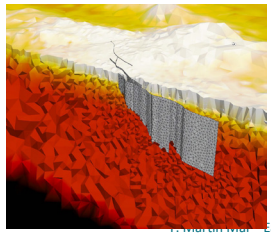
- 1992 M 7.3 Landers earthquake



Initial shear stress (MPa)
-47.3 0 100 171



(a)
(b)
(c)
(d)

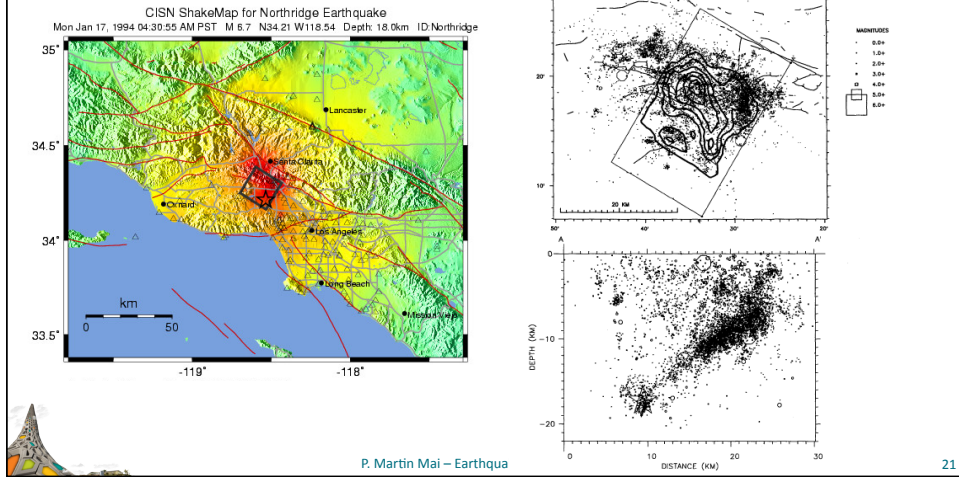


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Applications & Implications

Further early developments

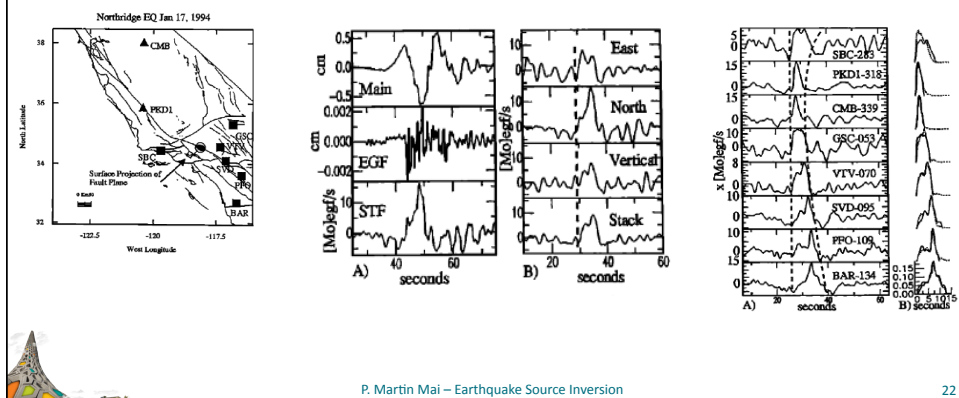
- 1994 M 6.7 Northridge earthquake (numerous studies ...) -- a "hidden" blind-thrust earthquake !



Applications & Implications

Further early developments

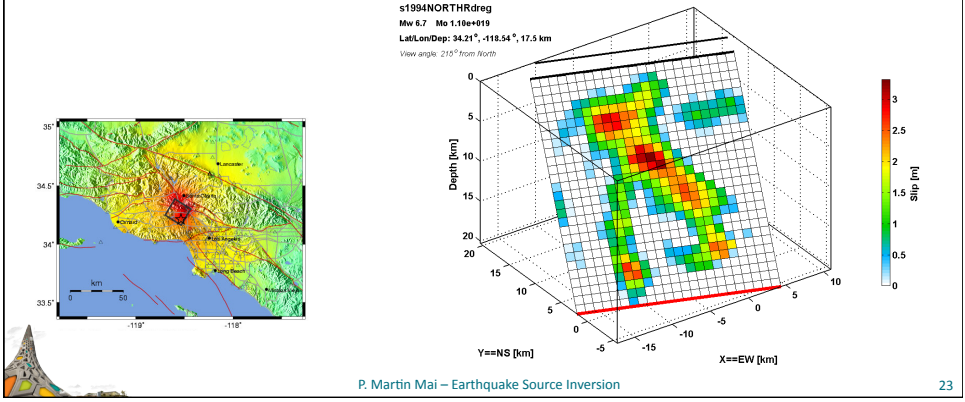
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 - Dreger (1994) used empirical Green's functions; deconvolved from the recorded seismograms to obtain local moment-rate functions for inversion



