Introduction to and State-of-the-Art in Earthquake Source Inversion

– Part 1 –

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Roadmap

Earthquake Source Inversion

(1) Introduction & Theory

(2) Applications & Implications

(3) Challenges, Developments, Opportunities

- **How to reach me:**
  - martin.mai@kaust.edu.sa
- **Material**
  - Slides will be made available
  - General theory based on Aki & Richards (2002), or similar text books (Udias et al., 2014; Stein and Wysession (2002) ….)
  - Examples and applications drawn from numerous research papers

Roadmap

Earthquake Source Inversion

(1) Introduction & Theory
- A brief overview
- Fundamentals
- From point-source to extended-fault modeling

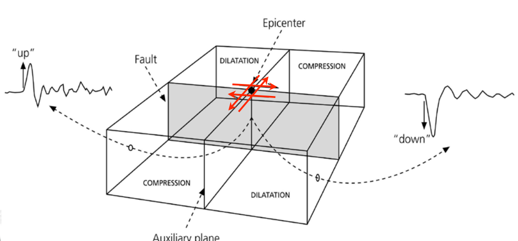
(2) Applications & Implications
- Early developments & case studies
- What can we extract from them?
- What to learn from finite-fault source models?

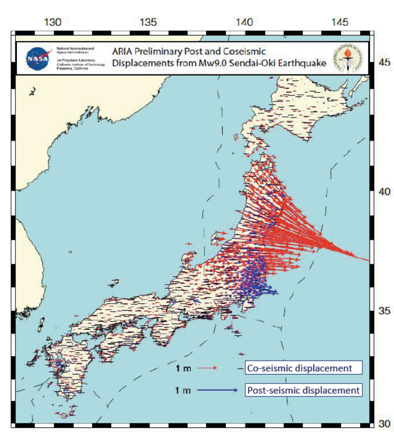
(3) Challenges, Developments, Opportunities
- Imaging versus inversion, or combination of both?
- Alternative methods
- Uncertainty quantification

Introduction & Theory

A brief overview

- Earthquakes occur – mostly – as shear failure on existing faults
- As the earthquake happens, it radiates seismic waves that we record and interpret
- If the earthquake is large enough, it may generate a measurable permanent offset the Earth surface that can be recorded with satellite-based data (GPS, InSAR)





Introduction & Theory

The first earthquake parameters we want to know

- Where and when did the earthquake occur?
  - Earthquake location in space and time
    - Numerous algorithms available to solve this weakly non-linear inverse problem (linearized; non-linear; MC ...)
- How large was the earthquake?
  - Magnitude estimation
    - Wave-amplitude based rapid estimation
    - Refined estimation requires detailed data analysis
- What were the consequences of the earthquake?
  - Damage / impact estimation
    - Macroseismic intensity
    - Engineering assessment

seconds

minutes

days/weeks

Introduction & Theory

Assume size and location are (approximately) known

- What happened exactly?
  - Determine which fault(s) ruptured
    - Fault-plane solution
    - Moment-tensor determination
  - Determine the dimensions of the rupture!
    - Aftershock data
    - GPS / InSAR data
    - Waveform analysis
  - Determine the space-time evolution of the rupture!
    - <u>Kinematic</u> finite-fault modelling/inversion
    - <u>Dynamic</u> rupture process

seconds

days

months

Introduction & Theory

Kinematic versus dynamic earthquake source models

Kinematic Earthquake Rupture Models

Characterized by time-dependent displacement field (slip vectors) on the rupture plane *without* considering the forces/stresses that cause the motions. The rupture process is entirely specified by the spatio-temporal distribution of the slip direction and the slip-rate function, and how rupture propagates over the fault plane.

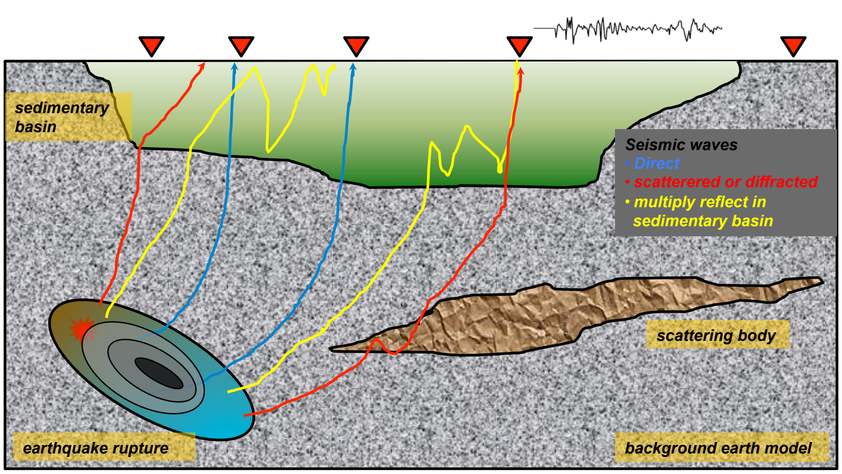
Dynamic Earthquake Rupture Models

Build a physical understanding of the earthquake rupture based on the material properties in the source volume, and the initial and boundary conditions for the forces/stresses acting on the fault plane. The slip-rate vector is obtained by solving the elasto-dynamic equations of motion under the assumption of some constitutive law.

Introduction & Theory

Seismic source & wave propagation = linear filter

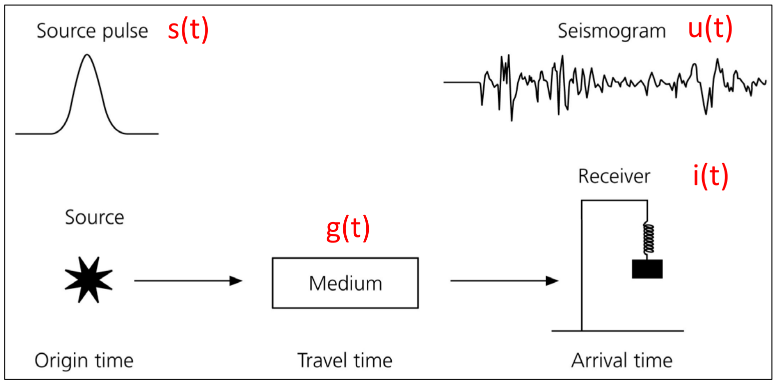
- Conceptual diagram of waves emitted from an elliptical source



Introduction & Theory

Seismic source & wave propagation = linear filter

- Conceptual diagram of seismic wave generation & recording



adapted after Stein & Wysession, 2002

Introduction & Theory

Seismic source & wave = linear filter

- Generation & propagation & recording of seismic wave as a linear filter

$$u_k(t) = s(t) * g_k(t) * i_k(t)$$

- In this notation:
  - u(t): observable ground displacement (or velocity) (~**known**)
  - s(t) : source term (**unknown**)
  - g(t): response of the medium (Green's function) (~**unknown**)
  - i(t) : instrument response (**known**)

