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The Response to Complex Ground Motions of Seismometers with Galperin Sensor Configuration

by Vladimir Graizer

Abstract Most seismological instruments recording ground motion use three sensors oriented north, east, and upward. In this cardinal configuration horizontal and vertical sensors differ in their construction because of the gravitational acceleration affecting the vertical sensor. An alternative sensor arrangement was first introduced by Galperin (1955) for petroleum exploration. In this arrangement three identical sensors are also positioned orthogonally to each other but are tilted at the same angle of 54.7° to the vertical axis (an orthogonal triaxial system of coordinates balanced on its corner). Records obtained using this sensor configuration must be rotated into an Earth referenced cardinal X, Y, Z coordinate system for most analyses. A number of recent seismological instruments (e.g., STS-2 and Trillium seismometers) use Galperin sensor configuration. In most seismological studies it is assumed that the rotational components of earthquake ground motion are small enough to be neglected. However, examples of significant rotational components have been noted (e.g., Bouchon and Aki, 1982; Graizer, 1991; Takeo, 1998; Huang, 2003; Zahradnik and Plesinger, 2005; Cochard *et al.*, 2006; Graizer, 2006a; Schreiber *et al.*, 2006; Spudich and Fletcher, 2008). The response of pendulums when installed in a cardinal configuration to input motions that include rotations has been studied in a number of publications (Golitzin, 1912; Rodgers, 1968; Wong and Trifunac, 1977; Graizer, 1991; Todorovska, 1998; Trifunac and Todorovska, 2001; Graizer, 2005, 2006b; Graizer and Kalkan, 2008). This article considers the response to input motions of pendulums in a Galperin sensor configuration as well as the resulting cardinal orientation system response. Given the benefits of identical designs for all three sensors in a Galperin configuration, this geometry may be useful for strong-motion measurements as well. The disadvantage of this sensor configuration is that if any of the sensors is not working properly or there are misalignments of sensor axes, then all three cardinal components are degraded.

Introduction

Seismic ground motion can be described as a vector in three-dimensional space. Common configurations of sensors in seismological measurements include three orthogonal sensors, two oriented horizontally and the third vertically in a cardinally-oriented Cartesian coordinate system. Most seismological sensors including seismometers and accelerometers use this cardinal configuration of sensors (north, east, and up). In this configuration the design of the vertical sensor must differ from that of the horizontal components requiring a compensating force because gravitational acceleration is continuously applied to the vertical sensor.

Another sensor configuration used for petroleum exploration since the 1960s is the so-called Galperin or symmetric configuration, originally developed as a tool for three-component borehole studies (Galperin, 1955, 1985). In Galperin's configuration the three sensors are also positioned orthogonally with respect to each other, but all three sensors are tilted at the same angle to the vertical axis (an orthogonal triaxial coordinate system balanced on its corner). This configuration ensures that each of the three identical, single-component sensors responds equally to gravity (Fig. 1). Such sensors are mounted at an angle of 35.26° to the horizontal (54.74° to the vertical axis) moving in a vertical plane. The sensor axes are at 120° spacing projected to the horizontal plane, usually at 0°, 120°, and 240° azimuth (Fig. 1). The Galperin configuration is also sometimes referred to in the literature as the "54° geometry" or the "symmetric configuration."

Most sensors used for seismological measurements are of the mass-on-rod pendulum type and use standard (N, E, and Z) configuration (e.g., Streckeisen STS-1 and Guralp CMG-3T). The sensor configuration developed by Galperin

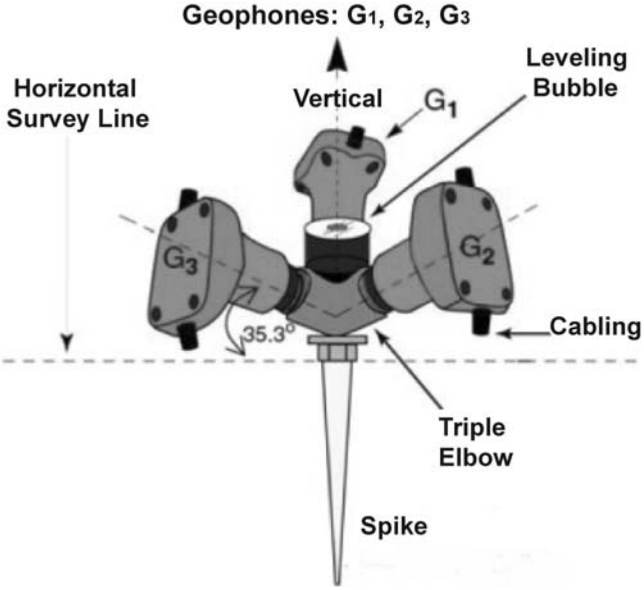

Figure 1. Three-component Galperin geophone configuration (modified from Ralston and Steeples, 2002).

for geophones was later introduced by Melton and Kirkpatrick (1970) for seismological measurements (Wielandt, 2002) under the name of homogeneous or symmetric triaxial arrangement. Apparently, Bradner and Dodds (1964) and Bradner *et al.* (1970) used a similar sensor configuration in their ocean-bottom seismometer. In recent years many broadband seismometers (e.g., Streckeisen STS-2 and Nanometrics Trillium) have adopted the Galperin configuration (Fig. 2). This configuration has the appearance of a three-arm chandelier. These three arms may be oriented upward or downward but always move in a vertical plane. In seismic prospecting, sensors are usually oriented upward as is the STS-2 (Figs. 1 and 2). For purposes of this article, the orientation of X, Y, Z relative to the Galperin U, V, W coordinate system is shown in Figure 3. In a prospective view shown in Figure 3 (bottom panel), V and W axes should overlap and are shown separately for clarity.

The term symmetric is often used to describe this sensor configuration in seismology but is not descriptive enough because it does not specify the angles of sensitivity of the three sensors. For example, the three-component system with sensitivity axes tilted at 45° to the horizontal can also be called symmetric (note that such a system is not Cartesian because the axes are not perpendicular to each other). I will follow the style adopted in seismic prospecting by calling the symmetric triaxial sensor configuration with axes of sensitivity oriented at 35.26° angles to the horizontal and 120° relative to each other in the horizontal plane the Galperin sensor configuration.

The advantage of the Galperin sensor configuration is that there is no need for a special design for a vertical sensor, and all the sensors are identical in design. Records obtained

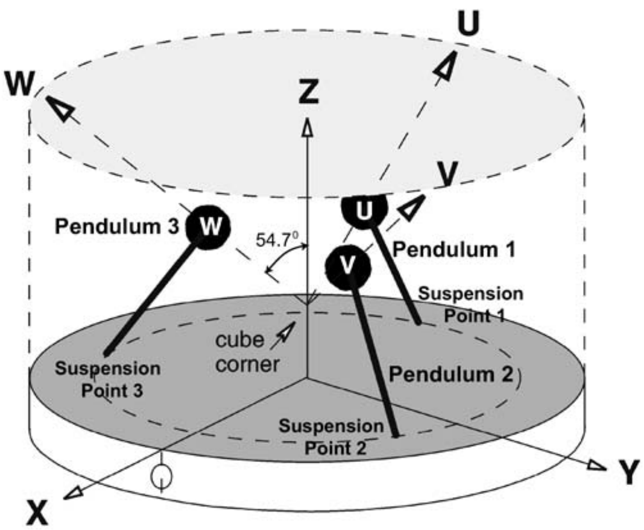

Figure 2. Triaxial geometry of the Streckeisen STS-2 seismometer (modified from Wielandt, 2002).