Applications & Implications

Further early developments

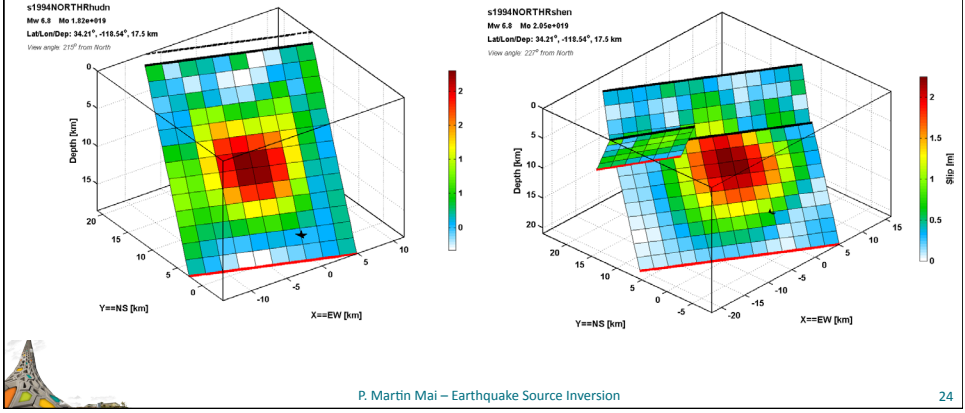
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 - Dreger (1994) used empirical Green’s functions; deconvolved from the recorded seismograms to obtain local moment-rate functions for inversion



Applications & Implications

Further early developments

- ⦿ **1994 M 6.7 Northridge earthquake** (numerous studies ...)
 - Hudnut et al (1996) and Shen et al (1996) used geodetic data

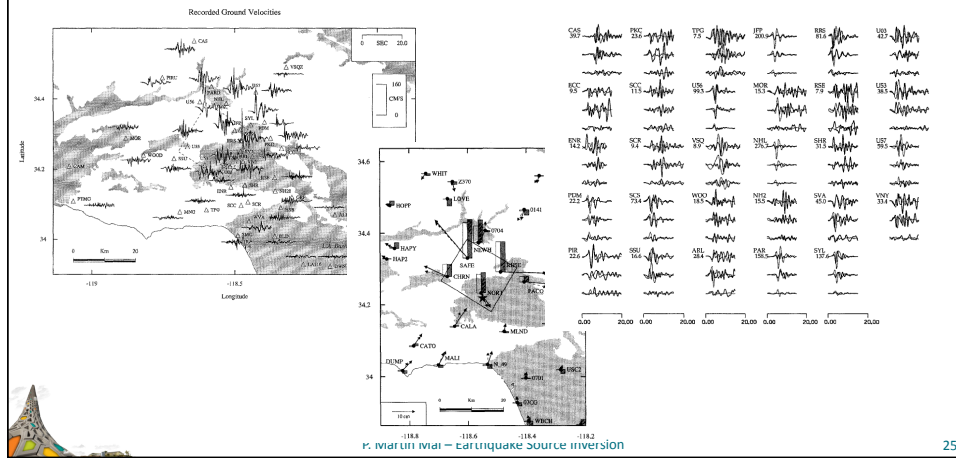


Applications & Implications



Further early developments

- 1994 M 6.7 Northridge earthquake (numerous studies ...)
- Wald et al (1996) use all available data, in the “standard” MTW approach



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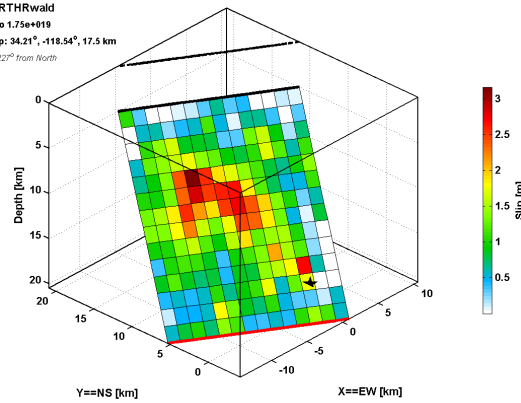
Applications & Implications



Further early developments

- 1994 M 6.7 Northridge earthquake (numerous studies ...)
- Wald et al (1996) use all available data, in the “standard” MTW approach

s1994NORTHrwald
 Mw 6.8 Mo 1.75e+019
 Lat/Lon/Dep: 34.21°, -118.54°, 17.5 km
 View angle: 227° from North