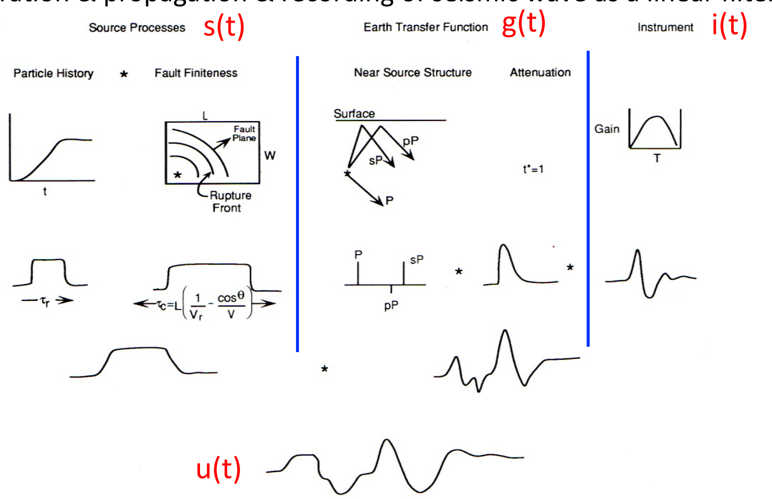
- In the frequency domain

$$U_k(\omega) = S(\omega) \cdot G_k(\omega) \cdot I_k(\omega)$$

Introduction & Theory

Seismic source & wave = linear filter

- Generation & propagation & recording of seismic wave as a linear filter



adapted after Lay & Wallace, 1995

Introduction & Theory

For earthquake source modeling / inversion

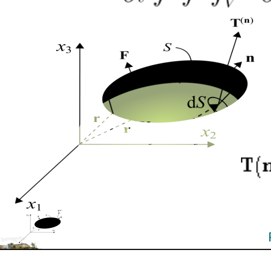
- Instrument response is (generally) known
  - Could be problematic when older earthquakes are studied
- How to get the Earth' transfer function (Green's function)?
  - Observational
    - "empirical" Green's function, from small nearby events
  - Theoretical / Numerical
    - Calculation for assumed/known Earth model
- How do we want to represent the source term **s(t)**?
  - Point-source
  - Extended-fault source

Introduction & Theory

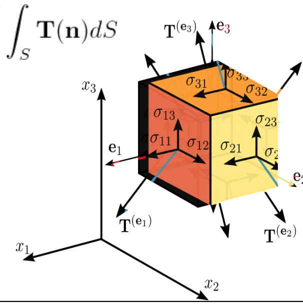
From equations of motion to Volterra's Theorem

- The particle displacements **u** = **u**(x̲,t) (in the Earth or at its surface) are derived as function of space and time in the volume *V* (with surface *S*) due to forces applied within and at the surface of that volume
- Assume small deformations (infinitesimal strain); use tractions ***T*** and stresses ***τ*** (or ***σ***) to analyze forces acting between adjacent particles in the volume.
- The momentum equation (*"F = ma")* places constraints on accelerations, body forces and tractions acting throughout the volume *V* with surface *S*.)

$$\frac{\partial}{\partial t}\int\int\int_V \rho\frac{\partial \mathbf{u}}{\partial t}dV = \int\int\int_V \mathbf{f}dV + \int\int_S \mathbf{T}(\mathbf{n})dS$$



$$\mathbf{T}(\mathbf{n}) = lim_{dS\to 0}\frac{\mathbf{F}}{dS}$$



Introduction & Theory

From equations of motion to Volterra's Theorem

- Given the stress tensor, τ_{kl} , the tractions on the infinitesimal small tetrahedron are related to the corresponding normal vectors

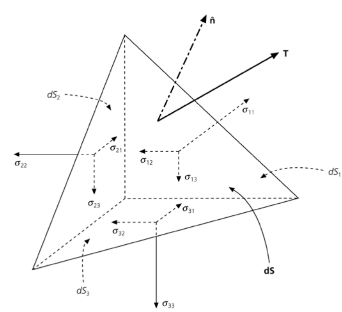
$$\tau_{kl} = \sigma_{kl} = T_l\left(\hat{x}_k\right)$$

- τ_{kl} is the lth component of the traction acting across the plane normal to the kth axis due to material with larger x_k acting upon material with smaller x_k ; the stress-tensor is symmetric

$$T_i = \tau_{ji} n_j$$

- Equations of motion, simplified

$$\rho \ddot{u}_i = f_i + \tau_{ij,j}$$



Introduction & Theory

From equations of motion to Volterra's Theorem

- Making a long story short ….
  - Include strain tensor *(internal deformation; no rigid body motion or rotation)*
    $$\varepsilon_{kl} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$
  - Write stress-strain relation
    $$\tau_{ij} = c_{ijkl}\varepsilon_{kl}$$
  - For isotropic, linear elastic media
    $$c_{ijkl} = \lambda\delta_{ij}\delta_{kl} + \mu\left(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\right)$$
    $$\tau_{ij} = \lambda\varepsilon_{kk}\delta_{ij} + 2\mu\varepsilon_{ij}$$

Introduction & Theory

From equations of motion to Volterra's Theorem

- Making a long story short ….
  - To parameterize seismic source we need additional ingredients, i.e. mathematical relations or theorems that connect the displacement ***u*(*x*,*t*)** to initial conditions and forces
  - Uniqueness Theorem: *The displacement field **u** = **u**(**x**,t) in the volume V with surface S is **uniquely** defined after time t_0 by initial values of displacement and particle velocity in V, and by values of the body forces **f**, the tractions **T** over any part S_1 of S for times $t \geq t_0$, and the displacements over the remainder S_2 of S ($S_1 + S_2 = S$)*
  - Betti's theorem: *Assume displacement field **u** = **u**(**x**,t) due to body forces **f** and boundary conditions on S and initial conditions at time t = 0. Assume further field **v** = **v**(**x**,t) due to body forces **g**. Using the notation **T**(**u**,**n**) and **T**(**v**,**n**)* for the tractions due to ***u*** and ***v***, respectively, the following reciprocal relation is given

$$\int\int\int_V (\mathbf{f} - \rho\ddot{\mathbf{u}}) \cdot \mathbf{v}\, dV + \int\int_S \mathbf{T}(\mathbf{u},\mathbf{n}) \cdot \mathbf{v}\, dS$$
$$= \int\int\int_V (\mathbf{g} - \rho\ddot{\mathbf{v}}) \cdot \mathbf{u}\, dV + \int\int_S \mathbf{T}(\mathbf{v},\mathbf{n}) \cdot \mathbf{u}$$

Introduction & Theory

From equations of motion to Volterra's Theorem

- Making a long story short ….
  - Betti's theorem does not involve any initial condition for ***u and v***, and remains true if all related quantities (particle velocities, tractions, body forces) are evaluated at two different times
  - Betti's theorem relates motions in the source region to some (far-)distant (surface) displacements, recorded as seismic waves, independent of when the corresponding forces have been applied
  - "Trick" : replace the general term ***v(x,***t) in Betti's theorem with a "simple" system response, i.e. the Green's function due to a unidirectional unit pulse precisely located in space and time, and acting in the *n*-direction

$$G_{in}\left(\underline{x},t;\underline{\xi},\tau\right) = A\delta\left(\underline{x}-\underline{\xi}\right)\delta\left(t-\tau\right)\delta_{in}$$