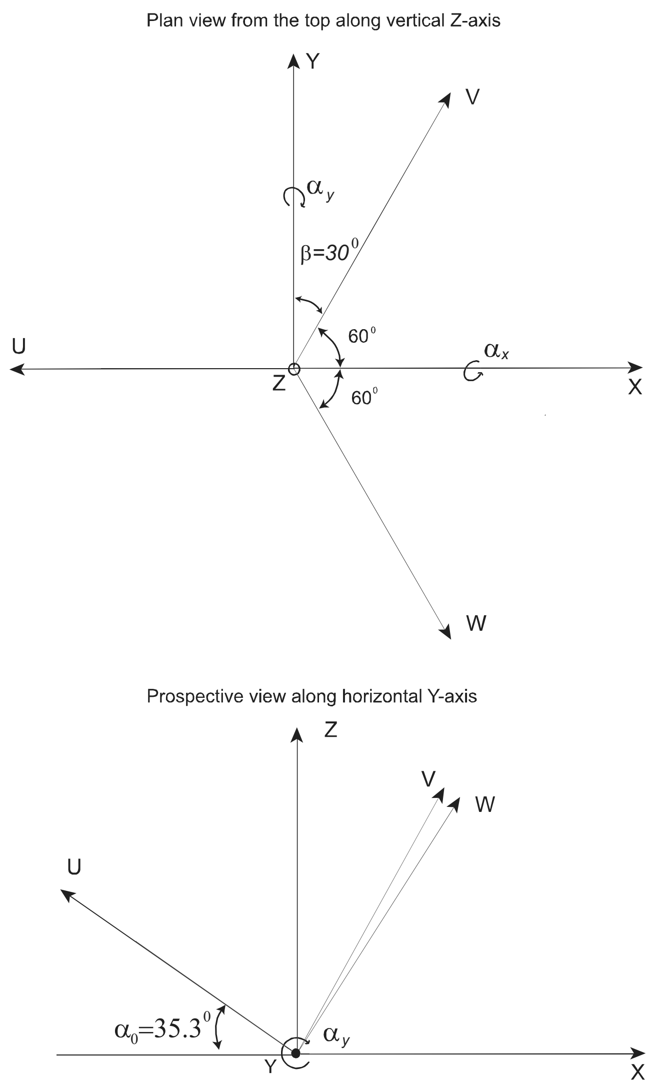

Figure 3. Schematic orientation of the X, Y, Z and U, V, W axes. In a prospective view shown in the bottom panel, V and W axes should overlap and are shown separately for clarity.

using Galperin's configuration generally are rotated into an Earth referenced X, Y, Z coordinate system by using trigonometric formulas. The disadvantage of this configuration is that if one of the sensors is not working properly or there is misalignment between sensors, degradation results in all three cardinal components X, Y, and Z.

Simplified Response of Seismometer

The response of a seismometer to input ground motion generally is described by an ordinary differential equation of second order as an elastic single degree of freedom oscillator (e.g., Golitzin, 1912)

$$\varphi_x'' + 2\omega_x D_x \varphi_x' + \omega_x^2 \varphi_x = -u_x'', \tag{1}$$

where:

φ_x is the recorded response of the instrument in the x direction,
ω_x and D_x are the natural circular frequency and fraction of critical damping of the oscillator, and
u_x'' is the translational ground-motion acceleration along the x axis.

As a first approximation, equation (1) is sufficient to describe the behavior of both a mass-on-spring pendulum (also called geophone in seismic prospecting) and a mass-on-rod pendulum oscillating in a vertical or horizontal plane about a pivot at one end of the rod.

Equation (1) correctly describes the response of either pendulum when the input ground motion is translational only. Historically, seismologists have generally assumed that rotational components of ground motion (also called rocking and torsion in engineering) are small enough to be neglected. However, recently examples of records with significant rotational components (relative to translational motion) were presented, and the effects of these should not be ignored (Bouchon and Aki, 1982; Graizer, 1991; Takeo, 1998; Huang, 2003; Zahradnik and Plesinger, 2005; Cochard *et al.*, 2006; Schreiber *et al.*, 2006; Spudich and Fletcher, 2008). The response of pendulums installed in a cardinal configuration (one vertical and two horizontals) to complex input motions including rotations has been studied by Golitzin (1912), Rodgers (1968), Wong and Trifunac (1977), Graizer (1989, 1991, 2005, 2006b), Todorovska (1998), Trifunac and Todorovska (2001), and Graizer and Kalkan (2008). Because the sensitivity of a tilted pendulum in the Galperin configuration differs from that of a vertical or a horizontal, the response to rotational motion should be considered. The goal of this article is to study the response of seismological sensors in the Galperin configuration to complex input ground motions, which include tilt. A comparison is made to the responses of cardinally oriented three-component systems. It should be noted that these responses would be identical if it were not for axis alignment and response errors. Nevertheless, such errors are inevitable in the construction of any seismometer and can manifest in unexpected ways in a Galperin system.

Equations of Pendulums in a Cardinal Configuration

I use the right-handed coordinate system (X, Y, Z) commonly used in physics and mathematics. In the right-hand system, rotational vector axes should point in the same directions as the translational axes. Figure 4 shows this sign convention for translational and rotational motion as recommended by the members of International Working Group on Rotational Seismology (Evans *et al.*, 2009). Note that a rotation vector along the upward Z axis represents counterclockwise rotation in the horizontal plane as viewed from above.

The complete equation for small oscillations of a horizontal pendulum of the mass-on-spring type can be expressed as (Graizer, 1989; Graizer and Kalkan, 2008)

$$\begin{aligned} \varphi_x'' + 2\omega_x D_x \varphi_x' + \omega_x^2 \varphi_x &= -u_x'' + g\alpha_y - u_z''\alpha_y - u_y''\alpha_z \\ \varphi_y'' + 2\omega_y D_y \varphi_y'' + \omega_y^2 \varphi_y &= -u_y'' - g\alpha_x + u_z''\alpha_x + u_x''\alpha_z, \end{aligned} \tag{2}$$

where g is acceleration due to gravity, α_x and α_y are rotations (tiltings) of the ground around the x or y axis, α_z is rotation of the ground around the vertical z axis (Fig. 5). The right-hand sides of equation (2) (forcing parts) include acceleration along the y axis u_y'', acceleration along the x axis u_x'', component of gravitational acceleration due to tilt of the base around the x axis $g\alpha_x$ or y axis $g\alpha_y$, and cross-axis sensitivities $u_z''\alpha_x$, $u_z''\alpha_y$, $u_y''\alpha_z$, and $u_x''\alpha_z$. Theoretically, centrifugal acceleration $a_c = l_y(\alpha_x')^2$ and $a_c = l_x(\alpha_y')^2$ also contribute to the oscillations of the pendulum. Yet, it is lower order term and can be neglected because the length of the spring of this type of seismometer usually does not exceed 20 cm and the highest angular velocity observed during earthquakes does not exceed 0.2 rad/sec (Graizer, 2006a).