P. Wald (UCLA) et al - Earthquake Source Inversion

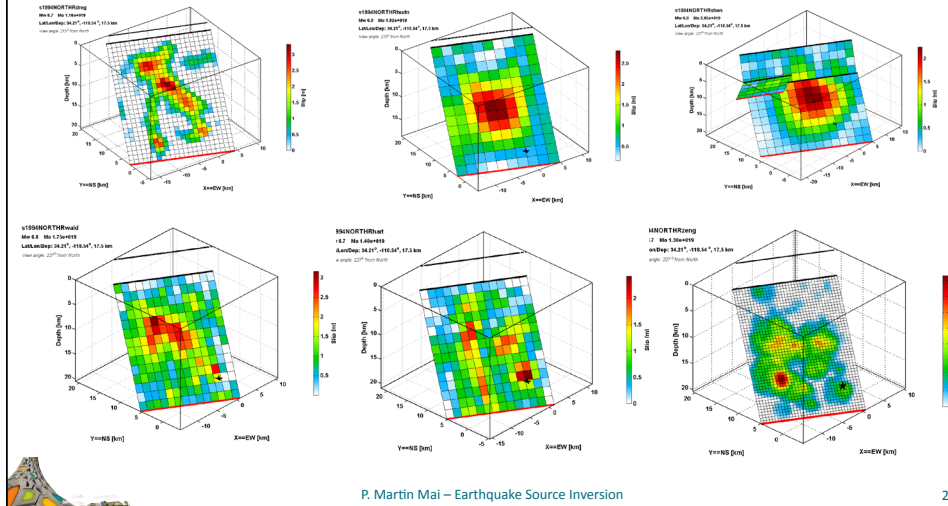
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Applications & Implications



Further early developments

1994 M 6.7 Northridge earthquake: a visual comparison



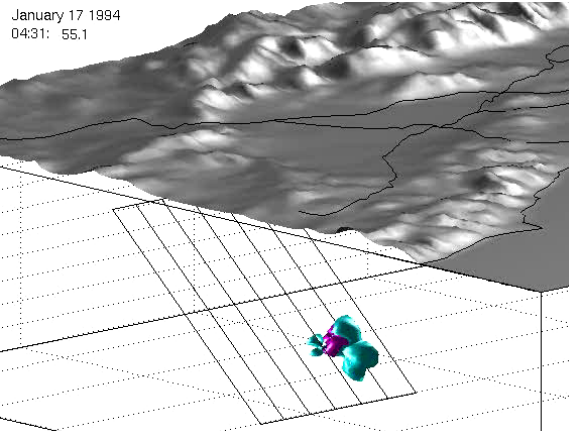
Applications & Implications



Using the inversion results

1994 M 6.7 Northridge earthquake: The story does not end here ...

- We still don't fully understand extreme shaking levels during the event, certain site, path, topography effects, or the precise dynamics of the rupture

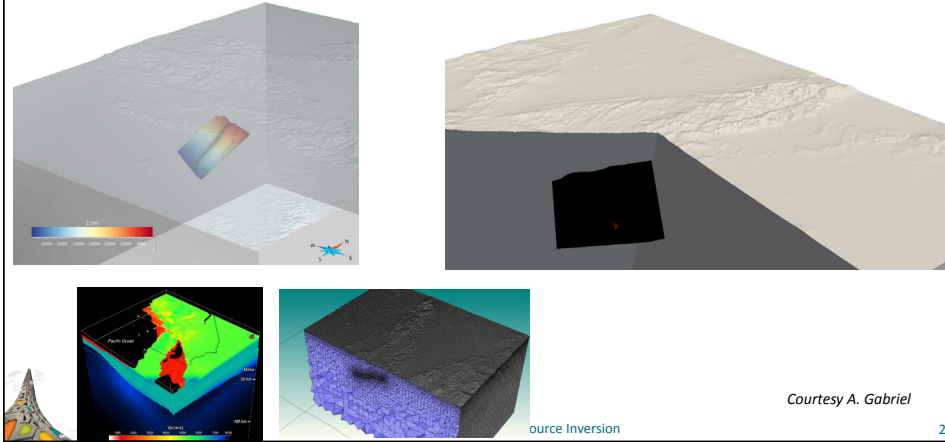


Courtesy K. Olsen

Applications & Implications

Using the inversion results

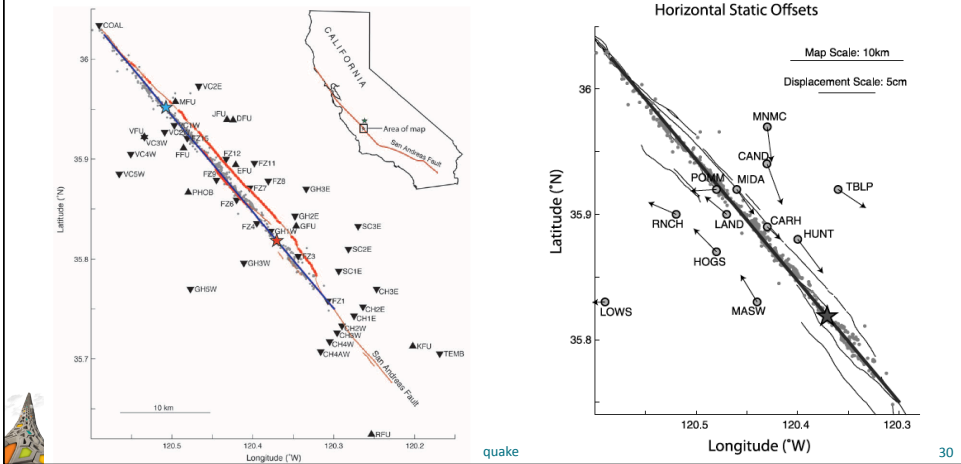
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 - We still don't fully understand extreme shaking levels during the event, certain site, path, topography effects, or the precise dynamics of the rupture