  - B.C's can be chosen such that $G\left(\underline{x},t;\underline{\xi},\tau\right)$ has spatial & temporal reciprocity

$$G_{in}\left(\underline{x},t;\underline{\xi},\tau\right) = G_{ni}\left(\underline{\xi},t;\underline{x},\tau\right) \qquad G_{in}\left(\underline{x},t-\tau;\underline{\xi},0\right)$$

Introduction & Theory

From equations of motion to Volterra's Theorem

- Making a long story short ….
  - Tractions, Green's function, Betti's theorem, Uniqueness

volume forces

$$u_n(\mathbf{x},t) = \int_{-\infty}^{\infty} d\tau \int\int\int_V f_i(\xi,\tau)\cdot G_{in}(\xi,t-\tau;\mathbf{x},0)\,dV \; +$$

stress glut

$$\int_{-\infty}^{\infty} d\tau \int\int_S [G_{in}(\xi,t-\tau;\mathbf{x},0)\cdot T_i(\mathbf{u}(,\xi,\tau),\mathbf{n})]\,dS \; -$$

dislocation

$$\int_{-\infty}^{\infty} d\tau \int\int_S [u_i(\xi,\tau)\cdot c_{ijkl}\cdot n_j\cdot G_{kn,l}(\xi,t-\tau;\mathbf{x},0)]\,dS$$

Introduction & Theory

Representation Theorem (Volterra's formula)

- Neglecting body forces acting on ***u***, we can write the representation theorem for internal dislocation sources

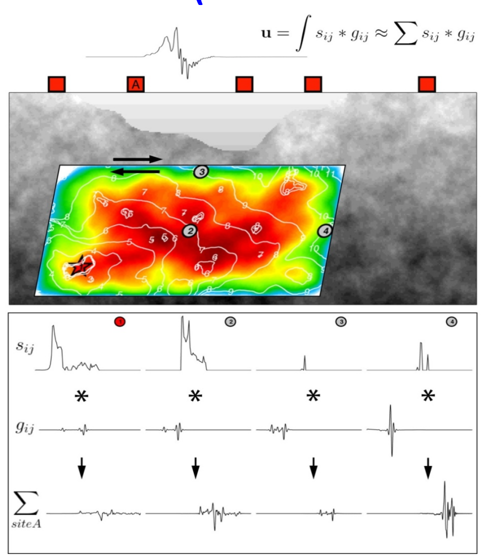
$$u_n(\mathbf{x},t) = \int_{-\infty}^{\infty} d\tau \int\int_{\Sigma} [u_i(\xi,\tau)] \cdot c_{ijpq} \cdot \nu_j \cdot \frac{\partial}{\partial \xi_q} G_{np}(\mathbf{x}, t-\tau; \xi, 0) d\Sigma$$

$u_n(\mathbf{x},t)$	n[th] component of the observed displacement field at ***x***, *t*
$[u_i(\xi,\tau)]$	i[th] component of the displacement at the source as a function of position ***x*** on the fault plane and time *t*
$G_{np}(\mathbf{x}, t-\tau; \xi, 0)$	system response in *n*-direction due to unit impulse in direction *p* on the fault plane at ***x***, *t*
$\frac{\partial}{\partial \xi_q} G_{np}(\mathbf{x}, t-\tau; \xi, 0)$	generalized force couple in x_q-direction and force in *p*-direction
$c_{ijpq} \cdot \nu_j$	elasticity tensor; normal to the rupture plane

Introduction & Theory

Representation Theorem (Volterra's formula)

- Conceptually



Introduction & Theory

Representation Theorem (Volterra's formula)

- In practice
  - The earthquake rupture process, here denoted as s_{ij}, can be very complex, occurring in a geologically complicated environment. The representation theorem allows us in practice to compute the ground motions
  - The rupture process could be occurring on an extended plane, or multiple planes, but the representation theorem still holds
  - To compute *u(<u>x</u>,t)* we simply need to sum up all contributions from all points on the fault plane(s), i.e. convolve the local source contribution with the local Green's function (valid for a given site of interest) to obtain a "local" seismogram, and then sum all these local seismograms.
  - **The representation theorem is simply a giant book-keeping device for computing ground motions for arbitrarily complex source models embedded in arbitrarily complex geologic structures.**

Introduction & Theory

Representation Theorem (Volterra's formula)

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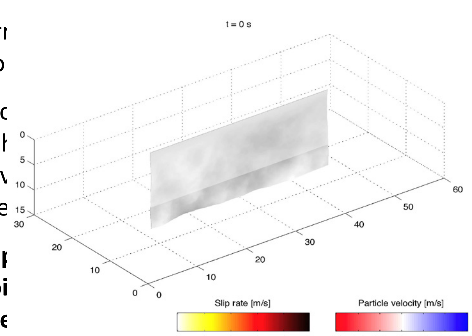

Introduction & Theory

Representation Theorem (Volterra's formula)

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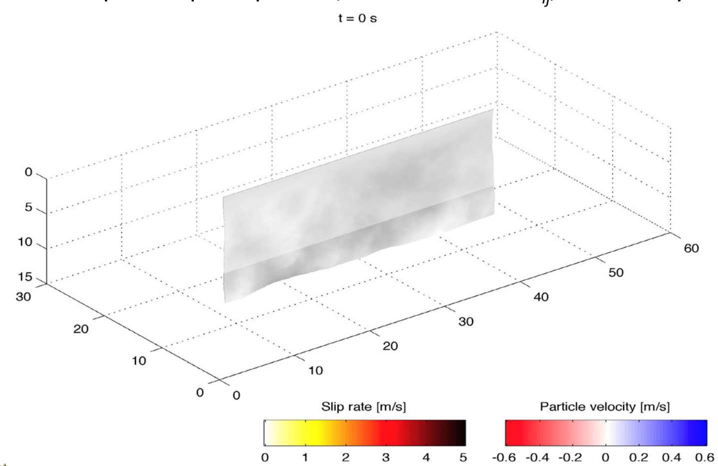

Introduction & Theory

Representation Theorem (Volterra's formula)

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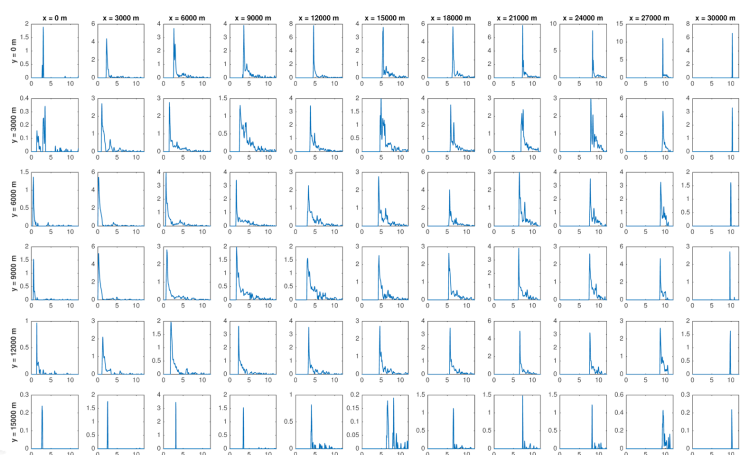