Sensitivity of the vertical pendulum to tilts is different. For small tilts it is proportional to $1 - \cos(\alpha_y)$, and because $\cos(\alpha_y) \approx 1 - \alpha_y^2/2$, it is proportional to $g\alpha_y^2/2$ and can be neglected. The complete equation of motion for a vertical pendulum of this type can be written as

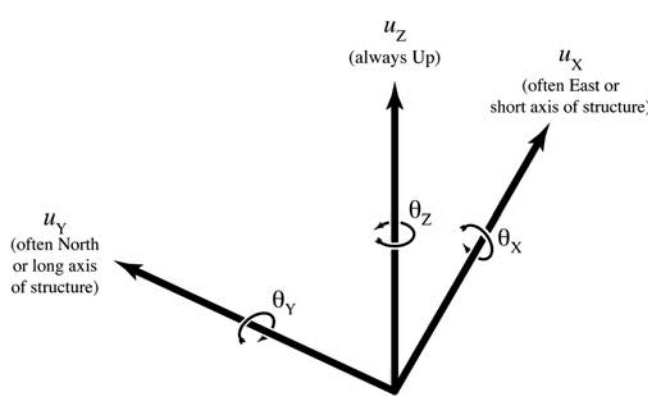

Figure 4. Sign convention for translational and rotational motion (from Evans *et al.*, 2009).

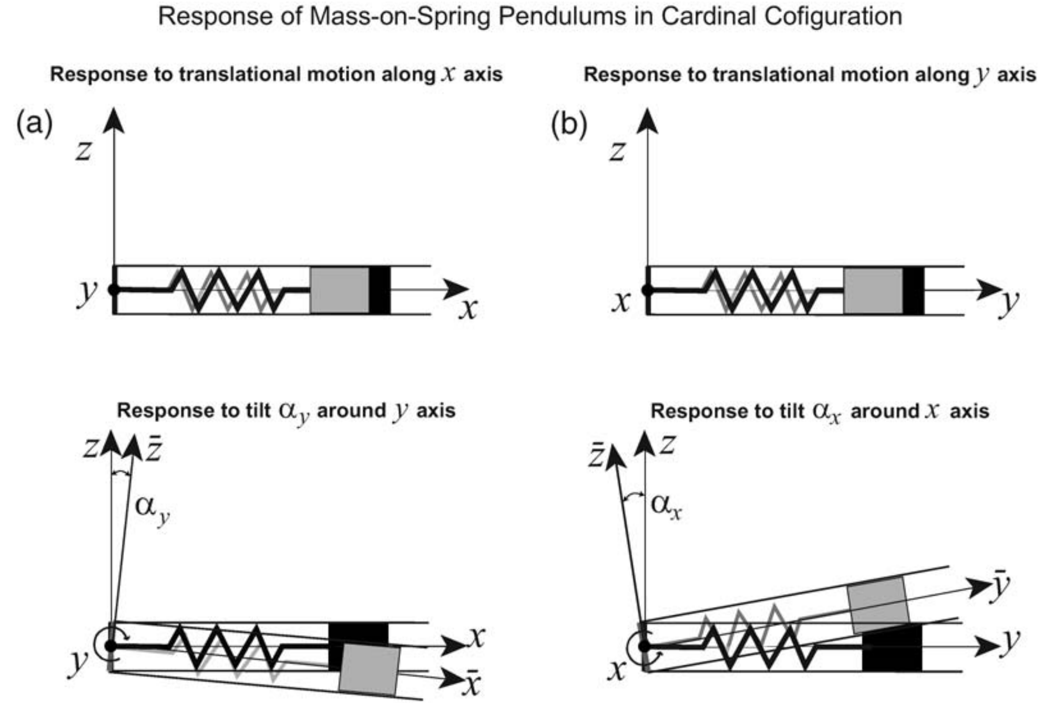

Figure 5. Response of a mass-on-spring pendulum in cardinal configuration to horizontal motions and tilts.

$$\varphi_z'' + 2\omega_z D_z \varphi_z' + \omega_z^2 \varphi_z = -u_z'' - u_x'' \alpha_y - u_y'' \alpha_x, \quad (3)$$

where u_z'' is acceleration in the vertical direction.

The complete equation for small oscillations (i.e., $\sin\theta \cong \theta$) of a horizontal pendulum of the mass-on-rod type (Fig. 6) can be expressed as

$$\begin{aligned} \varphi_x'' + 2\omega_x D_x \varphi_x' + \omega_x^2 \varphi_x &= -u_x'' + g\alpha_y - l_x \alpha_z'' - u_y'' \theta_z \\ \varphi_y'' + 2\omega_y D_y \varphi_y' + \omega_y^2 \varphi_y &= -u_y'' - g\alpha_x - l_y \alpha_z'' + u_x'' \theta_z, \end{aligned} \quad (4)$$

where θ_z is the deflection angle (around the z axis) of a pendulum relative to the frame of the seismometer from the position of equilibrium, l_y and l_x are the lengths of x and y pendulum arms, $\varphi_y = l_y \theta_z$ and $\varphi_x = l_x \theta_z$ for small angles of θ_z (note that the horizontal pendulum sensitive to ground motion in the y direction is rotating around the vertical z axis (Fig. 6). Note the difference between α_z rotation of the ground around the vertical z axis and θ_z deflection angle around the z axis of the pendulum relative to the frame of the seismometer.

Comparison of equations (2) and (4) shows some differences between the forcing (right-hand side) parts. The main difference is that the mass-on-spring pendulum is not sensitive to angular accelerations. In this article we are not considering the response of an inverted (astatic) mass-on-rod pendulum oscillating in a vertical plane around the horizontal axis (e.g., classical Wiechert's horizontal seismograph built around 1905 and still used at some seismological observatories and Guralp horizontal seismometers CMG-40T and CMG-3T). An inverted horizontal pendulum has a more complex response, and it was considered in detail by Graizer and Kalkan (2008).

The resulting complete equation of motion for a vertically sensitive pendulum of the mass-on-rod type (with its pivot along the y axis) can be written as

$$\varphi_z'' + 2\omega_z D_z \varphi_z' + \omega_z^2 \varphi_z = -u_z'' - l_z \alpha_y'' - u_x'' \theta_y. \quad (5)$$

While a horizontal pendulum (equation 4) is sensitive to the acceleration of linear motion, tilt, angular acceleration, and cross-axis excitations; a vertical pendulum is sensitive to the acceleration of linear motion, angular acceleration, and cross-axis excitations (equation 5) but not tilt for small tilt angles α_y.

The studies of Wong and Trifunac (1977), Graizer (1989), Todorovska (1998), Trifunac and Todorovska (2001), and Graizer (2006b) show that the last terms on the right-hand side of equations (2)–(5)) (cross-axis sensitivities) are relatively small and can be neglected in most applications. In most cases the length of pendulum arm is small (about 20–30 cm for a seismometer and 2–3 cm or less for an accelerometer). As a result, the angular acceleration terms $l_z \alpha_x''$ (or $l_y \alpha_z''$) in the right-hand side of equations (4) and (5) are also relatively small and can be neglected.