Applications & Implications

Another important case

- 2004 M 6.0 Parkfield earthquake (numerous studies ...)
 - Lots of data, but still ambiguities

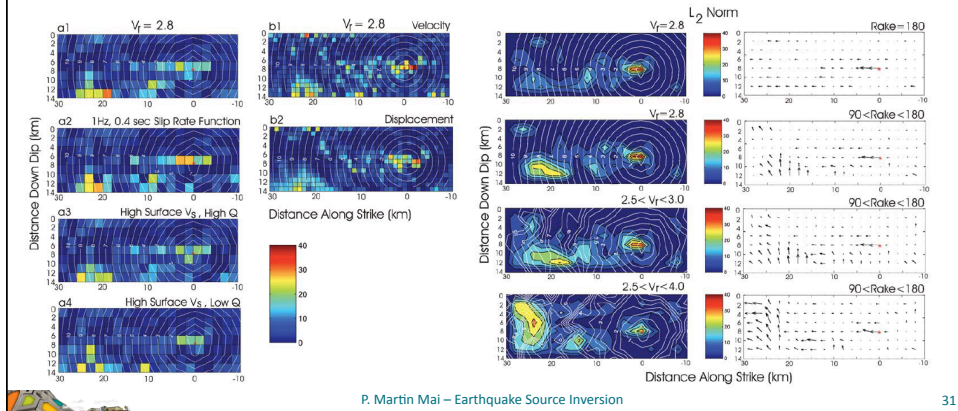


Applications & Implications

Another important case

○ 2004 M 6.0 Parkfield earthquake (numerous studies ...)

- Lots of data, but still ambiguities
- Hartzell et al (2007) tested various misfit norms & parameterizations

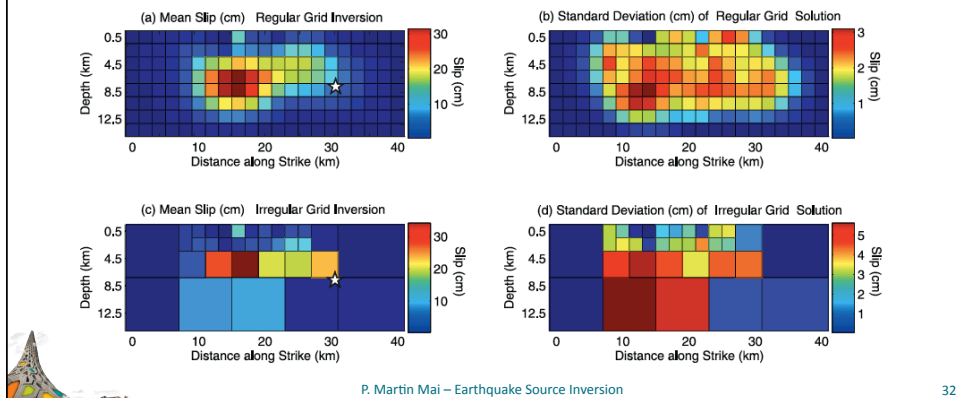


Applications & Implications

Another important case

○ 2004 M 6.0 Parkfield earthquake (numerous studies ...)

- Lots of data, but still ambiguities
- Page et al. (2009) used GPS data and various discretization

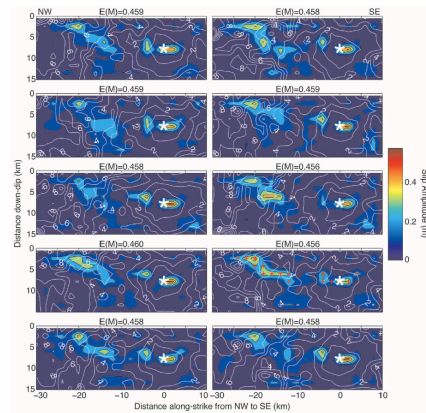
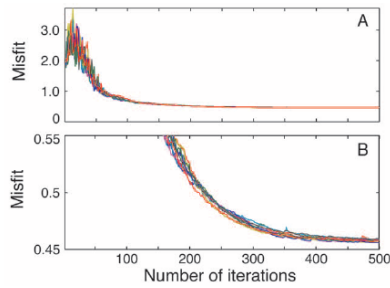


Applications & Implications

Another important case

2004 M 6.0 Parkfield earthquake (numerous studies ...)

- Lots of data, but still ambiguities
- Lui et al. (2006) conducted a non-linear inversion and did statistics on the 10 “best” models