Introduction & Theory

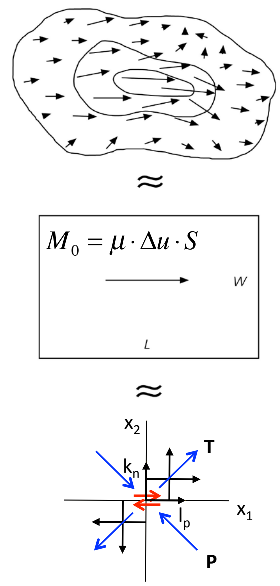
Simplifying the representation theorem

- From complex to point-source
  - Space-time heterogeneity of $[u_i(\xi,\tau)]$; equivalent time-dependent point-forces (moment density tensor m_{pq}) $$u_n(\underline{x},t) = \iint_\Sigma m_{pq} * \frac{\partial}{\partial \xi_q} G_{np} d\Sigma$$
  - Earthquake rupture process using space-time averaged source parameters (l: unit vector in direction of slip; k: normal to fracture plane)

$$u_n(\underline{x},t) = (l_n k_p + l_p k_n) \int_{-\infty}^{\infty} M_0(\tau) \cdot G_{np,q}(t-\tau) d\tau$$

  - Point-source approximation using the point-source double-couple approximation; M_{pq}: moment tensor

$$u_n(\underline{x},t) = M_{pq} * G_{np,q}$$



adapted after Stein & Wysession, 2002

Introduction & Theory

Point-source shear-fracture (dislocation)

- After much algebra, the displacement $u_n(x,t)$ for a shear-fault with time-dependent moment and moment-rate function $M_{pq}(t)$ and $dt\, M_{pq}(t)$

$$\begin{aligned} M_{pq} * G_{np,q} = &\left(\frac{15\gamma_n\gamma_p\gamma_q - 3\gamma_n\delta_{pq} - 3\gamma_p\delta_{nq} - 3\gamma_q\delta_{np}}{4\pi\rho}\right) \frac{1}{r^4} \int_{r/\alpha}^{r/\beta} \tau M_{pq}(t-\tau)\, d\tau \\ &+ \left(\frac{6\gamma_n\gamma_p\gamma_q - \gamma_n\delta_{pq} - \gamma_p\delta_{nq} - \gamma_q\delta_{np}}{4\pi\rho\alpha^2}\right) \frac{1}{r^2} M_{pq}\left(t - \frac{r}{\alpha}\right) \\ &- \left(\frac{6\gamma_n\gamma_p\gamma_q - \gamma_n\delta_{pq} - \gamma_p\delta_{nq} - 2\gamma_q\delta_{np}}{4\pi\rho\beta^2}\right) \frac{1}{r^2} M_{pq}\left(t - \frac{r}{\beta}\right) \\ &+ \frac{\gamma_n\gamma_p\gamma_q}{4\pi\rho\alpha^3} \frac{1}{r} \dot{M}_{pq}\left(t - \frac{r}{\alpha}\right) - \left(\frac{\gamma_n\gamma_p - \delta_{np}}{4\pi\rho\beta^3}\right) \gamma_q \frac{1}{r} \dot{M}_{pq}\left(t - \frac{r}{\beta}\right) \end{aligned}$$

Introduction & Theory

Point-source shear-fracture (dislocation)

- After much algebra, the displacement $u_n(x,t)$ for a shear-fault with time-dependent moment and moment-rate function $M_{pq}(t)$ and $dt\, M_{pq}(t)$

The displacement field due to a general 2nd-order moment tensor contains:

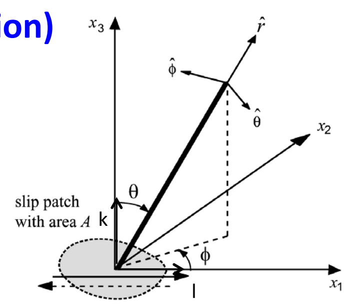
- near-field terms, proportional to $\frac{1}{r^4}\int_{r/\alpha}^{r/\beta} \tau M_{pq}(t-\tau)\, d\tau$
- Intermediate-field terms, $\frac{1}{r^2} M_{pq}\left(t - \frac{r}{\alpha}\right)$
- far-field term, proportional to $\frac{1}{r}\dot{M}_{pq}\left(t - \frac{r}{\beta}\right)$

Introduction & Theory

Point-source shear-fracture (dislocation)

- In spherical coordinates

$$\begin{aligned}
\boldsymbol{u}(\boldsymbol{x}, t) = & \frac{1}{4\pi\rho} \boldsymbol{A}^N \frac{1}{r^4} \int_{r/v_P}^{r/v_S} \tau M_0(t-\tau)\, d\tau \\
& + \frac{1}{4\pi\rho v_P^2} \boldsymbol{A}^{IP} \frac{1}{r^2} M_0(t - r/v_P) \\
& + \frac{1}{4\pi\rho v_S^2} \boldsymbol{A}^{IS} \frac{1}{r^2} M_0(t - r/v_S) \\
& + \frac{1}{4\pi\rho v_P^3} \boldsymbol{A}^{FP} \frac{1}{r} \dot{M}_0(t - r/v_P) \\
& + \frac{1}{4\pi\rho v_S^3} \boldsymbol{A}^{FS} \frac{1}{r} \dot{M}_0(t - r/v_S).
\end{aligned}$$



$$\begin{aligned}
\boldsymbol{A}^N &= 9 \sin 2\theta \cos\phi\, \hat{\boldsymbol{r}} - 6(\cos 2\theta \cos\phi\, \hat{\boldsymbol{\theta}} - \cos\theta \sin\phi\, \hat{\boldsymbol{\phi}}), \\
\boldsymbol{A}^{IP} &= 4 \sin 2\theta \cos\phi\, \hat{\boldsymbol{r}} - 2(\cos 2\theta \cos\phi\, \hat{\boldsymbol{\theta}} - \cos\theta \sin\phi\, \hat{\boldsymbol{\phi}}), \\
\boldsymbol{A}^{IS} &= -3 \sin 2\theta \cos\phi\, \hat{\boldsymbol{r}} + 3(\cos 2\theta \cos\phi\, \hat{\boldsymbol{\theta}} - \cos\theta \sin\phi\, \hat{\boldsymbol{\phi}}), \\
\boldsymbol{A}^{FP} &= \sin 2\theta \cos\phi\, \hat{\boldsymbol{r}}, \\
\boldsymbol{A}^{FS} &= \cos 2\theta \cos\phi\, \hat{\boldsymbol{\theta}} - \cos\theta \sin\phi\, \hat{\boldsymbol{\phi}},
\end{aligned}$$

SV **SH**

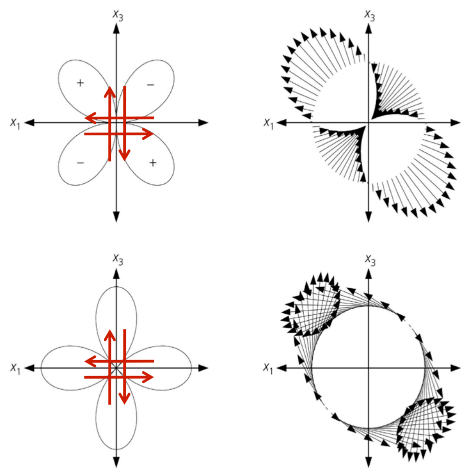
Introduction & Theory

Point-source shear-fracture (dislocation)

- The resulting far-field radiation patterns

P-waves

S-waves



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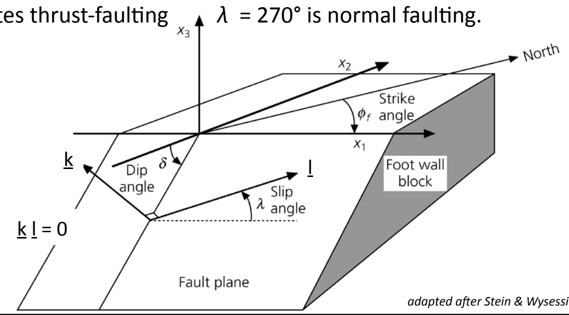
Introduction & Theory

The fault geometry

- Defining geographical parameters of the fault plane
  - Fault dip δ, measured from horizontal downward into fault plane
  - Strike Φ_f measured from north in a RHS coordinate system, hence $\delta \leq 90°$
  - Rake-angle λ measures the direction of slip in the fault plane as the movement of the hanging wall with respect to the foot wall.

