Bulletin of the Seismological Society of America, Vol. 99, No. 2B, pp. 1028–1039, May 2009, doi: 10.1785/0120080089

Tutorial on New Developments in the Physics of Rotational Motions

by Roman Teisseyre

Abstract We present a linear continuum theory incorporating asymmetric stress fields as well as symmetric strains and antisymmetric rotations. We discuss the related constitutive laws and balance equations. In this theory, the motion equation related to the balance of the antisymmetric part of stresses replaces that for the stress moments. Our theory proves that the rotation waves may exist even in a homogeneous elastic continuum.

Different kinds of extreme deformations are considered. The wave solutions, including the coaction of the rotation and twist fields, are presented and discussed. The dislocation density–stress relations are derived with the help of the symmetric and antisymmetric parts of stresses. The synchronization solution, rotation, and twist, shifted in phase by $\pi/2$, are presented for a material in an advanced deformation state with granulation and microcracking. Some examples of the spin and twist motion records are reported that confirm this synchronization hypothesis.

Introduction

The need for a new, reliable, and relatively simple approach to continuum theory comes from many aspects, different arguments, and many insufficiencies of the classical theories, notably the classical elasticity. Some problems found temporary solutions with the help of the nonlinear relations introduced; we may mention, for example, the soliton waves, important in ocean dynamics but also recently appearing in seismology (Mikhailov and Nikolaevskii, 2000; Bykov, 2006; Majewski, 2006; Bykov, 2008). However, our intention is to remain basically in the linear domain as we believe that a possible need for a nonlinear behavior shall be fulfilled in an extension of the adequate linear theory with the help of the Riemannian or nonRiemannian space geometry (e.g., Teisseyre, 1995).

The classical elasticity almost perfectly describes the small deformations, but it has many well-recognized insufficiencies (for instance, the angular motions are not coherently incorporated in it). Many attempts to improve the theory should be mentioned: first of all, the Cosserat brothers' theory of elasticity with displacements and rotations (see Cosserat [1909] and a number of articles related to the micropolar and micromorphic elastic theories, e.g., Eringen and Suhubi, 1964; Mindlin, 1965; Nowacki, 1986; for an advanced review, see Eringen, 1999). The micropolar and micromorphic theories constitute a very powerful tool for description of many complicated material problems (see Jones, 1973); however, these theories sometimes seem to be too complicated and difficult to manage for nonspecialists and therefore are not in common use in the seismological studies (for some examples of seismological application, see Teisseyre, 1973, 1974).

Some inadequacies and unsolved problems faced by the existing classical theories are as follows:

- In the classical elastic continuum theory, the balance of angular momentum holds when the stresses are assumed to be symmetric, while the angular motions can be introduced only artificially with the help of a characteristic length element and a reference rotation point; this problem is solved in the micromorphic theories.
- Searching for fault-slip solutions, we may rely on classical elasticity but only with the friction constitutive laws introduced additionally in accordance with the experimental data; the corresponding elastodynamic solutions describe slip propagation along a fault, include the friction effects, and solve some problems of seismic radiation.
- Transition to advanced deformation states of continuum, like plastic flow, can only be accomplished by changing the constitutive laws; however, a more serious problem is related to the description of granulation and fragmentation of material, which would need an inclusion of rotation processes in the frame of a coherent theory with asymmetric fields.
- Fracture geometry related to earthquake processes usually reveals an asymmetric pattern with the main slip plane; we may also believe that the premonitory processes as described by deformations in a continuum with defects develop in an asymmetric way, and such an approach cannot be included in the classical theory.
- Solution for an edge dislocation presents some asymmetry in relation to its strain components in the plane perpendicular to the dislocation line (wedge line); for a continuous distribution of dislocations this fact should result in con-

frontation with the symmetry of shears in an asymmetry of stresses.

- Direct differential relation between the density of edge dislocations and the related stresses cannot be found in the frame of the classical symmetric continuum.

These problems and the consideration on fundamental point deformations as discussed by Teisseyre and Górski (2009) can be the basis of a search for a linear continuum theory with asymmetric stresses. Moreover, we believe that separate balance laws exist for all the discussed point motions; some solutions might reveal a mutual interrelation between these motions.

Our aim was to construct a new, relatively simple theoretical approach in which the stress moments would be replaced by the antisymmetric part of stresses, and the introduced displacement and rotation motions could be shifted in phases when originated by the independent but correlated processes.

For the antisymmetric part of stresses and related rotation field, we will need to introduce a proper constitutive law; we will call it Shimbo law as we have followed Shimbo's (1975, 1995) consideration on the friction processes and rotation of grains. Of course, the equivalent constitutive laws have already been introduced in the micromorphic theories and in the Kröner approach to the continuum theory with the self-fields and related internal nuclei (Kröner, 1981). However, the existing theories either do not include all the point motions, or in an opposite case, assume a complete independence of these motions.

Thus, we need a theory incorporating the separate balance relations and constitutive laws for all the point deformations, which will permit us to arrive at different solutions together with those presenting mutual interrelation between the fields as well as a possible phase shift between the motions.

The existing technical facilities record the spin motion (Cochard *et al.*, 2006), but we shall be aware that the recorded rotations come from different source processes related to the independent or mutually related translations and rotations in their origin sites. We may also note that in the course of microfracturing under confining load, we are dealing with a significant rotation release process with the spin motion distinctly overpassing the displacement motions when observed at a very near field. For the strong near-ground motions, which include a tilting component, the rotation of displacements may be very important. Further on, we may even observe a magnification of horizontal rotation of displacements and an appearance of the rocking–tilting component of displacement rotation caused by the geometry of constructions, especially high buildings.

Symbols and Notation

The tensor notation and the summation convention for the repeating indexes are primarily used. For example, $A_k B_k$ means $\sum_k A_k B_k$. The symmetric and antisymmetric parts of asymmetric tensors are designated by brackets, (..) and [..], for indexes; for example, $S_{(ik)} = S_{(ki)}$ while $S_{[ik]} = -S_{[ki]}$. The curl operator is expressed with the help of the fully antisymmetric tensor ε_{nks} equal, respectively, to 1 and -1 for the even and odd permutation of indexes or to 0 when any repetition of indexes would occur, for example, for rotation of displacements, curl $\mathbf{u}$, we may write $\varepsilon_{nks}\frac{\partial u_s}{\partial x_k}$ (with the summation over the repeating indexes). The time derivative is designated by an overhead dot.

Asymmetric Continuum—Standard Theory

We outline in the following section the development of standard asymmetric theory presenting the motion equations for all the fundamental point deformations (see Teisseyre and Górski, 2007) and the related constitutive laws for the related fields.

Taking into account the arguments presented in the former section and the insufficiencies of the classical elasticity, we assume asymmetry of stresses; and therefore, our theory shall include the constitutive laws and motion equations for the symmetric and asymmetric fields; besides the asymmetric stresses, S_{kl}, we introduce the asymmetric deformation, D_{ik}, containing the symmetric strain, E_{kl}, and antisymmetric rotation, ω_{kl}

$$\begin{aligned} S_{kl} &= S_{(kl)} + S_{[kl]}, \qquad D_{ik} = E_{ik} + \omega_{ik}; \\ E_{kl} &= E_{(kl)}, \qquad \omega_{kl} = \omega_{[kl]}. \end{aligned} \tag{1}$$

We have two groups of relations, those for symmetric and antisymmetric fields.

We introduce a new parameter, the phase index χ^0, which combines the deformation fields, $D_{ik} = E_{ki} + \omega_{ik}$, with the derivatives of displacement motions in an independent way, $D_{kl} = E^0_{kl} + \chi\omega^0_{kl}$, and we get

$$\begin{aligned} E_{kl} &= E^0_{kl} = \frac{1}{2}\left(\frac{\partial u_l}{\partial x_k} + \frac{\partial u_k}{\partial x_l}\right), \\ \omega_{kl} &= \chi^0\omega^0_{kl} = \chi^0\frac{1}{2}\left(\frac{\partial u_l}{\partial x_k} - \frac{\partial u_k}{\partial x_l}\right), \qquad |\chi^0| = 1, \end{aligned} \tag{2}$$

where the classical elasticity is obtained for $\chi^0 = 0$.