Thus, first-order equations for small oscillations of a mass-on-spring and mass-on-rod pendulums can be written as (Graizer, 2006b)

$$\begin{aligned} &x\text{-direction} \quad \varphi_x'' + 2\omega_x D_x \varphi_x' + \omega_x^2 \varphi_x = -u_x'' + g\alpha_y, \\ &y\text{-direction} \quad \varphi_y'' + 2\omega_y D_y \varphi_y' + \omega_y^2 \varphi_y = -u_y'' - g\alpha_x, \\ &z\text{-direction} \quad \varphi_z'' + 2\omega_z D_z \varphi_z' + \omega_z^2 \varphi_z = -u_z''. \end{aligned} \quad (6)$$

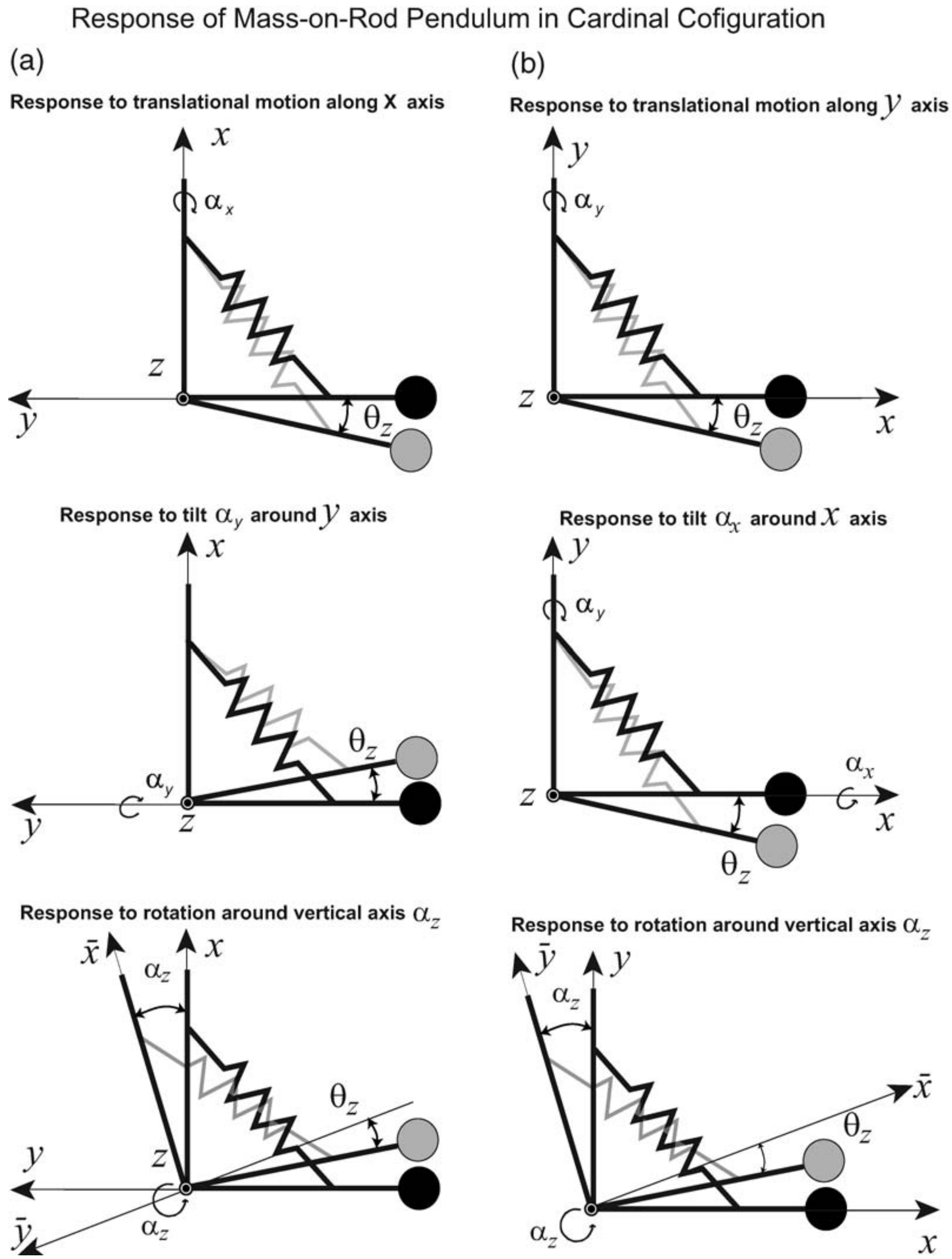

Figure 6. Response of a mass-on-rod pivoting pendulum in cardinal configuration to horizontal motion and rotations.

In other words, typical horizontal pendulums respond to horizontal accelerations and tilts, while the vertical sensor is responding to vertical acceleration.

Equation of Pendulum Motion in Galperin Configuration

Mass-on-Spring Pendulum

Galperin (1955, 1985) designed his noncardinal sensor configuration for use with geophones mass-on-spring pendulums. I first consider the response of a single mass-on-spring pendulum in the Galperin configuration to input motions that include rotations. Figure 7 shows the response of this type of pendulum to the positive (clockwise) tilting of the instrument base:

$$\varphi_u'' + 2\omega_u D_u \varphi_u' + \omega_u^2 \varphi_u = -u_u'' - g(\sin\alpha_m - \sin\alpha_0), \quad (7)$$

where U is the direction of sensitivity (normally tilted at the angle α_0 to the horizontal plane), $\alpha_y(t)$ is the angle of dy-

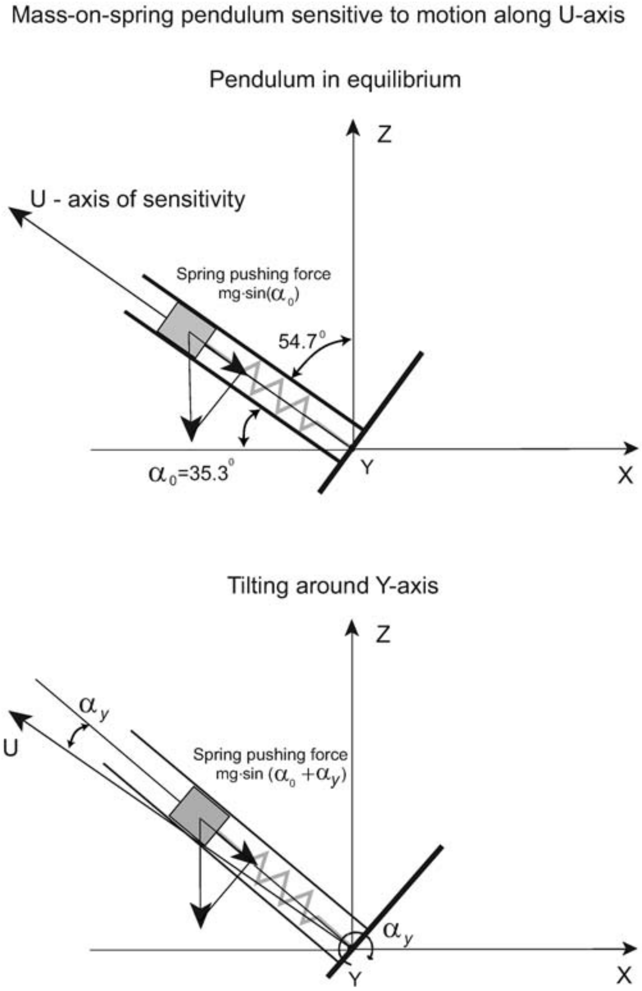

Figure 7. Response of a mass-on-spring translational sensor to tilt α_y.

namic tilting relative to the position of equilibrium, and the total angle of rotation $\alpha_m(t) = \alpha_0 + \alpha_y(t)$. Note that we are considering small tilt angles $\alpha_y(t)$, usually much less than 0.1° (2 mrad) in teleseismic and not more than 2°–3° (50 mrad) in strong-motion measurements.