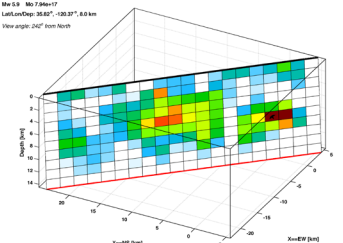
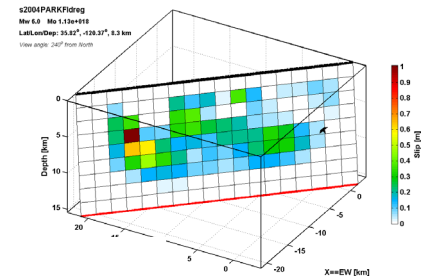
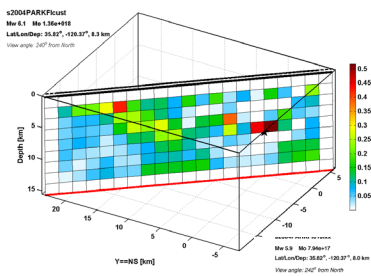


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Applications & Implications

Another important case

2004 M 6.0 Parkfield earthquake



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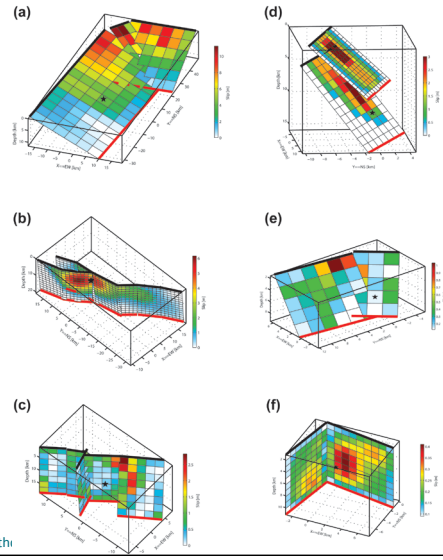
Applications & Implications



There are many more finite-fault inversions

From large to small earthquakes

- a) A model for the 1999 Chi-Chi (Taiwan) earthquake, $M_w = 7.6$
- b) The 1999 $M_w = 7.1$ Hector Mine (CA) earthquake
- c) Five segments for the $M_w = 6.6$ 2000 Tottori (Japan) earthquake
- d) Two overlapping planes in the 1971 $M_w = 6.6$ San Fernando (CA) event
- e) The 2003 Miyagi-hokubo (Japan) earthquake ($M_w = 6.1$)
- f) The 1997 Kagoshima (Japan) earthquake, $M_w = 6.0$



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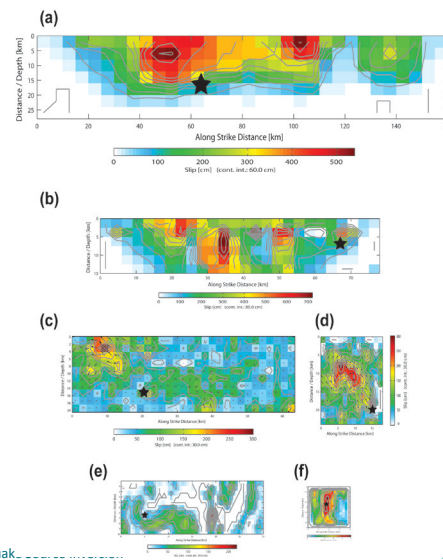
Applications & Implications



There are many more finite-fault inversions

From large to small earthquakes

- a) A model for the 1999 Izmit (Turkey) earthquake, $M_w = 7.6$, $L = 160$ km
- b) The 1992 $M_w = 7.3$ Landers (CA) earthquake ($L = 80$ km)
- c) The $M_w = 6.9$ 1995 Kobe (Japan) earthquake ($L = 60$ km, arrows indicated slip direction = rake angle)
- d) The 1994 $M_w = 6.7$ Northridge (CA) earthquake, $L = 20$ km, $W = 24$ km.
- e) The 1984 Morgan Hill earthquake ($M_w = 6.1$), $L = 30$ km, $W = 10$ km
- f) An event of the 1998 Hida Mountain sequence (Japan) $M_w = 4.5$, $L = W = 4$ km



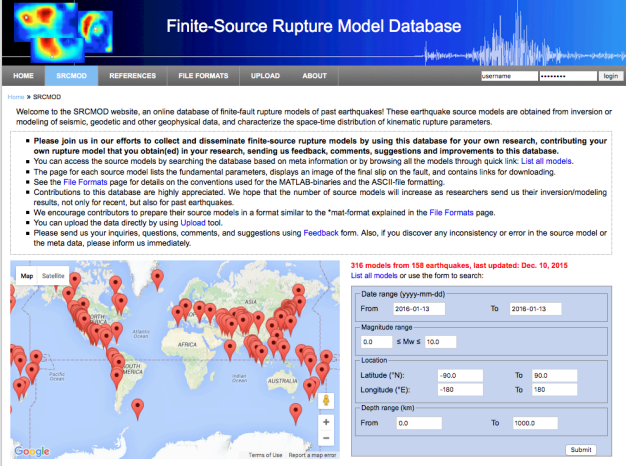
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Applications & Implications

A compilation: <http://equake-rc.info/srcmod>

- Database of finite-fault rupture models: > 300 rupture models for >150 earthquakes (*published in Seis. Res. Lett., Dec 2014*)



Finite-Source Rupture Model Database

HOME SRCMOD REFERENCES FILE FORMATS UPLOAD ABOUT

Welcome to the SRCMOD website, an online database of finite-fault rupture models of past earthquakes! These earthquake source models are obtained from inversion or modeling of seismic, geodesic and other geophysical data, and characterize the space-time distribution of kinematic rupture parameters.