For the internal energy stored in such a medium, we obtain

$$\mathsf{E} = S_{(ks)}E_{ks} + S_{[ks]}\omega_{ks}. \tag{3}$$

In a more general case, we may put

$$E_{kl} = e^0\frac{1}{2}\left(\frac{\partial u_l}{\partial x_k} + \frac{\partial u_k}{\partial x_l}\right), \qquad \omega_{kl} = \chi^0\frac{1}{2}\left(\frac{\partial u_l}{\partial x_k} - \frac{\partial u_k}{\partial x_l}\right), \tag{4}$$

where for particular values of indexes e^0 and χ^0, we will have:

- For $e^0 = 0$, we define a granular–crushed medium filled with rigid spherical grains with a friction interaction; when applying a torque load on its surface boundary (e.g., a cylindrical one), we will obtain only some angular deformation, and the torque energy stored will be given as $E = S_{[ks]}\omega_{ks}$.
- For $e^0 = 1$ and for χ^0 from $\chi^0 = 0$ to $|\chi^0| = 1$, we define solid continuum with friction and different kinds of internal defects (dislocation densities) and with partly granulated material.

We may also consider a continuum related not only to displacements but also to other physical fields (e.g., thermal, electric, and magnetic, properly arranged in their interaction roles); then for the asymmetric deformation tensor, $D_{ik} = D_{(ki)} + D_{[ik]}$, we can introduce instead of $D_{kl} = E_{kl} + \omega_{kl} = E^0_{kl} + \chi\omega^0_{kl}$ (equations 1 and 2), an expression containing the influence of other physical fields on deformations

$$\begin{aligned} D_{(kl)} &= E_{kl} = e^0 \frac{1}{2}\left(\frac{\partial u_l}{\partial x_k} + \frac{\partial u_k}{\partial x_l}\right) + eI_{(kl)}, \\ D_{[kl]} &= \omega_{kl} = \chi^0 \frac{1}{2}\left(\frac{\partial u_l}{\partial x_k} - \frac{\partial u_k}{\partial x_l}\right) + \chi I_{[kl]}, \end{aligned} \tag{5}$$

where $I_{ki} = I_{(ki)} + I_{[ki]}$ represents an ensemble of the interactive fields while an ensemble of the interaction constants, e and χ, may include the possible phase shift corrections related to the source processes.

Further on, we will concentrate only on the displacement and rotation counterparts leading to the asymmetric stress tensor as given in equations (1) and (2).

We shall note that the indexes introduced relate mainly to the phase shift between the strain and rotation fields not considered in any former theory; the different deformations are related to the mutual correlation between the strain and rotation fields' constants. These additional indexes are not additional material constants but determine the families of solutions or, in other words, the types of the considered deformations and processes. Our attempt probably represents the simplest approach; it seems more logical to first link the interactive fields with deformation tensor and then the asymmetric deformation with stresses because the introduced ensemble of constants may also represent the phase delay between the deformation caused by mechanical load and that related to the effects of interactive fields.

In Teisseyre and Górski (2009) we have noted that any rotation motion can be expressed by displacements; hence, outside the source we may get two independent yet sometimes correlated displacement fields, $U = u + \tilde{u}$, with $\tilde{u} = l^2 \operatorname{curl} \omega$ (l being the characteristic Cosserat length) emitted in the source as a rotation with some phase shift $\chi^0 = \pm i$.

Shear Field: Twist Pseudovector

A shear field is given by the deviatoric part of strains

$$\begin{gathered} E^D_{ik} = E_{ik} - \frac{1}{3}\delta_{ik}E_{ss}, \qquad E^A_{ik} = \frac{1}{3}\delta_{ik}E_{ss}, \\ E^D_{ss} = \sum_s E^D_{ss} = 0. \end{gathered} \tag{6}$$

We may present it either in the form related to the main shear axes, $(E^D_{11}, E^D_{22}, E^D_{33})$, or in the form related only to the off-diagonal components. However, we may note that the transformation from the deviatoric axial tensor to the off-diagonal system is not unique because in the latter we have only two independent components (Wiszniowski and Teisseyre, 2008). The situation considerably simplifies at the ground surface, $x_3 = 0$ with the condition $E^D_{23} = E^D_{31} = 0$, a particular transformation from an axial deviatoric tensor

$$\begin{bmatrix} E^D & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -E^D & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}$$

leads directly to the equivalent off-diagonal form

$$\begin{bmatrix} \mathbf{0} & E^D & \mathbf{0} \\ E^D & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}.$$

In general using the off-diagonal system, we can define the twist pseudovector,

$$(\tilde{\omega}_s) = (E^D_{23}, E^D_{31}, E^D_{12}) \quad \text{or} \quad E^D_{kl} = \varepsilon_{kls}\tilde{\omega}_s. \tag{7}$$

Note that the twist vector is related to the string–string point deformation (see Teisseyre and Górski, 2009); the defined twist motion, $\tilde{\omega}_s$, means the rotational oscillation of the off-diagonal shear axes of the deviatoric tensor and the amplitude related to it (due to disturbances of an external load by the inner microfracture processes and defects formed) as schematically shown in Figure 1.

However, we can present this pseudovector, $\tilde{\omega}_s$, as incorporated into the invariant tensor form with the help of the Dirac tensors. The invariant Dirac tensors, γ^α, are related to the coordinate system in such a way that their individual shapes are preserved in any coordinate system. The Dirac tensors fulfill the conditions

$$\frac{1}{2}(\gamma^\alpha\gamma^\beta + \gamma^\beta\gamma^\alpha) = \eta^{\alpha\beta},$$

where the space-time metric is

$$\eta^{\alpha\beta} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}.$$

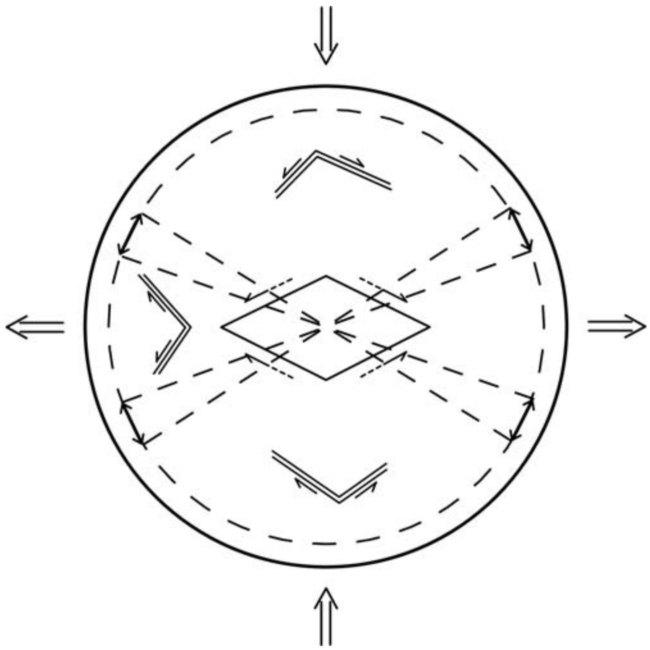

Figure 1. Twist motions: shear axes' oscillation (marked by two side arrows) and the amplitude as caused by the inner microfracture processes and defects (transformation of a square into a rhombus).

Both the symmetric and antisymmetric Dirac tensors exist. We may invariantly present the twist pseudovector with the help of the antisymmetric Dirac tensor. We make this choice as it also assures the invariant form of the related conservation law.