    λ = 0° indicates left-lateral slip, λ = 180° indicates right-lateral slip

    λ = 90° denotes thrust-faulting λ = 270° is normal faulting.



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Introduction & Theory

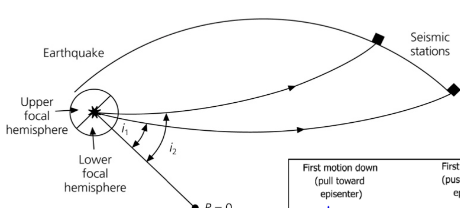
Assume size and location are (approximately) known

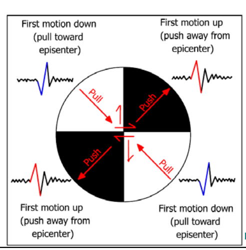
- What happened exactly?
  - Determine which fault(s) ruptured!
    - Fault-plane solution
    - Moment-tensor determination

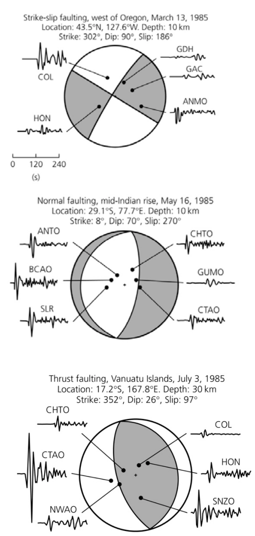
Introduction & Theory

Assume size and location are (approximately) known

- What happened exactly?
  - Determine which fault(s) ruptured!
    - Fault-plane solution from first motion polarities
    - Lower-hemisphere projection







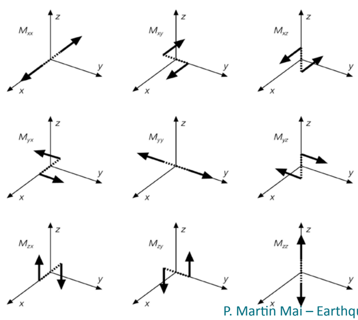
adapted after Stein & Wysession, 2002

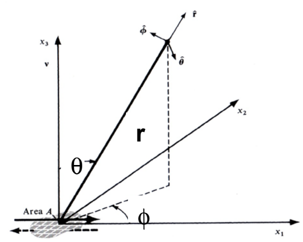
Introduction & Theory

Assume size and location are (approximately) known

- What happened exactly?
  - Determine which fault(s) ruptured!
    - Moment-tensor determination $u_n(\underline{x},t) = M_{pq} * G_{np,q}$

$$M_{pq} = \mathbf{M} = \begin{pmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{pmatrix} = \begin{pmatrix} M_{xx} & M_{xy} & M_{xz} \\ M_{yx} & M_{yy} & M_{yz} \\ M_{zx} & M_{zy} & M_{zz} \end{pmatrix} = \begin{pmatrix} M_{\theta\theta} & M_{\theta\phi} & M_{\theta r} \\ M_{\theta\phi} & M_{\phi\phi} & M_{-\phi r} \\ M_{\theta r} & M_{-\phi r} & M_{rr} \end{pmatrix}$$





Introduction & Theory

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  - Determine which fault(s) ruptured!
    - Moment-tensor determination $u_n(\underline{x},t) = M_{pq} * G_{np,q}$

$$M_{pq} = \mathbf{M} = \begin{pmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{pmatrix} = \begin{pmatrix} M_{xx} & M_{xy} & M_{xz} \\ M_{yx} & M_{yy} & M_{yz} \\ M_{zx} & M_{zy} & M_{zz} \end{pmatrix} = \begin{pmatrix} M_{\theta\theta} & M_{\theta\phi} & M_{\theta r} \\ M_{\theta\phi} & M_{\phi\phi} & M_{-\phi r} \\ M_{\theta r} & M_{-\phi r} & M_{rr} \end{pmatrix}$$

- Isotropic explosion/ implosion
- Vertical strike-slip
- Vertical dip-slip
- 45°-dipping thrust
- Two versions of non-double couple mechanisms

Moment tensor	Beachball	Moment tensor	Beachball
$\frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$		$-\frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	
$-\frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$		$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	
$\frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}$		$\frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}$	
$\frac{1}{\sqrt{2}}\begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$		$\frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	
$\frac{1}{\sqrt{6}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$		$\frac{1}{\sqrt{6}}\begin{pmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	
$\frac{1}{\sqrt{6}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$		$-\frac{1}{\sqrt{6}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$	

adapted after Stein & Wysession, 2002

Introduction & Theory

Assume size and location are (approximately) known

- What happened exactly?
  - Determine which fault(s) ruptured!
    - **Moment-tensor inversion**
      - 6 independent components; 5 if only deviatoric ($M_{11} + M_{22} + M_{33} = 0$)
      - Express as $M_{pq}(t) = M_0 \cdot f(t) \cdot m_{pq}$ and assume f(t)

$$u_n(\underline{x},t) = \sum_{i=1}^{6} g_i^n(r,t) * m_i(t) \longrightarrow \underline{u} = \boldsymbol{G}\underline{m}$$

      - Solution for ***m*** depends on the structure of ***G*** (the inverse problem / data)

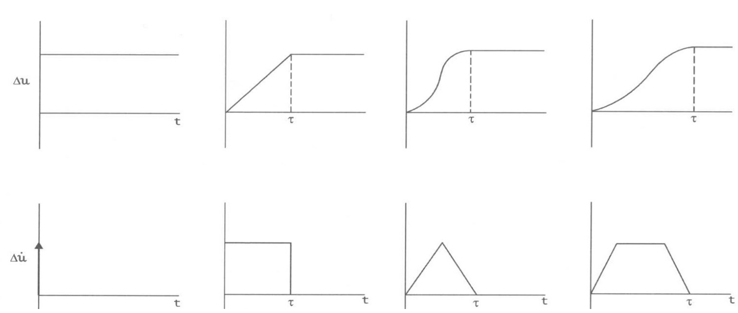
$$\underline{m} = \left(\boldsymbol{G}^T\boldsymbol{G}\right)^{-1}\boldsymbol{G}^T\underline{u} \qquad \underline{m} = \left(\boldsymbol{G}^T\boldsymbol{G} + \lambda\boldsymbol{R}^T\boldsymbol{R}\right)^{-1}\boldsymbol{G}^T\underline{u}$$

$$G = U\Lambda V^T \qquad \underline{m} = \left(V\Lambda^{-1}U^T\right)\underline{u}$$