Applying a well-known trigonometric formula to the equation (7) results in

$$
\begin{gathered}
\sin\alpha_m - \sin\alpha_0 = 2\sin\frac{\alpha_m - \alpha_0}{2}\cos\frac{\alpha_m + \alpha_0}{2}, \\
k = \cos\frac{\alpha_m + \alpha_0}{2} = \cos(\alpha_0 + \alpha_y/2) \approx \cos\alpha_0, \\
\sin\frac{\alpha_m - \alpha_0}{2} = \sin\left(\frac{\alpha_y}{2}\right) \approx \frac{\alpha_y}{2}, \\
\sin\alpha_m - \sin\alpha_0 \approx \alpha_y \cdot \cos\alpha_0 = k\alpha_y, \\
\varphi_u'' + 2\omega_u D_u \varphi_u'' + \omega_u^2 \varphi_u = -u_u'' - kg\alpha_y.
\end{gathered} \tag{8}
$$

For the Galperin orientation $\alpha_0 = 35.26°$ and for small dynamic tilt angles α_y,

$$
\begin{gathered}
k = \cos(35.26°) = 0.816, \\
\varphi_u'' + 2\omega_u D_u \varphi_u' + \omega_u^2 \varphi_u = -u_u'' - 0.816 g\alpha_y.
\end{gathered} \tag{9}
$$

In contrast to the cardinal configuration, where horizontal but not the vertical sensors are sensitive to small tilts, each of the three sensors in the Galperin configuration is sensitive to tilts. The sensitivity of a pendulum oriented at 54.74° from the vertical axis is lower than that of the horizontal (about 82% of the tilt sensitivity of a horizontal) but much higher than that of the vertical. Increasing the angle of a pendulum from the horizontal decreases its tilt sensitivity coefficient k.

Mass-on-Rod Pendulum

Because most seismological instruments are of the mass-on-rod type, let us consider the response of this type of pendulum to six degrees of freedom input motions. When the axis of sensitivity U is oriented at α_0 from the horizontal, the pendulum in its position of equilibrium is oriented at an angle $(90° - \alpha_0)$ from horizontal (Figs. 2 and 8):

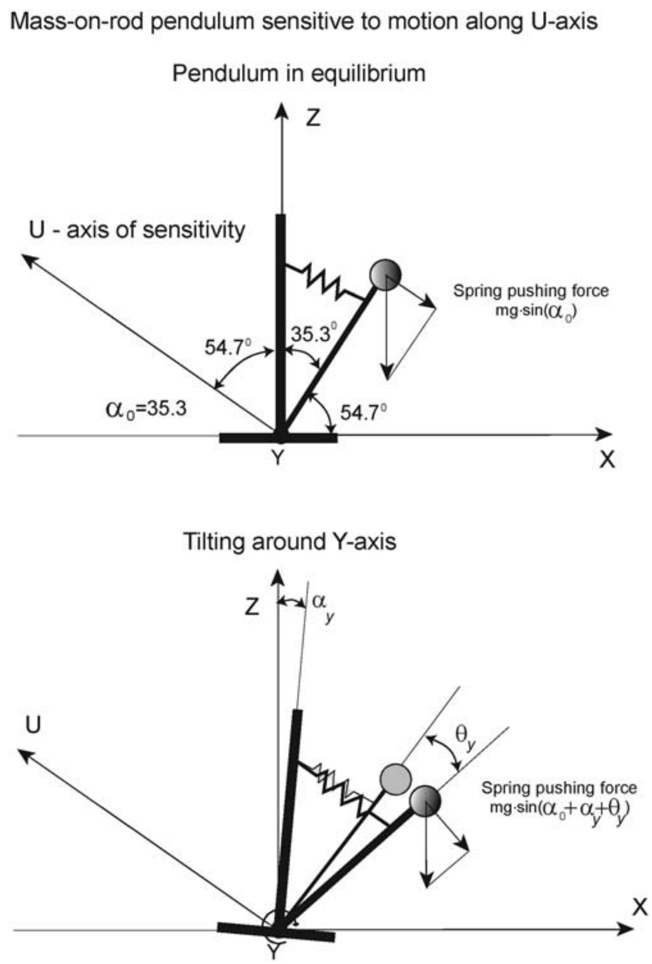

Figure 8. Response of a mass-on-rod pivoting sensor to tilt α_y around the y axis.

$$\varphi_u'' + 2\omega_u D_u \varphi_u' + \omega_u^2 \varphi_u = -u_u'' - g(\sin\alpha_{m1} - \sin\alpha_0) - l_u \alpha_y'', \quad (10)$$

where $\alpha_{m1} = \alpha_0 + \alpha_y + \theta_y$ and $\varphi_u = \theta_y l_u$ for small angles θ_y of pendulum deflection from its equilibrium position. After trigonometric transformations similar to those shown previous to equation (10) can be written

$$\varphi_u'' + 2\omega_u D_u \varphi_u' + \omega_u^2 \varphi_u = -u_u'' - kg(\alpha_y + \theta_y) - l_u \alpha_y''. \quad (11)$$

Assuming that the third term in the right-hand side of equation (11) is small for existing seismographs and accelerographs, equation (11) can be simplified,

$$\varphi_u'' + 2\omega_u D_u \varphi_u' + \omega_u^2 \varphi_u = -u_u'' - kg(\alpha_y + \theta_y),$$

and transferring deflection term θ_y of the sensor from its equilibrium to the left-hand side gives

$$\varphi_u'' + 2\omega_u D_u \varphi_u' + (\omega_u^2 + kg/l_u)\varphi_u = -u_u'' - kg\alpha_y. \quad (12)$$

In the case of Galperin orientation and for small angles α_y and θ_y,

$$\varphi_u'' + 2\omega_u D_u \varphi_u' + (\omega_u^2 + 0.816g/l_u)\varphi_u = -u_u'' - 0.816g\alpha_y. \quad (13)$$

As for the mass-on-spring pendulum, the mass-on-rod pendulum in a Galperin configuration is sensitive to tilts. But in contrast to the response of the mass-on-spring pendulum, the response of the mass-on-rod pendulum is nonlinear. This effect is larger for low-frequency systems and may result in increasing or decreasing (depending upon the angle of tilting) the effective frequency of an oscillator. For high-frequency accelerometers this effect is negligible because $\omega_u^2 \gg 0.816g/l_u$. Early designers of seismometers with Galperin sensor configuration (Melton and Kirkpatrick, 1970) recognized these potential problems. They specifically mentioned that the seismometer should be leveled at all times to avoid changes of its period.