We define the complex rotation tensor,

$$\Omega_{\alpha\beta} = \omega_{\alpha\beta} + i\tilde{\omega}_{\alpha\beta} = \begin{bmatrix} 0 & (\omega_3 + i\tilde{\omega}_3) & -(\omega_2 + i\tilde{\omega}_2) & -(\omega_1 + i\tilde{\omega}_1) \\ -(\omega_3 + i\tilde{\omega}_3) & 0 & (\omega_1 + i\tilde{\omega}_1) & -(\omega_2 + i\tilde{\omega}_2) \\ (\omega_2 + i\tilde{\omega}_2) & -(\omega_1 + i\tilde{\omega}_1) & 0 & -(\omega_3 + i\tilde{\omega}_3) \\ (\omega_1 + i\tilde{\omega}_1) & (\omega_2 + i\tilde{\omega}_2) & (\omega_3 + i\tilde{\omega}_3) & 0 \end{bmatrix}, \tag{8}$$

which includes the rotation and twist fields. Its invariant form can be built in the following way:

$$\begin{aligned} \Omega_{\alpha\beta} &= \omega_{\alpha\beta} + i\tilde{\omega}_{\alpha\beta} \\ &= i\gamma^1(\omega_1 + i\tilde{\omega}_1) + i\gamma^2(\omega_2 + i\tilde{\omega}_2) + \gamma^3(\omega_3 + i\tilde{\omega}_3), \end{aligned} \tag{9}$$

where we have introduced the Dirac tensors,

$$\gamma^1 = i\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}, \quad \gamma^2 = i\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}, \quad \gamma^3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

The definition, equation (8), serves for both the rotation and twist vectors and makes it possible to present the invariant conservation equations for the two fields. The related motion equations can be expressed by the standard conservation law in 4D and can be applied to the complex rotation motion defined in equation (8),

$$\frac{\partial \Omega_{\alpha\beta}}{\partial x^\beta} = 0 \quad \text{or} \quad \varepsilon_{nsk}\frac{\partial(\omega_k + i\tilde{\omega}_k)}{\partial x^s} - \frac{\partial(\omega_n + i\tilde{\omega}_n)}{iV\partial t} = 0, \tag{10}$$

where $x^\beta = (x^1, x^2, x^3, x^4)$; $x^4 = iVt$; and where V means a propagation velocity related to these fields.

The real and imaginary parts become quite similar in form to the Maxwell equations (provided that velocity V is transformed according to relativistic rules for the sum of velocities),

$$\varepsilon_{nsk}\frac{\partial \omega_k}{\partial x^s} - \frac{\partial \tilde{\omega}_n}{V\partial t} = 0, \qquad \varepsilon_{nsk}\frac{\partial \tilde{\omega}_k}{\partial x^s} + \frac{\partial \omega_n}{V\partial t} = 0. \tag{10a}$$

According to these equations the rotation and twist motions appear mutually connected; any rotation motion induces the twist-shear response with related displacements and vice versa. This is an important statement valid even for an advanced deformation state of continuum; thus, we may assume it valid before and after a macroscopic fracture. However, the experimental data mentioned later indicate that such mutual interactions are also observed in some detected wavelets originated at the fracture time domain.

The related wave fields derived from equation (10a) are as follows:

$$\frac{\partial^2 \omega_k}{\partial x^s \partial x^s} - \frac{\partial^2 \omega_k}{V^2 \partial t^2} = 0, \qquad \frac{\partial^2 \tilde{\omega}_k}{\partial x^s \partial x^s} - \frac{\partial^2 \tilde{\omega}_k}{V^2 \partial t^2} = 0. \tag{11}$$

Constitutive and Balance Laws for Asymmetric Deformations

Strains

First, considering the symmetric part of the deformation field, $D_{(ki)} = E_{ik}$ (equation 1), we will assume the classical constitutive law,

$$S_{(kl)} = \lambda \delta_{kl} E_{ss} + 2\mu E_{kl} \tag{12}$$

but there is no problem to include into it the appropriate linear deviations related to viscoplastic effects.

From the classical motion equation for the symmetric part of stresses

$$\frac{\partial}{\partial x_k} S_{(kl)} = \rho \frac{\partial^2}{\partial t^2} u_l + F_l \tag{13}$$

with the following scalar and vector potentials

$$\begin{aligned} u_l &= l^2 \frac{\partial}{\partial x_l} \varphi + l^2 \varepsilon_{lps} \frac{\partial}{\partial x_p} \psi_s, \\ F_l &= l^2 \frac{\partial}{\partial x_l} \Phi + l^2 \varepsilon_{lps} \frac{\partial}{\partial x_p} \Psi_s \end{aligned} \tag{14}$$

we have

$$\begin{aligned} (\lambda + 2\mu) \frac{\partial^2}{\partial x_k \partial x_k} \varphi &= \rho \ddot{\varphi} + \Phi, \\ \mu \frac{\partial^2}{\partial x_k \partial x_k} \psi_s &= \rho \ddot{\psi}_s + \Psi_s, \end{aligned} \tag{15}$$

where we have introduced the intrinsic length unit l (characteristic Cosserat length) and the potentials Φ and Ψ_s for the axial strain and body forces; we have also introduced the conditions $\frac{\partial}{\partial x_s} \psi_s = 0$ and $\frac{\partial}{\partial x_s} \Psi_s = 0$.

The strain tensor can be presented with the help of the introduced potentials as follows:

$$\begin{aligned} E_{lq} = {} & l^2 \frac{\partial^2}{\partial x_l \partial x_q} \varphi + \frac{1}{2} l^2 \varepsilon_{lps} \frac{\partial^2}{\partial x_p \partial x_q} \psi_s \\ & + \frac{1}{2} l^2 \varepsilon_{qps} \frac{\partial^2}{\partial x_p \partial x_l} \psi_s. \end{aligned} \tag{16}$$

Dividing this expression into the axial and deviatoric parts we obtain

$$\begin{aligned} E_{kk} &= l^2 \frac{\partial^2 \varphi}{\partial x_s \partial x_s}, \\ E^D_{lq} &= l^2 \left[\frac{\partial^2 \varphi}{\partial x_l \partial x_q} - \frac{\delta_{lq}}{3} \frac{\partial^2 \varphi}{\partial x_s \partial x_s} \right. \\ & \qquad \left. + \frac{1}{2} \frac{\partial}{\partial x_p} \left(\varepsilon_{lps} \frac{\partial}{\partial x_q} + \varepsilon_{qps} \frac{\partial}{\partial x_l} \right) \psi_s \right], \end{aligned} \tag{17}$$

and the motion equations become

$$(\lambda + 2\mu) \Delta E_{kk} - \rho \frac{\partial^2 E_{kk}}{\partial t^2} = l^2 \Delta \Phi \tag{18a}$$

$$\begin{aligned} & (\lambda + \mu) \left(\frac{\partial^2 E_{ss}}{\partial x_l \partial x_q} - \frac{\delta_{lq}}{3} \frac{\partial^2 E_{ss}}{\partial x_k \partial x_k} \right) + \mu \frac{\partial^2 E^D_{lq}}{\partial x_k \partial x_k} - \rho \frac{\partial^2 E^D_{lq}}{\partial t^2} \\ & \quad = l^2 \left(\frac{\partial^2 \Phi}{\partial x_l \partial x_q} - \frac{\delta_{lq} \Delta \Phi}{3} \right) + \frac{l^2 \partial}{2 \partial x_p} \left(\varepsilon_{lps} \frac{\partial \Psi_s}{\partial x_q} + \varepsilon_{qps} \frac{\partial \Psi_s}{\partial x_l} \right). \end{aligned} \tag{18b}$$

The latter is the wave equation for the deviatoric strain tensor.