Introduction & Theory

Assume size and location are (approximately) known

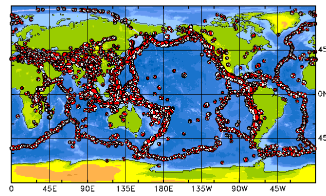
- What happened exactly?
  - Determine which fault(s) ruptured!
    - Moment-tensor determination $u_n(\underline{x},t) = M_{pq} * G_{np,q}$
    - Temporal dependence, f(t) $M_0(t-\tau) = M_{pq}(t-\tau)$



Introduction & Theory

Some simple cases studies

- CMT (Centroid Moment Tensor) Solution
  - Adjust for the fact that the seismic moment is not released at the rupture nucleation point (hypocenter)

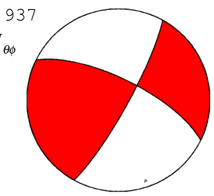


(http://www.globalcmt.org/)

```
110302J CENTRAL ALASKA
Date: 2002/11/ 3 Centroid Time: 22:13:28.0 GMT
Lat= 63.23 Lon=-144.89
Depth= 15.0 Half duration=23.5
Centroid time minus hypocenter time: 47.0
Moment Tensor: Expo=27 0.513 -6.038 5.525 0.183 2.615 -3.937
```

M_{rr} $M_{\theta\theta}$ $M_{\phi\phi}$ $M_{\theta r}$ $M_{\phi r}$ $M_{\theta\phi}$

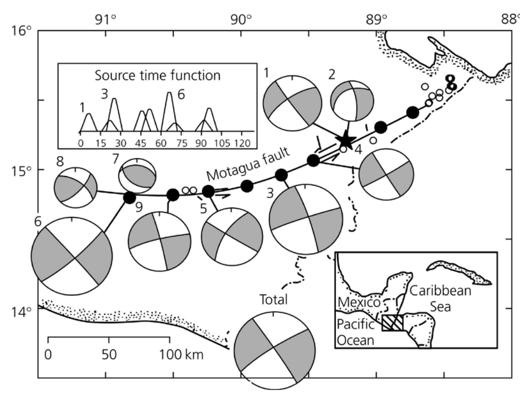
```
Mw = 7.8 mb = 7.0 Ms = 8.5 Scalar Moment = 7.48e+27
Fault plane: strike=296 dip=71 slip=171
Fault plane: strike=29 dip=82 slip=19
```

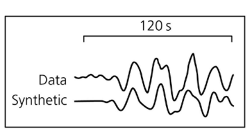


Introduction & Theory

Some simple cases studies

- Wave-form modeling
  - Add multiple moment-tensor contributions to fit waveforms for a complex earthquake





adapted after Stein & Wysession, 2002

Introduction & Theory

Standard finite-fault earthquake source inversion

- The earthquake-source term is space-time dependent, i.e. each "point" on the fault is activated to slip when it is reached by the expanding rupture front
- Rupture speed and the local slip-rate function depend on initial & boundary conditions (stress, friction etc.); both may vary strongly over the rupture plane
- The earthquake rupture process itself is a highly nonlinear process
- However, standard/classical earthquake source inversions approaches "linearize" the problem
  - What exactly do we mean by "linearize" here?
  - Which assumptions do we have to make?

Introduction & Theory

Standard finite-fault earthquake source inversion

- The representation theorem is linearized, using a number of assumptions (e.g. Olson & Apsel, 1982; Hartzell & Heaton, 1983):

$$u_n(\mathbf{x},t) = \int_{-\infty}^{\infty} d\tau \int\int_{\Sigma} [u_i(\xi,\tau)] \cdot \nu_j \cdot c_{ijpq} \cdot \frac{\partial}{\partial \xi_q} G_{np}(\mathbf{x}, t-\tau; \xi, 0) d\Sigma$$

Seismic data observations — ***what we have***

Space-time dependent rupture process — ***what we want***

Earth structure and related Green's functions — ***what we need to compute***

$$d = g(m) \qquad \underline{d} = \underline{\underline{G}}\ \underline{m}$$

- Assuming we "know" **Earth structure & fault geometry**, we can examine seismic observations to learn about **earthquake source properties** given **seismological and/or geodetic data**.
- This is a kinematic representation; we cannot infer rupture physics here!

Introduction & Theory

Standard finite-fault earthquake source inversion

- The representation theorem is "linearized" using a number of assumptions (e.g. Olson & Apsel, 1982; Hartzell & Heaton, 1983):
  - Source geometry is known, and approximated by one or more rectangular planes that are again subdivided into a set of "subfaults" (nodes)
  - An elementary identical slip function acts on each point on the fault
  - The slip-history at each point can be represented by a summation of elementary basis functions, lagged in time (multi-time window)
  - The rupture velocity and is constant

Introduction & Theory

Standard finite-fault earthquake source inversion

- The representation theorem is linearized, using a number of assumptions (e.g. Olson & Apsel, 1982; Hartzell & Heaton, 1983):

$$u_n(\mathbf{x},t) = \int_{-\infty}^{\infty} d\tau \int\int_{\Sigma} [u_i(\xi,\tau)] \cdot \nu_j \cdot c_{ijpq} \cdot \frac{\partial}{\partial \xi_q} G_{np}(\mathbf{x}, t-\tau; \xi, 0) d\Sigma$$

- Discretized version:

$$u_n(\mathbf{x},t) = \sum_{itm=1}^{\mathrm{ntm}} \sum_{is=1}^{\mathrm{ns}} \sum_{if=1}^{\mathrm{nf}} m(if, is, itm) \times \int \left[u_{\mathrm{unit}_{is}} (\tau - \Delta t_{\mathrm{trig}}) \right]$$
$$\times\, c_{i(is)jkl}(\xi) n_j \mathrm{G}_{kn,l}(\mathbf{x}, t-\tau; \xi(if), 0)\, \mathrm{d}\tau$$

Here, $m(if, is, itm)$ is the amount of slip in the isth direction at the itmth time window on the ifth subfault, nf is the number of subfaults, ns is the number of slip directions, ntm is the number of time windows, R is the hypocentral distance of the ifth subfault, and $[u_{\mathrm{unit}_{is}}(\tau)]$ is the unit slip function.