Equation (13) corresponds to the response of a conventional passive seismometer in which the inertial force produced by ground motion deflects the mass from its position of equilibrium, and the displacement or velocity of the mass is converted into an electric signal. Most current broadband seismometers and accelerometers now use a force-balance design. In a force-balance instrument, the inertial force is compensated by an electrically generated force so that the mass moves as little as possible (e.g., Wielandt, 2002). Force balancing minimizes the deflection θ of the mass, so equations (12) and (13) become the same as equations for the mass-on-spring pendulum. That is, its nonlinearity is negligible.

Converting from Galperin to Cardinal Systems of Coordinates

For mass-on-rod pendulums, transformation from response in the coordinate system U, V, W to the response in the X, Y, Z coordinate system can be written

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} -\cos\alpha_0 & \cos\alpha_0 \sin\beta & \cos\alpha_0 \sin\beta \\ 0 & \cos\alpha_0 \cos\beta & -\cos\alpha_0 \cos\beta \\ \sin\alpha_0 & \sin\alpha_0 & \sin\alpha_0 \end{pmatrix} \times \begin{pmatrix} U \\ V \\ W \end{pmatrix}, \quad (14)$$

where β is the angle between the Y axis and the projection of the V axis onto the horizontal plane (e.g., $\beta = 30°$ in Fig. 3, top panel). In particular, for the Galperin system (Wielandt, 2002)

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} -\sqrt{2/3} & 1/\sqrt{6} & 1/\sqrt{6} \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \end{pmatrix} \begin{pmatrix} U \\ V \\ W \end{pmatrix}. \quad (15)$$

And the inverse transformation can be written

$$\begin{pmatrix} U \\ V \\ W \end{pmatrix} = \begin{pmatrix} -\sqrt{2/3} & 0 & 1/\sqrt{3} \\ 1/\sqrt{6} & 1/\sqrt{2} & 1/\sqrt{3} \\ 1/\sqrt{6} & -1/\sqrt{2} & 1/\sqrt{3} \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}. \quad (16)$$

These transformations differ only in sign from these of Galperin written for the mass-on-spring pendulums (Galperin, 1955; Hardage, 2000).

Effect of Tilt on Galperin Three-Component Systems

Equations (9) and (13) demonstrate the effect of tilt on the response of a single mass-on-spring pendulum oriented at the Galperin angle of 35.3° to the horizontal plane or a mass-on-rod pendulum oriented at the same angle relative to the vertical axis. Let us consider the effect of tilt on the response of a three-component Galperin system when transformed into an X, Y, Z coordinate system (Figs. 3, 7, and 8). First assume that the input signal contains only tilt and no translational motion. Tilt has two components α_y and α_x, the first representing rotation in the U–X plane (around the Y axis) and the second one representing rotation in the Y–Z plane (around the X axis).

Positive α_y results in an increase of gravitational force on the U sensor of $0.816g\alpha_y$ (equation 9); thus, with the U sensor moving in a negative direction (Fig. 7, mass-on-spring sensor). The component α_y will also result in the decrease of gravitational force on sensors V and W. This decrease is $0.816g\alpha_y \sin 30° = 0.816g\alpha_y/2$ and results in both V and W sensors moving in positive directions.

Transformation to X, Y, and Z axes using equation (15) results in a signal being proportional to

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} -\sqrt{2/3} & 1/\sqrt{6} & 1/\sqrt{6} \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \end{pmatrix} \times \begin{pmatrix} -0.816g\alpha_y \\ 0.816g\alpha_y \cdot \sin 30° \\ 0.816g\alpha_y \cdot \sin 30° \end{pmatrix} = \begin{pmatrix} g\alpha_y \\ 0 \\ 0 \end{pmatrix}.$$

As expected, tilting α_y of the base of a seismograph around the Y axis does not produce any Y-axis signal, and the effect of tilt α_y on a vertical channel is properly compensated.

Tilt α_x around the X axis produces the opposite effects on W and V sensors $\pm 0.816g\alpha_x \cdot \cos 30°$ and negligible effect on the U sensor. Transformation to X, Y, Z coordinates results in a signal being

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} -\sqrt{2/3} & 1/\sqrt{6} & 1/\sqrt{6} \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \end{pmatrix} \times \begin{pmatrix} 0 \\ -0.816g\alpha_x \cdot \cos 30° \\ 0.816g\alpha_x \cdot \cos 30° \end{pmatrix} = \begin{pmatrix} 0 \\ -g\alpha_x \\ 0 \end{pmatrix}.$$

As might be expected tilting α_x of the base of a seismograph around the X axis does not produce any X-axis signal. Similarly to the effect of tilt α_y, the effect of α_x on the vertical channel is compensated.

Therefore, the response of the Galperin triaxial system recording ground motion on U, V, and W axes to translational and tilting motions can be written as

$$\begin{aligned} \varphi_u'' + 2\omega_u D_u \varphi_u' + \omega_u^2 \varphi_u &= -u_u'' - g\alpha_y \cos\alpha_0, \\ \varphi_v'' + 2\omega_v D_v \varphi_v' + \omega_v^2 \varphi_v &= -u_v'' + g\alpha_y \cos\alpha_0 \sin\beta \\ &\quad + g\alpha_x \cos\alpha_0 \cos\beta, \\ \varphi_w'' + 2\omega_w D_w \varphi_w' + \omega_w^2 \varphi_w &= -u_w'' + g\alpha_y \cos\alpha_0 \sin\beta \\ &\quad - g\alpha_x \cos\alpha_0 \cos\beta, \end{aligned} \tag{17}$$

where α_0 and β are fixed angles corresponding to the system design, and α_y and α_x are dynamic tilts occurring during ground shaking. One consequence of the Galperin sensor arrangement (because each pendulum is oscillating in the vertical plane) is that it is not sensitive to rotation around the vertical axis (called torsion or twisting in engineering).

The system of equations describing the response of a Galperin sensor configuration ($\alpha_0 = 35.26°$ and $\beta = 30°$) to translational motion and tilts α_y and α_x can be written as

$$\begin{aligned} \varphi_u'' + 2\omega_u D_u \varphi_u' + \omega_u^2 \varphi_u &= -u_u'' - 0.816g\alpha_y, \\ \varphi_v'' + 2\omega_v D_v \varphi_v' + \omega_v^2 \varphi_v &= -u_v'' + 0.408g\alpha_y + 0.707g\alpha_x, \\ \varphi_w'' + 2\omega_w D_w \varphi_w' + \omega_w^2 \varphi_w &= -u_w'' + 0.408g\alpha_y - 0.707g\alpha_x. \end{aligned} \tag{18}$$

If permanent tilt occurs during ground motion, each of the three sensors in Galperin configuration will experience a baseline shift while in the cardinal X, Y, Z configuration the vertical channel is not sensitive to small tilts. Any deviations from the designed symmetry of the Galperin system will require recalculating the effect on axes transformations. Otherwise, the transformation to a cardinal system will be contaminated.

Applying the transformation matrix (15) results in zero tilt effect (compensation) on the vertical cardinal axis. In other words, as with a cardinal configuration, the output signal in the vertical direction is not sensitive to tilt.