However, as we already mentioned, the deviatoric strains might also be described by the twist vector field, $\tilde{\omega}_s$; the related homogeneous wave equation has been derived (equation 11). Now, despite the fact that the strain shear tensor is symmetric, and the twist tensor is antisymmetric (equations 8 and 9), we may see their mutual correspondence, $E^D_{lq} \leftrightarrow \tilde{\omega}_{lq}$. The wave equation for twist (equation 11) may be written (similarly to equations 8 and 9) in the 4D form

$$\begin{aligned} & \mu \frac{\partial^2 \tilde{\omega}_{\lambda\kappa}}{\partial x_k \partial x_k} - \rho \frac{\partial^2 \tilde{\omega}_{\lambda\kappa}}{\partial t^2} = Y_{\lambda\kappa}; \\ & \qquad \tilde{\omega}_{\lambda\kappa} = i \tilde{\omega}_{12} \gamma^1 + i \tilde{\omega}_{31} \gamma^2 + \tilde{\omega}_{23} \gamma^3, \end{aligned} \tag{19}$$

where $\tilde{\omega}_{12} = E^D_{12}$, $\tilde{\omega}_{23} = E^D_{23}$, and $\tilde{\omega}_{31} = E^D_{31}$. We have included the source field given by the external 4D antisymmetric tensor

$$\begin{aligned} Y_{\lambda\kappa} &= i Y_{12} \gamma^1 + i Y_{31} \gamma^2 + Y_{23} \gamma^3 \\ &= \begin{bmatrix} 0 & Y_{12} & -Y_{31} & -Y_{23} \\ -Y_{12} & 0 & Y_{23} & -Y_{31} \\ Y_{31} & -Y_{23} & 0 & -Y_{12} \\ Y_{23} & Y_{31} & Y_{12} & 0 \end{bmatrix} \end{aligned} \tag{20}$$

defined similarly to equation (8) with the help of the Dirac tensors and the following scalars:

$$\begin{aligned} Y_{12} &= l^2 \left[\frac{\partial^2}{\partial x_1 \partial x_2} \Phi + \frac{\partial}{2 \partial x_p} \left(\varepsilon_{1ps} \frac{\partial}{\partial x_2} + \varepsilon_{2ps} \frac{\partial}{\partial x_1} \right) \Psi_s \right], \\ Y_{31} &= l^2 \left[\frac{\partial^2}{\partial x_3 \partial x_1} \Phi + \frac{\partial}{2 \partial x_p} \left(\varepsilon_{3ps} \frac{\partial}{\partial x_1} + \varepsilon_{1ps} \frac{\partial}{\partial x_3} \right) \Psi_s \right], \\ Y_{23} &= l^2 \left[\frac{\partial^2}{\partial x_2 \partial x_3} \Phi + \frac{\partial}{2 \partial x_p} \left(\varepsilon_{2ps} \frac{\partial}{\partial x_3} + \varepsilon_{3ps} \frac{\partial}{\partial x_2} \right) \Psi_s \right]. \end{aligned}$$

Equation (18b) for the symmetric deviatoric strain tensor is equivalent to that for the antisymmetric twist tensor (equation 19); the latter helps us understand another kind of motion, namely the twist motion.

As we already mentioned according to its definition, the twist, $\tilde{\omega}_{\lambda\kappa}$, means the rotational oscillation of the off-diagonal shear axes of the deviatoric tensor, E^D_{lq}, with the related changes of the shear magnitude as caused by the internal microfracture and granulation processes (Fig. 1).

Rotations

Instead of the balance of moment of momentum, we will introduce the equations for the antisymmetric part of stresses. First, we consider the appropriate constitutive law between the antisymmetric stress and rotation related to the friction–fracture processes; we will call it Shimbo law (Shimbo, 1975, 1995)

$$S_{[kl]} = 2\mu\omega_{kl}. \tag{21}$$

Here, in place of rotation rigidity we put the value equal to rigidity modulus, μ, as we believe that the rotation motions are closely related to the S waves. The S-wave amplitudes reach maxima along the off-diagonal axes of shear tensor while the P-wave amplitudes reach maxima along the main shear axes; an axial deformation also contributes to the P waves with the displacements evenly distributed in space.

The motion equation related to the balance of antisymmetric stresses $S_{[ni]}$ replaces that for the stress moments; in this balance there enters (1) the divergence of rotation force moment acting on a body element due to the antisymmetric stresses (rotational moment of forces) and (2) the acceleration related to angular momentum (Teisseyre and Boratyński, 2003, 2006)

$$\varepsilon_{lki}\frac{\partial^2}{\partial x_k \partial x_n} S_{[ni]} = \rho\varepsilon_{lki}\frac{\partial^2}{\partial t^2}\omega_{ki} + \varepsilon_{lki}\rho K_{[ki]}, \tag{22}$$

where we have introduced the body force couples, $K_{[ki]}$, or the body force moment $K_{[l]} = \varepsilon_{lki}\rho K_{[ki]}$. With the compatibility condition for the antisymmetric fields,

$$\varepsilon_{imk}\varepsilon_{jns}\frac{\partial^2}{\partial x_m \partial x_n}\omega_{ks} = 0$$

introduced in a similar way as for the symmetric fields, we get

$$\begin{gathered}\frac{\partial^2 S_{[ki]}}{\partial x_s \partial x_s} = 2\rho\frac{\partial^2\omega_{ki}}{\partial t^2} + 2\rho K_{[ki]}, \quad \text{or} \\ \mu\frac{\partial^2\omega_{ki}}{\partial x_s \partial x_s} - \rho\frac{\partial^2\omega_{ki}}{\partial t^2} = \rho K_{[ki]}.\end{gathered} \tag{23}$$

These relations become equivalent to the balance of the moment of momentum when putting

$$\frac{1}{l^2}\frac{\partial M_{lk}}{\partial x_k} = \varepsilon_{lki}\frac{\partial^2 S_{[ni]}}{\partial x_k \partial x_n}, \quad \text{or} \quad M_{lk} = 2\mu l\varepsilon_{lki}\frac{\partial l\omega_{ni}}{\partial x_n},$$

where M_{lk} relates to the derivatives of rotation, and thus, the space differences in rotation field distribution lead to rotation moments, and constant $2\mu l$ relates for a given material to the rotation modulus.

In this way we have shown that the motion equation for the antisymmetric stresses can replace the balance law for the stress moments. In the asymmetric elastic continuum, the displacements and rotations are treated as independent fields; in a natural way the axial deformation field (e.g., thermal field) also enters, and a possible phase shift between displacement velocity and spin, or equivalently between twist motion and spin, originated in a source zone.

Concluding, we can write again the basic motion relations for the considered rotation and twist fields (according to equation 10a with the defective current term, J_k),

$$\varepsilon_{kps}\frac{\partial\omega_s}{\partial x_p} - \frac{1}{V}\frac{\partial}{\partial t}\tilde{\omega}_k = \frac{4\pi}{V}J_k, \qquad \varepsilon_{kps}\frac{\partial\tilde{\omega}_s}{\partial x_p} + \frac{1}{V}\frac{\partial}{\partial t}\omega_k = 0, \tag{24a}$$

their wave forms for the rotation vector (see equation 23)

$$\mu\frac{\partial^2}{\partial x_k \partial x_k}\omega_l - \rho\frac{\partial^2}{\partial t^2}\omega_l = \rho K_{[l]}, \tag{24b}$$

and the 3D form of equations (19) and (20) with $\tilde{\omega}_s = \{\tilde{\omega}_{23}, \tilde{\omega}_{31}, \tilde{\omega}_{12}\}$, $Y_s = \{Y_{23}, Y_{31}, Y_{12}\}$:

$$\mu\frac{\partial^2\tilde{\omega}_s}{\partial x_k \partial x_k} - \rho\frac{\partial^2\tilde{\omega}_s}{\partial t^2} = Y_s. \tag{24c}$$

Solutions of the Maxwell-like equation (24a) lead us to the geometrical orthogonality of the rotation and twist vectors or to another phase-type orthogonality assured by the phase shift of $\pi/2$,

$$\omega_s = \pm i\tilde{\omega}_s, \tag{25}$$

where $\omega_s = \omega_s^0 \exp[i(k_i x_i - \varpi t)$, $\tilde{\omega}_s = \tilde{\omega}_s^0 \exp[i(k_i x_i - \varpi t)]$ with the complex constants $\omega_s^0 = \mathrm{abs}(\omega_s^0)\exp(i\psi_s)$, $\tilde{\omega}_s^0 = \mathrm{abs}(\tilde{\omega}_s^0)\exp(i\varphi_s)$, and angular frequency ϖ.