- Please join us in our efforts to collect and disseminate finite-source rupture models by using this database for your own research, contributing your own rupture model that you obtain(ed) in your research, sending us feedback, comments, suggestions and improvements to this database.
- You can access the source models by searching the database based on meta information or by browsing all the models through quick link. [List all models.](#)
- The page for each source model lists the fundamental parameters, displays an image of the final slip on the fault, and contains links for downloading.
- See the [File Formats](#) page for details on the conventions used for the MATLAB-binaries and the ASCII-file formatting.
- Contributions to this database are highly appreciated. We hope that the number of source models will increase as researchers send us their inversion/modeling results, not only for recent, but also for past earthquakes.
- We encourage contributors to prepare their source models in a format similar to the *.mat-format explained in the [File Formats](#) page.
- You can upload the data directly by using [Upload](#) tool.
- Please send us your inquires, questions, comments, and suggestions using [Feedback](#) form. Also, if you discover any inconsistency or error in the source model or the meta data, please inform us immediately.

316 models from 158 earthquakes, last updated: Dec. 10, 2015

List all models or use the form to search:

Date range (yyyy-mm-dd)
From 2016-01-13 To 2016-01-13

Magnitude range
0.0 ≤ Max ≤ 10.0

Location
Latitude (°N): -90.0 To 90.0
Longitude (°E): -180 To 180

Depth range (km)
From 0.0 To 1000.0

Submit

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Applications & Implications

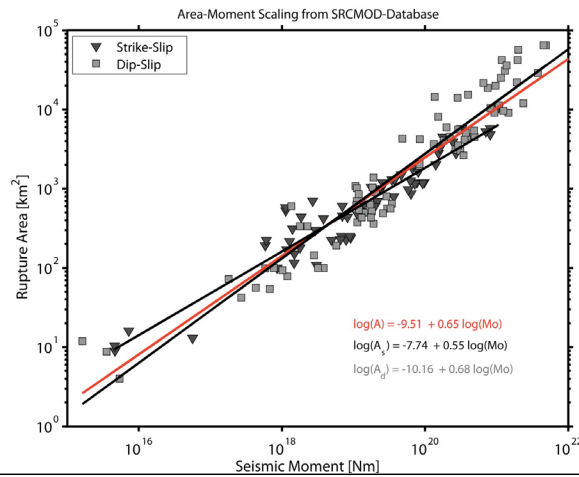
So, what can we extract from all this?

- Earthquake source inversion is a tough problem, with non-unique solutions that depend on many aspects:
 - available data, data distribution, data selection, data processing
 - inversion methodology (linearized, non-linear)
 - inversion parameterization (e.g. gridding, smoothing) and added constraints
- Despite the variability in the solutions, we can extract some common features
 - Source-scaling relations
 - Slip heterogeneity (in fact: rupture complexity in general)

Applications & Implications

Source-scaling relations

- The overall scaling follows the classic $\log(M_0) \approx 2/3 \log(\text{area})$
 but important differences exist for large strike-slip earthquakes ...

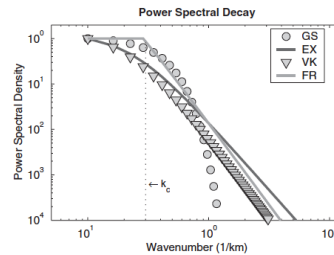
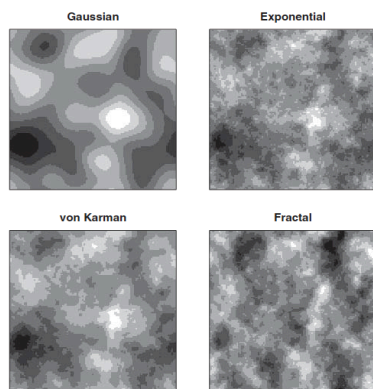


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Applications & Implications

Slip is heterogeneous, and shows characteristic behavior

- Slip complexity as a spatial random field (Mai & Beroza, 2002; Lavallee and Archuleta, 2003, 2004)



<i>self-affine fractal</i>		<i>auto-correlation</i>
$P(k) \propto \frac{1}{k^{3+1}} \propto \frac{1}{(k_x^2 + k_y^2)^{4-D}}$		$C(r) \quad P(k)$
$D = E + 1 - H.$		GS $e^{-r^2} \quad \frac{a_s a_z}{2} e^{-\frac{1}{2}k^2}$
		EX $e^{-r} \quad \frac{a_s a_z}{(1+k^2)^2}$
		VK $\frac{G_H(r)}{G_H(0)} \quad \frac{a_s a_z}{(1+k^2)^{H+1}}$

Mai and Beroza (2002)