However, the result of the transformation shows that the two horizontal components are sensitive to tilt and tilt sensitivity along the horizontal direction is $g\alpha$, the same as the sensitivity of a horizontal sensor in cardinal orientation. The sensitivity of a Galperin system when transformed to cardinal is the same as in the cardinal configuration.

As noted, Galperin configuration requires the system to be carefully oriented, leveled, and sensors to be identical

$$\begin{aligned} \omega &= \omega_u = \omega_v = \omega_w, \\ D &= D_u = D_v = D_w, \\ l &= l_u = l_v = l_w. \end{aligned} \tag{19}$$

Thus, the equations for the three cardinal components transformed from Galperin configuration are

$$\begin{aligned} \varphi_x'' + 2\omega D \varphi_x' + \omega^2 \varphi_x &= -u_x'' + g\alpha_y, \\ \varphi_y'' + 2\omega D \varphi_y' + \omega^2 \varphi_y &= -u_y'' - g\alpha_x, \\ \varphi_z'' + 2\omega D \varphi_z' + \omega^2 \varphi_z &= -u_z''. \end{aligned} \tag{20}$$

This system of equations corresponds to the response of Galperin and cardinal instruments with minimal deflection θ of pendulum masses from their positions of equilibrium (e.g., force balance). Despite visual similarity of results after reduction to the X, Y, Z configuration, the Galperin system in contrast to the classical cardinal configuration is not sensitive to rotation around a vertical axis. Of course, it is correct only if the system is properly leveled.

Pillet and Virieux (2007) demonstrated that sensitivity to rotations around the vertical axis resulted in an unexpected 45° N polarization of long-period background noise recorded by the STS-1 seismometer (with cardinal sensor configuration). Recordings from the STS-2 seismometer with Galperin configuration demonstrated local variations in polarization of long-period background noise. In this example insensitivity of Galperin configuration to rotation around the vertical axis is an advantage over the cardinal configuration. Not all sensors with cardinal configurations have this sensitivity to α_z.

For example, Guralp horizontal seismometers (CMG-40T and CMG-3T) employ inverted pendulums and are not sensitive to rotation around the vertical axis.

Let us estimate typical effects of tilt on the response of a seismometer, assuming amplitude of tilt of 10^{-6} rad, and $g \approx 10^3$ cm/sec^2 will result in an effect similar to the effect of acceleration of the order of 10^{-3} cm/sec^2. Such amplitudes are comparable to those of translational motions at regional distances. This example shows that tilt may affect the output of seismometers, especially at long periods. Equation (20) shows that a horizontal channel is sensitive to translational acceleration and tilt. Thus, double integration of this mixed record will produce the sum of displacement and double integrated tilt. This contamination may look like low-frequency signal (Graizer, 2005). In contrast, the vertical channel is not sensitive to tilts. Fourier amplitude spectra of the vertical and horizontal components of uncontaminated translational acceleration in the frequency domain from zero to a few hertz should be similar to one another. Thus, comparison of vertical and horizontal spectra can be used as an indicator of tilt contamination. If the two horizontal channels demonstrate higher spectral amplitudes at lower frequencies than these of the vertical, it may mean that the low-frequency signals of the horizontal channels are contaminated by tilt. A method of tilt discrimination based on this idea was successfully applied to a number of strong-motion records by Graizer (1991, 2006a).

Potential Errors Due to Galperin Configuration and Implications for Strong-Motion Registration

The record from each channel of a cardinal configuration is independent of the other channels and can be processed independently for amplitude and phase correction. In contrast to most seismometers recording velocity, almost all existing strong-motion instruments are accelerometers. Strong-motion accelerometers are usually much smaller and less expensive and are installed in many locations much noisier than the instruments used in teleseismic investigations. Accelerometers often are leveled and calibrated with less precision than teleseismic instruments (broadband velocimeters).

At an early stage of data processing recorded strong-motion data generally are corrected for instrument response and then integrated to velocity and displacement using parameters of the channels obtained from calibration. Different procedures varying in basic assumptions about frequency range limitations and noise correction have been developed to perform displacement and velocity calculations and also to produce corrected acceleration (e.g., Trifunac, 1971; Graizer, 1979; Iwan *et al.*, 1985; Converse and Brady, 1992). All of them treat each channel independently.

A problem can arise when amplitudes of strong ground motion on any one of the components are clipped. In case of the cardinal configuration clipping can be identified immediately from the character of the record. However, for the Galperin configuration, if signals on two other components are not-clipped, transformation (reduction) to the X, Y, Z coordinate system can obstruct visibility of this effect. Figure 9 demonstrates this effect. Figure 9 (top left-hand panel) shows the unclipped signals recorded by channels U and V and the result of their reduction to the Z direction. Figure 9 (bottom left-hand panel) demonstrates the effect of clipped signal along the U direction on the reduced signal along the Z direction. Reduced signal along the Z direction demonstrates ~8% lower amplitude than the ideal one as well as a rounded shape.

Figure 9 (top right-hand panel) shows the unclipped signals recorded by channels U and V and the result of their reduction to the X direction. Figure 9 (bottom right-hand panel) demonstrates the effect of clipped signal along the U direction on the reduced signal along the X direction. Reduced signal along the X direction demonstrates lower amplitude than the ideal one and a different shape. This issue is very important because the strongest records are sometimes exceeding the limits of instrument. Of course, there is a possibility to check if the record at one channel is clipped by doing inverse transformation to the original U, V, W coordinate system using the matrix in equation (16).

If the sensors in a Galperin configuration are not properly calibrated (in other words, the condition in equation 19 is not satisfied) reduction to the X, Y, Z directions can result in increased errors. For example, 2% error in sensitivities of each channel can result in 18% error in amplitude of the signal along the X direction and 0.5% error in amplitude along the Z direction. Reputable producers of strong-motion sensors perform calibration of their sensors generally to about 0.1% accuracy, so that calibration errors do not contribute much to records.

As mentioned before, misalignment of the sensor's true axes is another source of errors. To study the influence of this effect I performed the following numerical experiment: let us assume that one of the sensors (U) is oriented at 34.26° to the horizontal instead of 35.26°. In other words, the orientation of the U sensor is 1° off the desired position. If this fact is not known and the transformation in equation (15) is applied, the result is a 2% error (overestimation) for X and 1.4% error (underestimation) for Z with no affect on Y. Leveling of the instrument is very important; if the instrument is not properly leveled, then X, Y, and Z will all be contaminated.

The cumulative effect of misalignment, imprecise leveling, and calibration errors can increase the final error and contaminate ground motion in all three reduced channels. Considering the benefits for equal design of all three sensors in a Galperin configuration including potentially lower costs, the Galperin configuration may be useful for strong-motion sensors not requiring high-resolution signal processing. Otherwise, for earthquake source studies and engineering investigations using the Galperin sensor configuration with reduction to the cardinal orientations places limitations on further processing of such records.