The solution in equation (25) will be called the synchronization wave solution important at the extreme phenomena.

Extreme Deformations: Rotation—Twist Coaction

Now we can consider the extreme displacement deformation; according to equation (2) we have

$$\begin{aligned}D_{kl} &= E_{kl} + \omega_{kl} = E^0_{kl} + \chi^0\omega^0_{kl} \\ &= \frac{1}{2}\left(\frac{\partial u_l}{\partial x_k} + \frac{\partial u_k}{\partial x_l}\right) + \chi^0\frac{1}{2}\left(\frac{\partial u_l}{\partial x_k} - \frac{\partial u_k}{\partial x_l}\right),\end{aligned} \tag{26}$$

and we will consider, among others, the cases $\chi^0 = \pm 1$ and $\chi^0 = \pm i$.

Burgers and Frank Vectors

A fundamental approach to both the defect fields and dislocation and disclination densities has been made by Kossecka and De Witt (1977) based on the Kröner theory of continuum in which the total fields, S^T_{mq}, E^T_{mq}, ω^T_{mq}, are related directly to displacement derivatives,

$$E^T_{mq} = 1/2(\partial u_q/\partial x_m + \partial u_m/\partial x_q) \quad \text{and}$$
$$\omega^T_{mq} = 1/2(\partial u_q/\partial x_m - \partial u_m/\partial x_q).$$

In the Kröner theory the total fields are formed by the sums of the elastic (physical) fields and the self-fields (density of internal nuclei),

$$E^T_{mq} = E_{mq} + E^S_{mq}, \qquad \omega^T_{mq} = \omega_{mq} + \omega^S_{mq}.$$

Kossecka and De Witt (1974) have defined the total twist-bend tensor as based on the gradient of rotations,

$$\chi^T_{mq} = \frac{\partial \omega^T_q}{\partial x_m}; \qquad \chi^T_{mq} = \chi_{mq} + \chi^S_{mq}$$

and applied its self-part into the expressions for the disclosures given by the Burgers and Frank vectors

$$B_l = -\oint [E^S_{kl} - \varepsilon_{lqr}\chi^S_{kq} x_r] dl_k,$$
$$\Omega_q = -\oint \chi^S_{kq} dl_k = \theta_{pq} ds_p.$$

However, with this definition of the twist-bend tensor, both the Burgers and Frank vectors will vanish, and the dislocation and disclination fields would not exist when defining their densities directly from these disclosures (Kossecka and De Witt, 1977).

Instead, in our asymmetric theory with strains and rotations related to displacements via equation (2), we take another definition of the twist-bend tensor,

$$\chi_{mq} = \varepsilon_{qsk} \frac{\partial \omega_{mk}}{\partial x_s} \tag{27}$$

and accordingly, we obtain directly (see Teisseyre and Boratyński, 2003)

$$\begin{aligned} B_l &= \oint [E_{(kl)} - \varepsilon_{lqr}\chi_{kq} x_r]\, dl_k = \oint [E_{kl} + \omega_{kl}]\, dl_k, \\ \Omega_q &= \oint \chi_{kl}\, dl_k = \iint \theta_{pq}\, ds_p, \end{aligned} \tag{28}$$

where we may note that for the continuum with the asymmetric part of stresses, we are not restricted to the compatibility condition for the twist-bend tensor (Kleman, 1980).

Fields Coincidence

For the case of $\chi^0 = 1$, we are dealing with wave propagation governed by equation (24), and we have no contribution to defect content (equations 2 and 28),

$$B_l = \oint [E_{kl} + \omega_{kl}]\, dl_k = \frac{1}{2}\oint [E^0_{kl} + \omega^0_{kl}] = \oint \frac{\partial u_l}{\partial x_k}\, dl_k = 0,$$
$$B_l = \iint \left(\alpha_{pl} - \frac{1}{2}\delta_{pl}\alpha_{ss} \right) ds_p = 0, \qquad \alpha_{pl} = 0. \tag{29}$$

For the case $e^0 = 1$ and χ^0 in the range (0, 1), we may arrive at the extreme deformation for the displacement motions; let us take the following expression:

$$\begin{aligned} D_{12} &= E_{12} + \chi^0 \omega_{12} \\ &= \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1} \right) + \chi^0 \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2} - \frac{\partial u_2}{\partial x_1} \right), \end{aligned}$$

where for $\chi^0 = 0$ we have the shear deformation described by the double couples, while for $\chi^0 = 1$ a full coincidence of motions leads to the deformation expressed by the single couples.

Dislocations

In the next case, $\chi^0 = -1$, and with equation (2) we obtain

$$B_l = \oint [E_{kl} + \omega_{kl}] dl_k = \frac{1}{2}\oint [E^0_{kl} - \omega^0_{kl}]$$

arriving at the definition of density of dislocations, α,

$$\begin{aligned} \alpha_{pl} - \frac{1}{2}\delta_{pl}\alpha_{ss} &= \varepsilon_{pmk}\left(\frac{\partial E_{kl}}{\partial x_m} + \varepsilon_{klq}\chi_{mq} \right) \\ &= \varepsilon_{pmk}\left(\frac{\partial E_{kl}}{\partial x_m} + \frac{\partial \omega_{kl}}{\partial x_m} \right) \\ &= \varepsilon_{pmk}\left(\frac{\partial E^0_{kl}}{\partial x_m} - \frac{\partial \omega^0_{kl}}{\partial x_m} \right). \end{aligned} \tag{30}$$

This definition is adequate to those for the discrete edge and screw dislocation fields.

By virtue of the compatibility condition the disclination density vanishes (see equation 28),

$$\theta_{pq} = \varepsilon_{pmk} \frac{\partial \chi_{kq}}{\partial x_m} = \varepsilon_{pmk}\varepsilon_{qns} \frac{\partial^2 \omega_{ks}}{\partial x_m \partial x_n} = 0.$$

The formula for the dislocation density takes the form

$$\alpha_{pl} - \frac{1}{2}\delta_{pl}\alpha_{ss} = \varepsilon_{pmk} \frac{\partial^2 u_k}{\partial x_m \partial x_l}. \tag{31}$$

The constitutive relation of equation (15) supplemented with that for the antisymmetric stresses and rotations of equation (21) leads us from equation (30) to the following relation between the dislocation density and asymmetric stresses,

$$\alpha_{pl} - \frac{1}{2}\delta_{pl}\alpha_{ss} = \varepsilon_{pmk}\left(\frac{\partial E_{kl}}{\partial x_m} + \frac{\partial \omega_{kl}}{\partial x_m}\right) = \frac{\varepsilon_{pmk}}{2\mu}\frac{\partial}{\partial x_m}\left[\left(S_{(kl)} - \frac{\nu}{1+\nu}\delta_{kl}S_{ii}\right) + S_{[kl]}\right], \tag{32}$$

where the relations between the edge and screw dislocations and asymmetric stresses become (no summation over the uppercase indexes, e.g., P and PP)

$$\alpha_{pl} = \frac{\varepsilon_{pmk}}{2\mu}\frac{\partial}{\partial x_m}\left[\left(S_{(kl)} - \frac{\nu}{1+\nu}\delta_{kl}S_{ii}\right) + S_{[kl]}\right] \quad \text{and}$$
$$\alpha_{PP} = -\frac{\varepsilon_{Pmk}}{\mu}\frac{\partial}{\partial x_m}\left(S_{(kP)} + S_{[kP]}\right),$$

where ν is the Poisson coefficient. Both cases considered may lead to the formation of the respective slip discontinuities. Note that for the edge and screw dislocations we deal with different rotation nuclei.