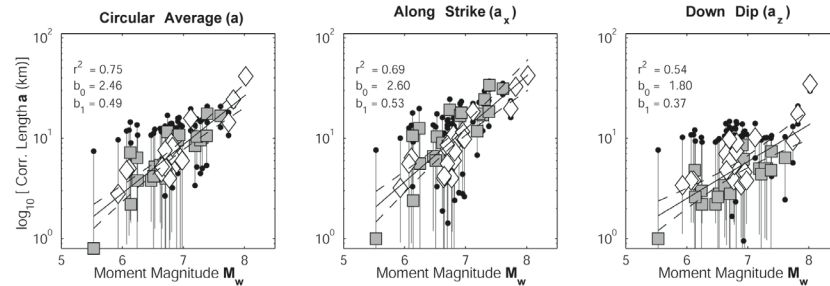
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Applications & Implications

Slip is heterogeneous, and shows characteristic behavior

- Slip complexity as a spatial random field (Mai & Beroza, 2002; Lavallee and Archuleta, 2003, 2004)
- a van Karman ACF with magnitude-dependent correlation lengths best fits the PSD of the slip functions



$$P(k) = \frac{4\pi H}{K_0(0)} \cdot \frac{a_x a_z}{(1+k^2)^{H+1}}; \quad k = \sqrt{a_x^2 k_x^2 + a_z^2 k_z^2}$$

$$a_x \approx 2.0 + \frac{1}{3} L_{eff}; \quad \log(a_x) \approx -2.5 + \frac{1}{2} M_w$$

$$a_z \approx 1.0 + \frac{1}{3} W_{eff}; \quad \log(a_z) \approx -1.5 + \frac{1}{3} M_w$$

Mai and Beroza (2002) P. Martin Mai – Earthquake Source Inversion 41

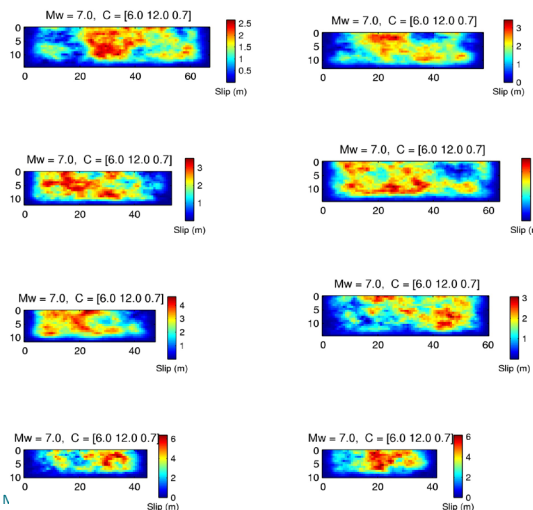
Applications & Implications

Slip is heterogeneous, and shows characteristic behavior

- This allows to generate “realistic” slip distributions for shaking simulations

Realizations for an M 7 strike-slip earthquake:

- Slip as a “constrained” stochastic process (Mai and Beroza, 2002)
- correlation lengths are computed from scaling with seismic moment
- Fault dimension based on source-scaling relations (Wells & Coppersmith, 1994; Mai and Beroza, 2000).



Applications & Implications

Slip is heterogeneous → stress drop even more so

- Slip inversion reveal how variable stress-drop is on the fault

$$\epsilon_{xx} = \frac{\partial u_x}{\partial x} \approx \frac{\bar{D}}{L_c}$$

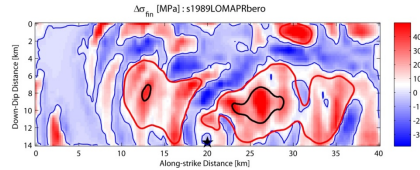
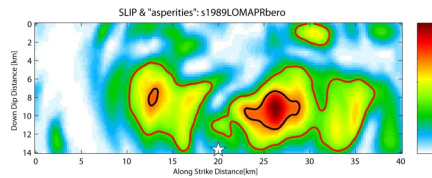
$$\Delta\sigma = \mu\epsilon_{xx} = \mu \frac{\bar{D}}{L_c}$$

$$\Delta\sigma_{\parallel}(\mathbf{k}) = K_{\parallel}(\mathbf{k})D(\mathbf{k})$$

$$K_{\parallel}(\mathbf{k}) = -\frac{1}{2} \frac{\mu}{\sqrt{k_{\parallel}^2 + k_{\perp}^2}} \left[\frac{2(\lambda + \mu)}{\lambda + 2\mu} k_{\parallel}^2 + k_{\perp}^2 \right]$$

$$\Delta\sigma_{\perp}(\mathbf{k}) = K_{\perp}(\mathbf{k})D(\mathbf{k}),$$

$$K_{\perp}(\mathbf{k}) = -\frac{1}{2} \frac{\mu}{\sqrt{k_{\parallel}^2 + k_{\perp}^2}} \left[\frac{2(\lambda + \mu)}{\lambda + 2\mu} - 1 \right] k_{\parallel} k_{\perp}$$