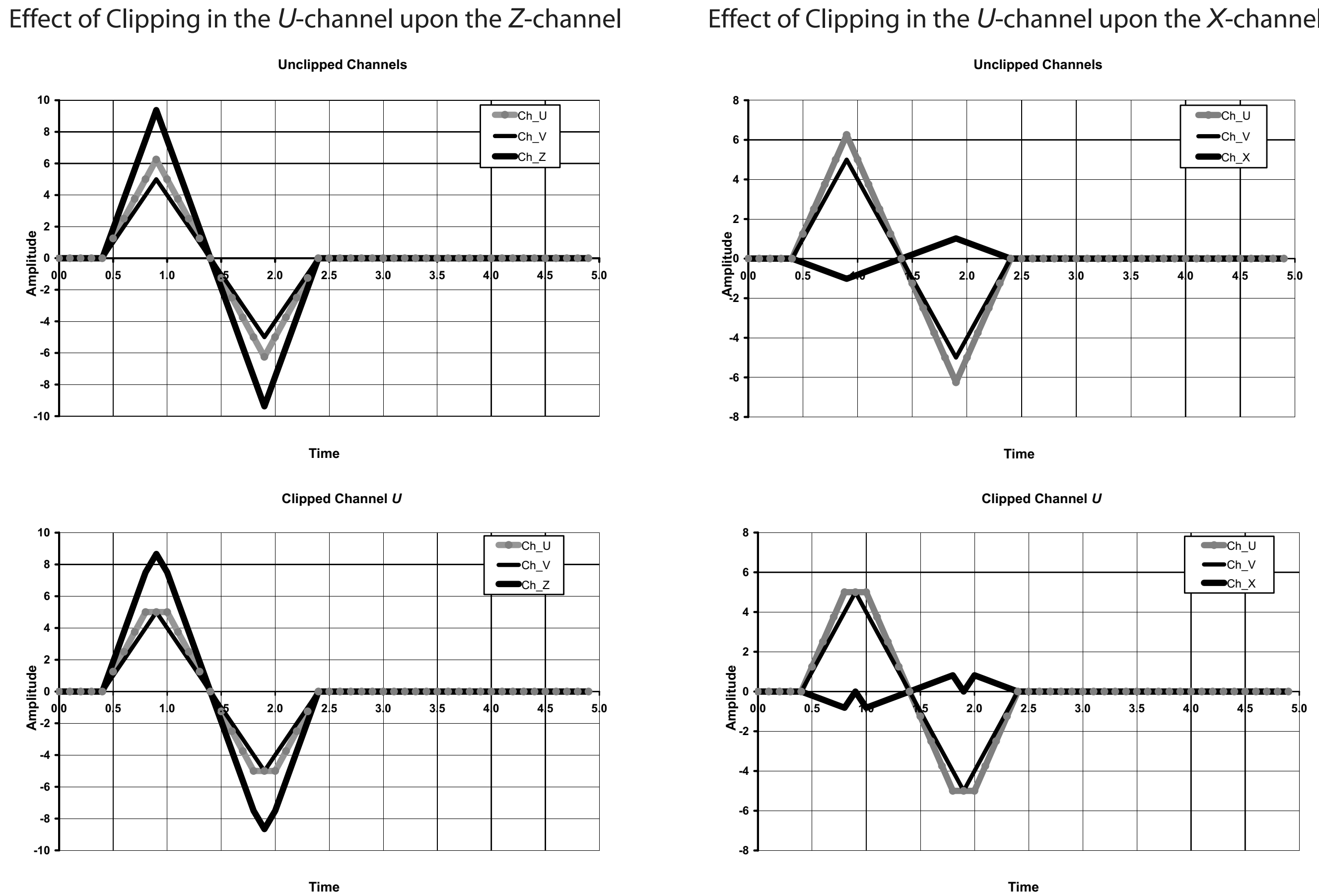

Figure 9. Results of clipping in the U channel upon the transformed Z and X channels. The top left-hand panel shows the unclipped signals on channels U and V and the result of their reduction to the Z direction. The bottom left-hand panel demonstrates the effect of a clipped signal along the U direction, which distorts softening the signal along the Z direction. The top right-hand panel shows the unclipped signals on channels U and V and the result of their reduction to the X direction. The bottom right-hand panel demonstrates the effect of a clipped signal along the U direction, which distorts the signal along the X direction.

If the Galperin sensor configuration makes its way into strong-motion seismology, it will be desirable to allow recording directly from the original accelerations along U, V, and W without reducing accelerations to the cardinal X, Y, and Z directions. As is done in strong-motion processing for cardinal sensors, instrument corrections should first be applied to accelerograms from individual U, V, and W sensors. Reduction to the cardinal directions can be accomplished after both correction and integration, though this would not protect against errors due to misalignment and improper leveling of the system.

A question for developers and users to answer before using Galperin configuration in strong-motion seismology is whether the benefits of using three identical sensors overcome issues of calibration, alignment, leveling, and data processing?

Conclusions

Most seismic sensors use the cardinal configuration of sensors (vertical and orthogonal horizontals). In this configuration the design of the vertical sensor is different from that of the horizontal components because of gravitational acceleration applied to the vertical sensor. Another arrangement (often used for petroleum exploration) is the so-called Galperin configuration. In Galperin's configuration the three sensors are also positioned orthogonally with respect to each other, but all three sensors are tilted at the same angle from the vertical axis (Cartesian system of coordinates balanced on the origin). This configuration ensures that each of the three identical, single-component sensors responds equally to gravity. Effective equations of mass-on-spring and mass-on-rod pendulums in Galperin configuration are presented. Seismological sensors in Galperin configuration are sensitive to translational acceleration and tilt. Tilt sensitivity of both types of pendulums in this geometry is 82% from that of the horizontal pendulum but when rotated to cardinal are the same as cardinal sensors.

A Galperin system should be leveled at all times to avoid changes in the periods of the sensors. A mass-on-rod pendulum, in contrast to a mass-on-spring pendulum, demonstrates nonlinearity when the base is tilted because this tilt changes the effective periods of the sensors. This effect is greater for the low-frequency pendulums and is negligible in high-frequency accelerometers; it can be almost eliminated by using a force-balance feedback system because feedback minimizes deflections of the pendulums from the position of equilibrium (so not really important to modern broadband either).

The reduction of U, V, W coordinates to X, Y, Z shows that the X, Y, Z are sensitive to tilt. Tilt sensitivities for Galperin components rotated to X and Y directions are $g\alpha$, the same as the sensitivities of horizontal sensors in cardinal orientation. Coordinate rotations also result in compensation of tilt effects for Z, which is not sensitive to tilt over small angles. The tilt sensitivity of the Galperin sensor rotated to cardinal coordinates is the same as tilt sensitivity of a cardinal system. Because the three sensors of a Galperin system are oscillating in a vertical plane, this system, in contrast to the cardinal system, is not sensitive to rotations around the vertical axis.

The Galperin configuration is used mainly in seismometers for teleseismic measurements (broadband weak motion seismometry) and not yet for recording strong motion. Implications of using the Galperin configuration for strong-motion studies include malfunction or clipping of the sensors that can go undetected if only output X, Y, and Z signals are considered. Misalignment, differences in sensors (sensors response was not identical) and imprecise leveling of an accelerograph can contaminate output signals X, Y, and Z. The need for correcting and processing strong-motion records from U, V, and W rather than X, Y, and Z can make use of the Galperin system difficult, particularly for acceleration because it is routinely integrated to displacement and this process is exquisitely sensitive to sensors imperfections.

Data and Resources

All of the data used in this study were collected by the author.

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