Other definitions of dislocation density might be achieved through modification of the twist-bend tensor definition when introducing some additional parameter κ to equation (27)

$$\chi_{mq} = \kappa\varepsilon_{qsk}\frac{\partial \omega_{mk}}{\partial x_s};$$

note that in the classical theory with defects, one may encounter different definitions for the dislocation field (e.g., the Burgers and Nye dislocations).

The related dislocation field formed by the edge and screw dislocation densities presents a kind of extreme shear deformation.

Wave Fields and Synchronization

The twist and rotation wave fields lead to a common displacement field only for $\chi^0 = 1$ and $\chi^0 = \pm i$. The cases of $\chi^0 = \pm i$, representing the phase shift by $\pi/2$ between twist and rotation (displacement strain and displacement rotation), relate to the dynamic origin of wave processes. When considering the wave solution of equation (25) for the strain and rotation, we arrive at the synchronization solutions for the displacement strain and displacement rotation, and vice versa; starting with the synchronization solutions for the strain and rotation, we arrive at the wave solutions for displacement strain and displacement rotation.

Granulation and Fracture Processes

At an advanced deformation process, material properties change, and we shall modify the constitutive laws expressing the strain and rotation responses to asymmetric stresses (equations 15 and 21); first, by including the time rates of the respective fields and, in the final stage, by entirely replacing these fields by their time rates. The structure phase indexes shall be adequately introduced similarly to those discussed for the strain and rotation (equations 2 and 4). Such changes also occur due to an increase of defects and an onset of the material granulation processes. These changes run in different ways for the compressive and shear loads: in the first case the fragmentation processes efficiently lead to granulation of the material, while at a shear load there appears a remarkable increase of dislocations, dislocation arrays, and microfracture density. However, in both cases there is an effective interaction between the rotation and twist fields, leading to a kind of synchronization of the granulation and microfracture processes. That means that the rotation and twist wave propagation may to some extent assure a synchronization of fracture processes.

Under a compressive load the energy release relates to the fragmentation revealed in rotation and granulation processes, while the induced intrinsic shear motions attenuate each other. Reversely, under a shear load a common shear deformation along one zone attenuates the opposite rotation motions along the perpendicular zones; in this way a shear progress will prevail at only one shear fracture zone, leading to concentration of dislocations and microcracks. Thus, at the compressive load the total shear stress drop will be relatively small, while the rebound rotations will be released in fragmentation and radiation of rotation energy. At the shear load, a release of shear stresses prevails leading to microcracks and radiation of shear-strain energy.

Together with the microfracture processes, we recognize the importance of granulation processes related to rotations in the intermediate scale between the bond breaking processes (the microscale) and material fragmentation (a somewhat greater scale). Thus, under the action of shear load, a material granulation leads in a spectacular way to the formation of a narrow, long, mylonite zone. The coaction of spin and twist shear motions in bond breaking, granulation, and formation of mylonite material also helps us to understand the fracture transport phenomena; based on the standard asymmetric continuum theory, we can consider the material progressive crashing and granulation processes leading to the conditions that are more similar to fluid material; thus, we finally enter into a domain of the Navier–Stokes transport equations.

Thus, we may assume that the constitutive laws for rock asymmetric continuum, relations in equations (15) and (21), written for the deviatoric and the antisymmetric fields, will gradually change during fracture and granulation to those including the time-dependent processes,

$$\begin{aligned} \sigma S^D_{(ik)} + \tau \dot{S}^D_{(ik)} &= 2\mu E^D_{ik} + 2\eta \dot{E}^D_{ik}, \\ \sigma S_{[ik]} + \tau \dot{S}_{[ik]} &= 2\mu E_{ik} + 2\eta \dot{\omega}_{ik}, \end{aligned} \tag{33}$$

where the material constants may become related to the magnitudes of slip, u, and its rate, υ.

Near a final stage these changes might lead to the melted and granulated part of the mylonite material and to the constitutive laws including only the time rates,

$$\dot{S}_{(ik)} = 2\eta \dot{E}_{ik}, \qquad \dot{S}_{[ik]} = 2\eta \dot{\omega}_{ik}, \tag{34}$$

where we assume (for the sake of simplicity) that the mylonite material is incompressible.

In our reasoning the shear load creates the dynamic angular deformations, which may lead to the bond breaking processes and, finally, to the fracturing transport process. When studying the fault-slip solutions, we usually assume the friction constitutive law additionally introduced to the classical theory; we hope that with this new approach it will be possible to replace the friction constitutive law by the constitutive law joining the asymmetric stresses with spin and twist fields in the appropriate fracture and fragmentation regime.

In the mylonite zone we further assume that the bond breaks, and related slips precede the rebound spin and a release of rotation energy retarded in phase. We can expect an increasing granulation and changes in friction, which may be included in the antisymmetric part of the constitutive law.

This spin rebound motion retarded in phase in relation to the acting shears (twist motion) can be described by the synchronization solution for twist motion and spin (equation 25). In this way we may consider the synchronization of microfracture and granulation processes as caused by an influence of the synchronization waves inside an earthquake preparation domain; the microfracture processes under compression assure a common sense of the induced twist and spin motions revealed in granulation and fragmentation of material, while under shear load the appearing microfractures will prevail. We may expect that the twist and rotation waves (equation 24a) generated inside a preseismic source would be revealed in the different wavelets describing a coaction of these motions and deformations; as discussed in the former section, this coaction may be revealed in a common displacement field for $\chi^0 = 1$ (accumulation phase) and for $\chi^0 = \pm i$ (release phase). With a counterpart of the time rates in the constitutive relations, we arrive at a much more complicated system. Thus, we suppose that the fracture process could proceed with the alternating accumulation and release microprocesses. In such a situation the related twist and spin motions will appear alternately as pairs of wavelets being in phase, antiphase, or shifted in phase by $\pm\pi/2$.

Some Experimental Evidence

To compare the rotation and twist waves, we may use a system of rotation seismometers (Moriya and Marumo, 1998; Teisseyre *et al.*, 2003; Teisseyre, 2007; Wiszniowski and Teisseyre, 2008). At the ground surface these complex rotation fields are limited to motions around the vertical axis, and we may rely on the data from two perpendicularly oriented rotation seismographs. From each system consisting of two oppositely oriented horizontal pendulums with related sensors, we can find the spin motion as the mean value of data from these sensors and the twist motion from their difference; however, we should be aware that the twist amplitudes obtained in this way are exact only for the orientation of seismographs coinciding with the main shear axes. Other-