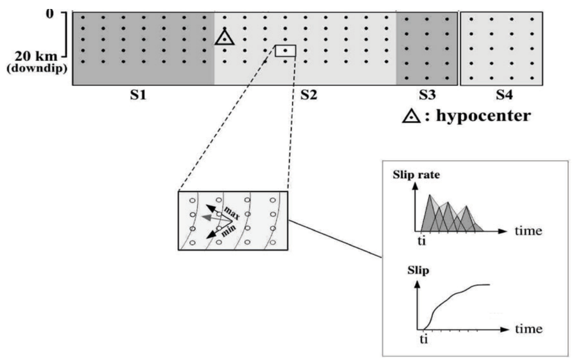
$$\Delta t_{\mathrm{trig}} = \frac{R}{Vr} + \Delta tw \cdot (itm - 1)$$

Introduction & Theory

Standard finite-fault earthquake source inversion

- Conceptual sketch
  - Geometry assumed or known
  - Rupture speed V_r assumed
  - Elementary slip (or slip-rate) function assumed

$$\Delta t_{\text{trig}} = \frac{R}{Vr} + \Delta tw \cdot (itm - 1)$$



From Delouis et al, 2002

Introduction & Theory

Standard finite-fault earthquake source inversion

- The system of equations then looks like

$$\underline{d} = \boldsymbol{G}\underline{m} = \sum_j G_{ij} m_j$$

include smoothing

$$\begin{bmatrix} \underline{d} \\ \underline{0} \end{bmatrix} = \begin{bmatrix} \boldsymbol{G} \\ \lambda \boldsymbol{R} \end{bmatrix} \underline{m}$$

R *: smoothing (regularization) matrix to account for variations in model parameters with distance and/or time (the farther apart subfaults are, the larger a difference is allowed);*
λ *has to be determined by trial-and-error, or some statistical techniques*

$$\begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_1 \\ d_2 \\ \vdots \\ d_1 \\ d_2 \\ \vdots \end{bmatrix} \cong \begin{bmatrix} G_{11} & G_{12} & \cdots & G_{1m} \\ G_{21} & G_{22} & \cdots & G_{2m} \\ \vdots & \vdots & & \\ G_{11} & G_{12} & \cdots & G_{1m} \\ G_{21} & G_{22} & \cdots & G_{2m} \\ \vdots & \vdots & & \\ G_{11} & G_{12} & \cdots & G_{1m} \\ G_{21} & G_{22} & \cdots & G_{2m} \\ \vdots & \vdots & & \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ \vdots \\ s_m \end{bmatrix}$$

(time) data points station 1
(time) data points station 2
(time) data points station N
Subfault 1 synthetics
Subfault 2 synthetics
Subfault m synthetics
s_1 Dislocation in subfault 1
s_2 Dislocation in subfault 2
s_m Dislocation in subfault m

Introduction & Theory

Standard finite-fault earthquake source inversion

- Solution strategies (least-squares) $$\underline{m} = \left(G^T G\right)^{-1} G^T \underline{d}$$
  - Include data covariance, $\sigma^2 C_d$ $$\underline{m} = \left(G^T C_d^{-1} G\right)^{-1} G^T \underline{d}$$
  - Covariance of model parameters $$C_m = \sigma^2 \left(G^T C_d^{-1} G\right)^{-1}$$
  - Difficult to use; need constraints (non-negativity; smoothness)

$$\underline{m} = \left(G^T C_d^{-1} G + \lambda^2 R^T R\right)^{-1} G^T \underline{d} \qquad C_m = \sigma^2 \left(G^T C_d^{-1} G + \lambda^2 R^T R\right)^{-1}$$

σ^2 and λ^2 may be chosen by Akaike-Bayesian Information Criterion (ABIC)

ABIC (λ) = N_d ln Res($\underline{m}$) – N_m ln λ^2 + ln | C_m | + b

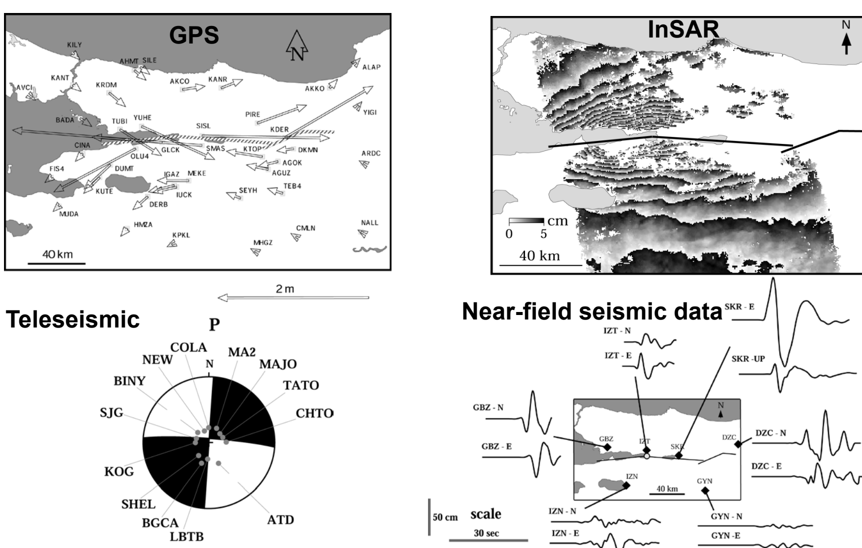
$$\text{ABIC} = -2(\text{log marginal likelihood}) + 2(\text{number of hyperparameters})$$

- Apply singular-value decomposition to ***G***

$$G = U \Lambda V^T \qquad \underline{m} = V_s \Lambda_s^{-1} U_s^T \underline{d}$$