Ripperger & Mai (2004)

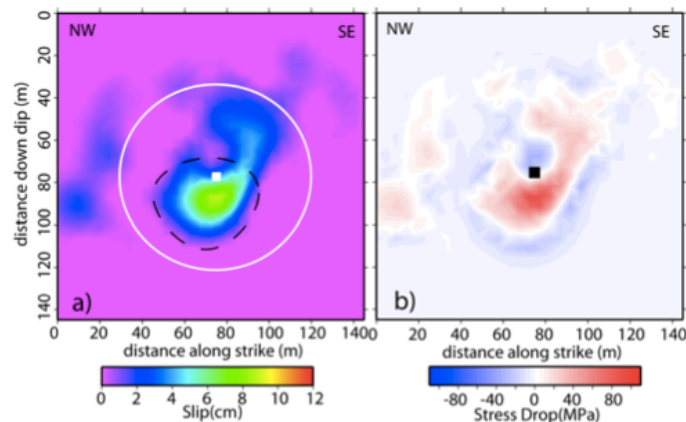
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Applications & Implications

Slip is heterogeneous → stress drop even more so

- Slip inversion reveal how variable stress-drop is on the fault, and shows much larger amplitudes than the “typical” value $\Delta\sigma \sim 3\text{MPa}$



Dreger et al. (2007)

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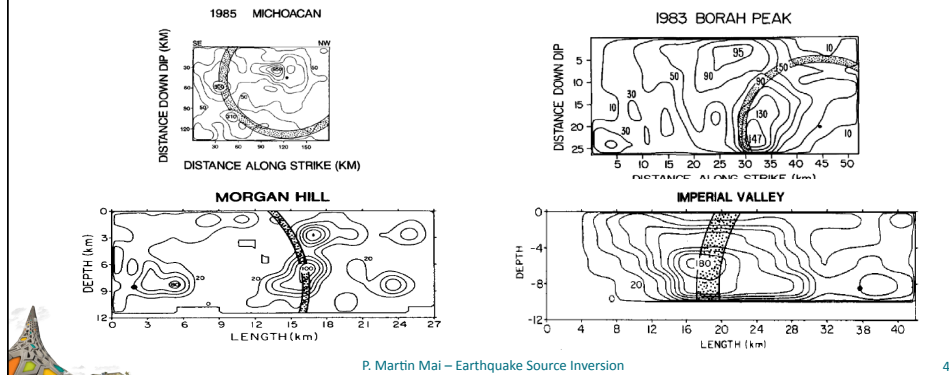
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Applications & Implications



So, what can we learn from finite-fault source inversions

- The rupture process is highly complicated in space and time
 - These result informed early studies of rupture dynamics, which in fact were based on constant stress drop models
 - The “Heaton” slip-pulse (Heaton, 1990) used source inversions



Applications & Implications



So, what can we learn from finite-fault source inversions

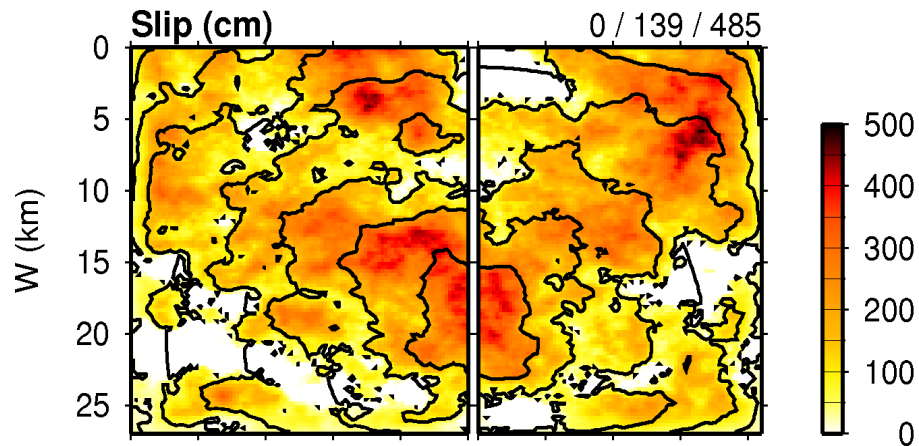
- The rupture process is highly complicated in space and time
 - These result informed early studies of rupture dynamics, which in fact were based on constant stress drop models
 - The “Heaton” slip-pulse (Heaton, 1990) used source inversions
 - Dynamic rupture simulations now use randomized stress on the fault almost routinely
- Slip-heterogeneity characterizations are now common-practice (“state-of-the-art”) in ground-motion simulations, using so-called “rupture-generators (Gutierrez et al, 2002, 2003; Graves & Pitarka, 2004, 2010; and others)



Applications & Implications



So, what can we learn from finite-fault source inversions



Scenario-rupture for a M 7.2 on Puente Hills Fault, L.A. area (by Rob Graves)

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Introduction & Theory



Next ?

(3) New developments, new challenges, new opportunities

- Imaging versus inversion; combination of both?
- Alternative methods
- Uncertainty quantification

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