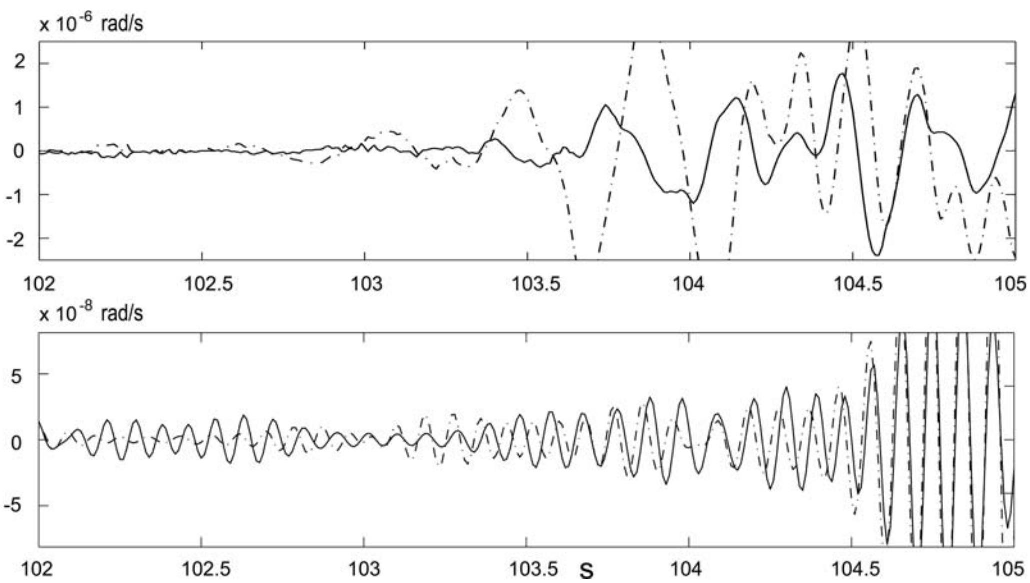

Figure 2. Seismic records of spin (broken lines) and twist motion (solid lines) at the Ioanina Observatory, Greece, 5 hr 14 min, 14 August 2003; the presented time interval, 102–105 sec, starts at the beginning of the event and displays the spin and twist motion as recorded and transformed from the two rotation seismographs of perpendicular orientation (the own period of pendulums is 1 sec); lower panel: the same fields in the frequency interval of 10–12.5 Hz (the beginning of the event ceases to be clearly visible).

wise, the twist data contain an uncertain scale of amplitudes. Nevertheless, such a system still permits us to compare the recorded rotation and twist wavelets as far as their shape in different frequency ranges is concerned; the twist wavelets present only the angular variations of the off-diagonal axes of shears.

In our examples we present the results related to some events observed in the Ioanina Observatory in Greece for the very near-field records (Fig. 2). In Figure 3 we show a typical example of the observed coincidences of the spin and twist motions, including the direct coincidences and those obtained after the Hilbert transformation of the twist record

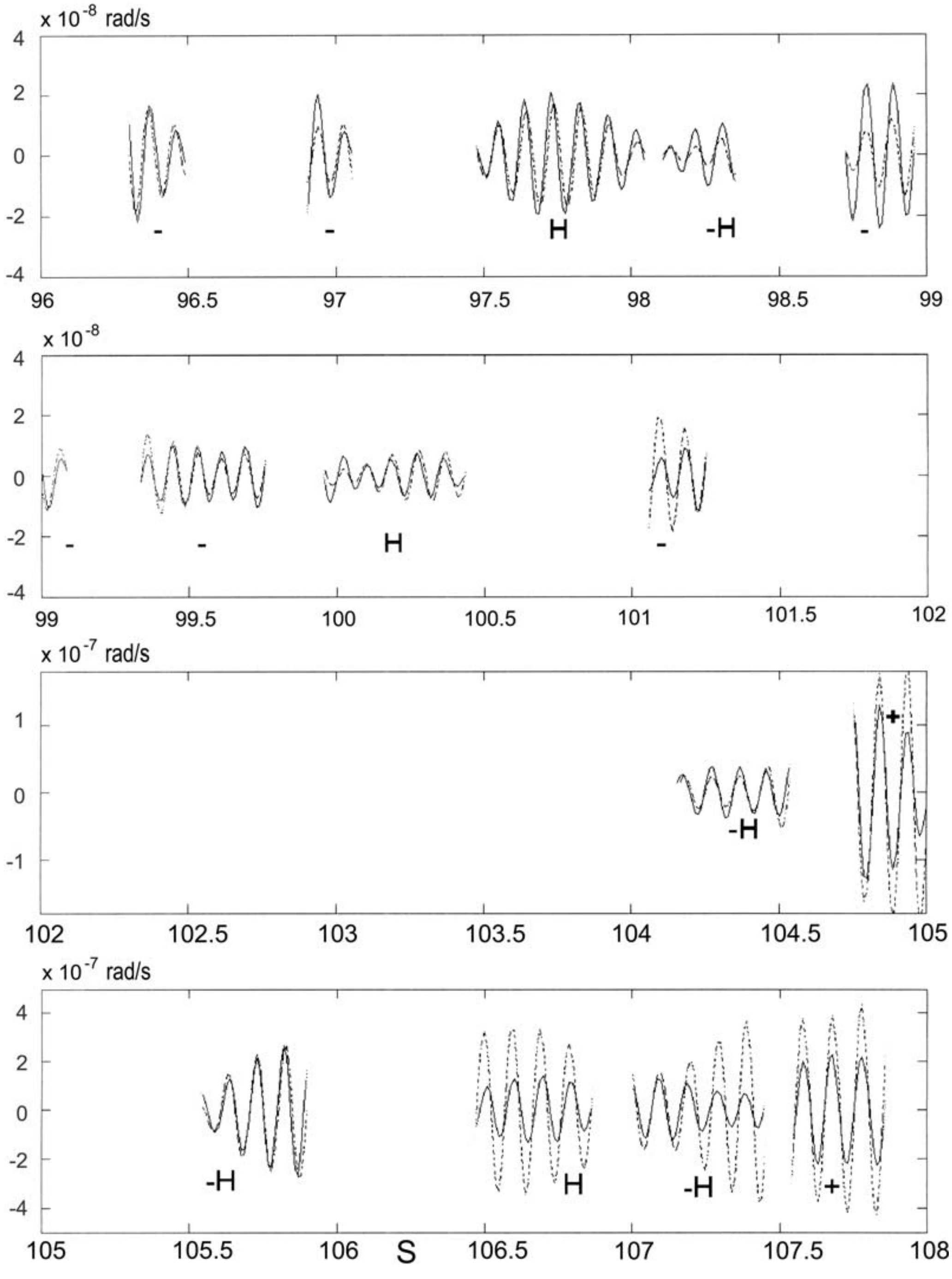

Figure 3. The fragments showing coincidences between the original spin records (broken lines) and the original and transformed twist motion records (solid lines) in the four time intervals arranged up–down as follows: before the event, (96–99 sec) and (99–102 sec), and just after its beginning, (102–105 sec) and (105–108 sec); the 102–105 sec interval corresponds to the lower panel in Figure 2. The fragments with the original twist records are marked with symbol $+$, those with the opposite sign by symbol $-$, while those after the Hilbert transformation by H (phase shift of $\pi/2$) and $-H$ (phase shift of $-\pi/2$).

(the phase shift of the record is $\pi/2$) and correspondingly, those with reverse signs.

The studied examples reveal certain intriguing results: some longer series of the synchronized wavelets of spin and twist directly and with the constant phase shifts of π, $\pi/2$, and $-\pi/2$ appear about a few seconds before the event and just in the first 1–2 sec from its beginning, while some wavelets with a direct correlation between twist and spin motions appear later (see Fig. 3). For a seismic noise, such coincidences are rare and seem to be only accidental. Numerous examples we studied confirm these rules; and therefore, some synchronized series we observed, with or without a phase shift (0, π, and $\pm\pi/2$), might be the candidates for immediate precursors of the very near events (e.g., rockbursts).

Conclusions

We have proven the existence of rotation waves using the continuum theory with an additional constitutive law relating rotations with an antisymmetric part of stresses; this remains valid even in a homogeneous elastic continuum.

We have defined the twist motion as the rotational oscillations of the main shear axes including the shear magnitude variations.

The wave equations derived for the twist and rotation motions have been considered in relation to the processes in seismic sources; these rotations at source zones help us understand the physics and geometry of fracture and the stress release processes in the precursory and rebound time domains. In particular, our considerations lead to a new description of the source processes including rotational effects. A microfracture process with synchronizing role of specific waves may be explained by a hypothesis of the twist-shear release followed by the rebound spin motion of the internal particles or grains constituting a continuum.

We have derived the relations between the asymmetric stresses and the dislocation density field. The conservation laws for the rotation and twist show the mutual relation of these fields; thus, the spin and twist motions are not completely independent but remain mutually correlated. Moreover, we may note that the derived wave equations for rotation and twist motions are similar to the electromagnetic wave equations.