Introduction & Theory

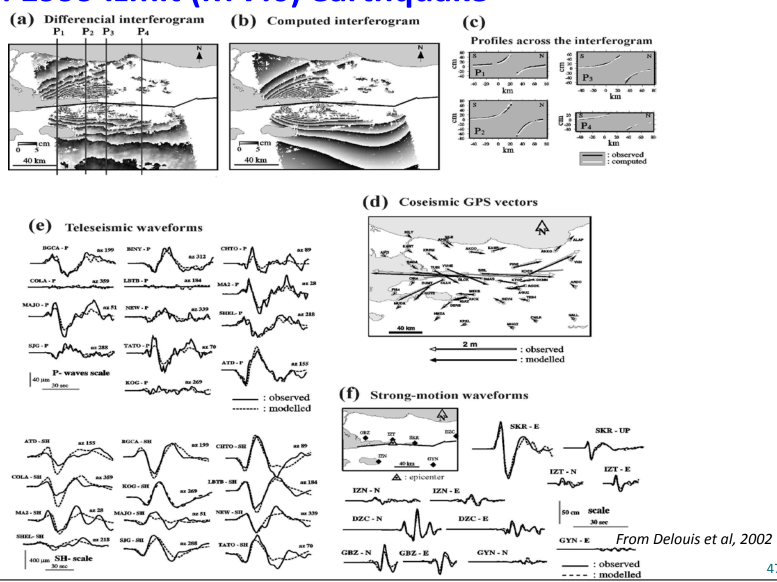
Example: 1999 Izmit (M 7.6) earthquake



From Delouis et al, 2002

Introduction & Theory

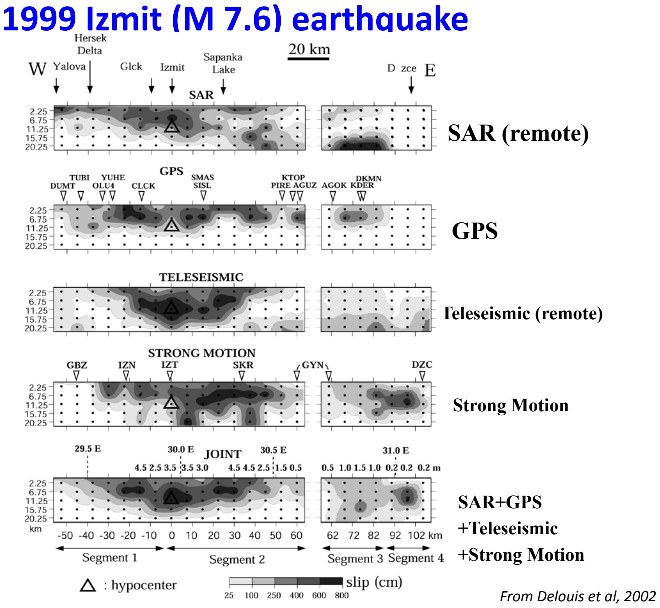
Example: 1999 Izmit (M 7.6) earthquake



From Delouis et al, 2002

Introduction & Theory

Example: 1999 Izmit (M 7.6) earthquake

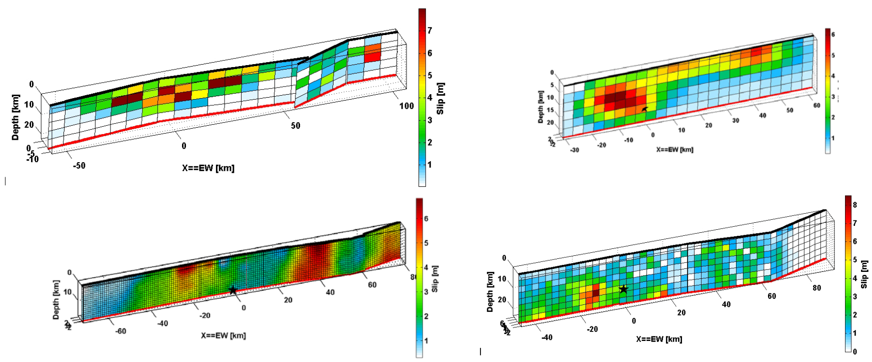


From Delouis et al, 2002

Introduction & Theory

Example: 1999 Izmit (M 7.6) earthquake

- Four solutions as available on the SRCMOD database (equake-rc.info/SRCMOD)



From Delouis et al, 2002

Introduction & Theory

Using permanent (static) surface displacements

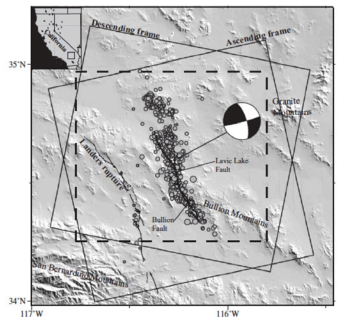
- Geodetic data (GPS/InSAR) for source inversions
  - Use analytical Okada (1989; 1992) solutions (elastic half-space) to compute the response of the medium
  - Non-linear inversion for fault geometry (with constant fault): strike, dip, rake, depth, width, length, position (x,y)
  - Once the geometry is fixed, the inversion is linear to find distributed slip on the fault

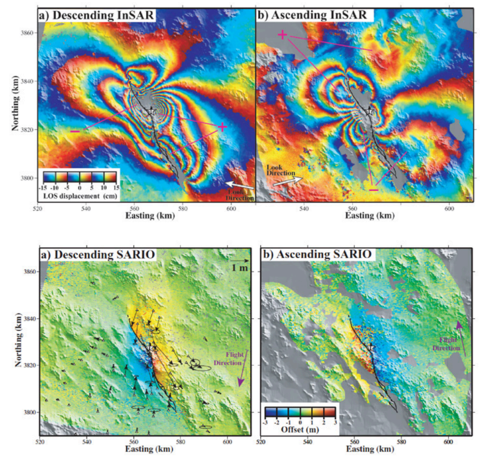
- If possible, different data sets are combined
  - Geodetic data to constrain geometry and shallow slip
  - Teleseismic to constrain seismic moment and overall characteristics
  - Strong motion data to constrain temporal rupture evolution
  - How to choose the weights for each data set?

Introduction & Theory

Using permanent (static) surface displacements

- Geodetic data (GPS/InSAR) for source inversions: **Hector Mine, 1999**





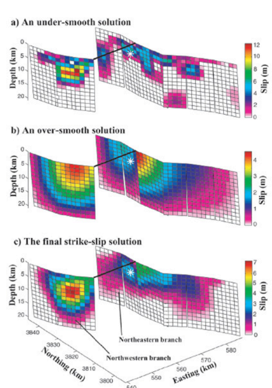
Fault Geometry & Data

Introduction & Theory

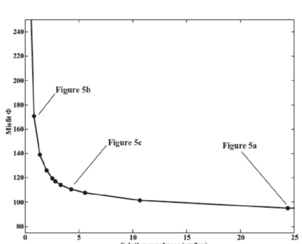
Using permanent (static) surface displacements

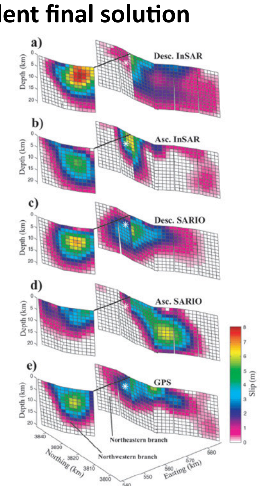
Geodetic data (GPS/InSAR) for source inversions: Hector Mine, 1999

"optimal" Laplacian smoothing (R) and data-dependent final solution



$$\underline{m} = \left(G^T G + \lambda^2 R^T R\right)^{-1} G^T \underline{d}$$





after Jonsson et al, 2002

Introduction & Theory

Where are the problems? (I)

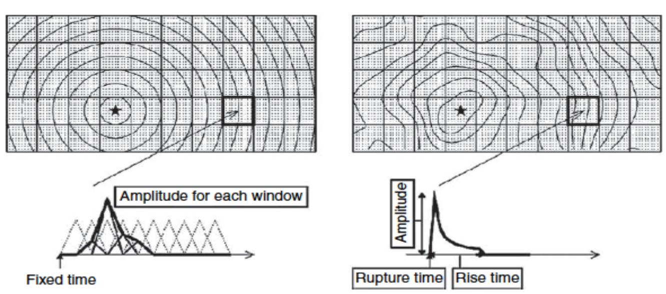
- Green's functions
  - Do we know Earth structure well enough?
  - Do we compute the Green's function correctly?
  - Should we include a formal error term for the Green's function?

- Fault geometry
  - Unknown a priori, often constrained from aftershocks and – if available – surface-faulting information
  - Slight variations in the geometry will affect the Green's function and hence the entire inversion
  - Different parameterizations of the fault plane

Introduction & Theory

Where are the problems? (II)

- Parameterization
  - Slip function on each point? How to be more physical?
  - Relaxing the rupture-speed assumption



Introduction & Theory

Where are the problems? (III)

- Solution to the inverse problem
  - Various approaches to solve the linearized problem
    - Optimal choice of smoothing / regularization?
  - Many methods to do a non-linear inversion
    - Building in physical constraints into the search space
  - Formal uncertainty quantification of the resulting model parameters
  - Parameterization of the inversion

- Data
  - What is the data distribution? Is that sufficient, or biased somehow?
  - Data selection / trimming / processing
  - Data weighting
  - Data uncertainties

Introduction & Theory

Next ?

(2) Applications & Implications
- Early developments & case studies?
- What can we extract from them?
- What to learn from finite-fault source models?