Also worth mentioning is the fact that some methods of the continuum mechanics can be easily applied to a material with regular lattice structure. Varotsos and Alexopoulos (1986) have obtained many important results considering thermodynamics of the point defects in the regular lattice; an extension of their approach is the consideration on the thermodynamic functions related to the line defect density in a continuum with an additional super lattice related to these defects (Teisseyre and Majewski, 2001). Such an approach might also be useful in the continuum subjected to advanced deformation processes close to fracture.

Finally, it is easy to show how to construct the asymmetric fluid theory with the help of the molecular stresses (in our approach: asymmetric stress rates) and the related strain and rotation rates; in this way, various extreme phenomena could be theoretically explained (see Teisseyre, 2008).

Data and Resources

Seismograms used in Figures 2 and 3 were obtained during the experimental measurements in Ioannina, Greece by a team from the Institute of Geophysics, Polish Academy of Sciences, Warsaw, Poland, and are available at the Institute's archives.

Acknowledgments

Thanks are due to W.H.K Lee for his encouragement and two anonymous referees for thorough reviews and constructive comments, which helped the author to clarify and improve the article as well as to Anna Dziembowska for her assistance in preparing the manuscript for publication. The work was supported by the International Association for the promotion of cooperation with scientists from the New Independent States of the former Soviet Union Project 05-1000008-7889.

References

Bykov, V. G. (2006). Solitary waves in crustal faults and their application to earthquakes, in *Earthquake Source Asymmetry, Structural Media and Rotation Effects*, R. Teisseyre, M. Takeo, and E. Majewski (Editors), Springer, Berlin, 241–254

Bykov, V. G. (2008). Stick-slip and strain waves in the physics of earthquake rupture: Experiments and models, *Acta Geophys.* **56,** 270–285.

Cochard, A., H. Igel, B. Schuberth, W. Suryanto, A. Velikoseltsev, U. Schreiber, J. Wassermann, F. Scherbaum, and D. Vollmer (2006). Rotational motions in seismology: theory, observation, simulation, in *Earthquake Source Asymmetry, Structural Media and Rotation Effects*, R. Teisseyre, M. Takeo, and E. Majewski (Editors), Springer, Berlin, 391–411.

Cosserat, E., and F. Cosserat (1909). *Theorie des Corps Déformables*, A. Hermann, Paris.

Eringen, A. C. (1999). *Microcontinuum Field Theories I: Foundations and Solids*, Springer, Berlin, 325.

Eringen, A. C., and E. S. Suhubi (1964). Non-linear theory of simple micro-elastic solids, *Int. J. Eng. Sci.* **2,** 189–203.

Jones, W. L. (1973). Asymmetric wave-stress tensors and wave spin, *J. Fluid Mech.* **58,** 737–747.

Kleman, M. (1980). The general theory of disclinations, in *Dislocations of Solids*, Other Effects of Dislocations: Disclinations, F. R. N. Nabarro (Editor), Vol. **5**, North-Holland, Amsterdam, 243–297.

Kossecka, E., and R. DeWitt (1977). Disclination kinematic, *Arch. Mech.* **29,** 633–651.

Kröner, E. (1981). Continuum theory of defects, in *Physique des Defauts/Physics of Defects*, Les Houches, Session XXXV, 1980, R. Balian, M. Kleman, and J. P. Poirer (Editors), North Holland, Dordrecht.

Majewski, E. (2006). Seismic rotation waves: Spin and twist solitons, in *Earthquake Source Asymmetry, Structural Media and Rotation Effects*, R. Teisseyre, M. Takeo, and E. Majewski (Editors), Springer, Berlin, 255–272.

Mikhailov, D. N., and V. N. Nikolaevskii (2000). Tectonic waves of the rotational type generating seismic signals, *Izvestiya Phys. Sol. Earth* **36,** no. 11, 895–902.

Mindlin, R. D. (1965). On the equations of elastic materials with microstructure, *Int. J. Solids Struct.* **1,** 73.

Moriya, T., and T. Marumo (1998). Design for rotation seismometers and their calibration, Hokkaido University, *Geophys. Bull.* **61,** 99–106.

Nowacki, W. (1986). *Theory of Asymmetric Elasticity*, PWN and Pergamon Press, Warszawa, Oxford, 383 pp.

Shimbo, M. (1975). A geometrical formulation of asymmetric features in plasticity, Hokkaido University, *Bull. Fac. Eng.* **77,** 155–159.

Shimbo, M. (1995). Non-Riemannian geometrical approach to deformation and friction, in *Theory of Earthquake Premonitory and Fracture Processes*, R. Teisseyre (Editor), PWN (Polish Scientific Publishers), Warszawa, 520–528.

Teisseyre, K. P. (2007). Analysis of a group of seismic events using rotational components, *Acta Geophys.* **55,** 535–553.

Teisseyre, R. (1973). Earthquake processes in a micromorphic continuum, *Pure Appl. Geophys.* **102,** 15–28.

Teisseyre, R. (1974). Symmetric micromorphic continuum: Wave propagation, point source solutions and some applications to earthquake processes, in *Continuum Mechanics Aspects of Geodynamics and Rock Fracture Mechanics'*, P. Thoft-Christensen (Editor), D., Reidel Publ. Comp., Dordrecht, Holland/Boston, U.S., 201–244.

Teisseyre, R. (1995). Deformation and geometry, in *Theory of Earthquake Premonitory and Fracture Processes*, R. Teisseyre (Editor), PWN (Polish Scientific Publishers), Warszawa, 504–511.

Teisseyre, R. (2008). Introduction to asymmetric continuum: Dislocations in solids and extreme phenomena in fluids, *Acta Geophys.* **56,** 259–269.

Teisseyre, R., and W. Boratyński (2003). Continua with self-rotation nuclei: Evolution of asymmetric fields, *Mech. Res. Commun.* **30,** 235–240.

Teisseyre, R., and W. Boratyński (2006). Deviations from symmetry and elasticity: Asymmetric continuum mechanics, in *Earthquake Source Asymmetry, Structural Media and Rotation Effects*, R. Teisseyre, M. Takeo, and E. Majewski (Editors), Springer, Berlin, 31–42.

Teisseyre, R., and M. Górski (2007). Physics of basic motions in asymmetric continuum, *Acta Geophys.* **55,** 119–132.

Teisseyre, R., and M. Górski (2009). Fundamental deformations in asymmetric continuum, *Bull. Seismol. Soc. Am.* **99,** no. 2B, 1132–1136.

Teisseyre, R., and E. Majewski (2001). Thermodynamics of line defects and earthquake Thermodynamics, in *Earthquake Thermodynamics and Phase Transformations in the Earth's Interior*, International Geophysical Series, R. Teisseyre and E. Majewski (Editors), Vol. **76,** Academic Press, San Diego, 261–278.

Teisseyre, R., J. Suchcicki, K. P. Teisseyre, J. Wiszniowski, and P. Palangio (2003). Seismic rotation waves: Basic elements of the theory and recordings, *Ann. Geophys.* **46,** 671–685.

Varotsos, P. A., and K. D. Alexopoulos (1986). *Thermodynamics of Point Defects and their Relation with Bulk Properties*, North-Holland, Amsterdam, New York, 474 pp.

Wiszniowski, J., and R. Teisseyre (2008). Field invariant representation: Dirac tensors, in *Physics of Asymmetric Continuum: Extreme and Fracture Processes—Earthquake Rotation and Soliton Waves*, R. Teisseyre, H. Nagahama, and E. Majewski (Editors), Springer, Berlin, 85–94.

Institute of Geophysics
Polish Academy of Sciences
Księcia Janusza 64
01-452 Warszawa, Poland
rt@igf.edu.pl

Manuscript received 30 May 2008