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Short Note

Rotational Motions of Seismic Surface Waves in a Laterally Heterogeneous Earth

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Abstract A number of recent studies have analyzed seismic rotational data using a relationship between transverse acceleration and rotation rate derived for a homogeneous full space. In this study we explore this relationship further theoretically by presenting a full ray theory (FRT) method to simulate rotational motions of fundamental mode seismic surface waves in smooth, laterally heterogeneous Earth models. In the ray picture of wave propagation the vertical component of the rotational rate motion of fundamental mode Love waves is obtained by dividing the transverse component of ground acceleration by the Love-wave local phase velocity beneath the seismic recording station. We illustrate the method with examples of theoretical calculations of $T \approx 40$ sec rotational rate ground motions of fundamental Love waves using the crust model CRUST2.0 combined with the mantle model S20RTS for the 25 September 2003 M 8.1 Tokachi-oki earthquake, Japan. FRT rotation synthetics match complete calculations using the spectral-element method very well and fit real data reasonably well. Furthermore, we show that the effect of realistic local structure beneath receivers on rotational motions is strong enough to be observable. FRT calculations could potentially help to determine Love-wave local dispersion curves and, thus, to estimate the 1D local shear velocity structure beneath seismic stations from point measurements of rotational rate and acceleration ground motions.

Introduction

With the ever-increasing amount of seismic broadband data assembled from high-quality global networks (Incorporated Research Institutions for Seismology [IRIS], GEOSCOPE, MedNet, Geofone, and other national networks) 3D global images of the Earth's mantle have greatly progressed over the last 30 yr. However, standard seismic observations are restricted to three components of translations, despite the fact that in theory the recovery of the complete ground motion requires the observation of three additional components of rotations and six components of strain (e.g., Trifunac and Todorovska, 2001; Aki and Richards, 2002).

In past years, rotation sensor technology has been improving in a way that may lead to the development of routine sensors for three additional rotational motion components useful for seismological applications. Using ring-laser technology, rotation rates as small as 10^{-10} $\mathrm{rad sec^{-1}}/\sqrt{\mathrm{Hz}}$ can be observed (e.g., Schreiber *et al.*, 2006). These recent observations of rotational motions showed that the rotational measurements are consistent with collocated observations of translations within the framework of linear elasticity (e.g., Igel *et al.*, 2005; Cochard *et al.*, 2006; Igel *et al.*, 2007), following the earlier pioneering observations of earthquake-induced rotational motions by McLeod *et al.* (1998) and by Pancha *et al.* (2000).

Assuming plane wave propagation in a homogeneous and isotropic full space, one can show that at a given station the transverse acceleration and rotation rate (around a vertical axis) are in phase, and their ratio is proportional to the horizontal phase velocity (e.g., Pancha *et al.*, 2000; Igel *et al.*, 2005). A similar relationship between strain and displacements can be used to determine horizontal phase velocities (e.g., Mikumo and Aki, 1964; Gomberg and Agnew, 1996). Igel *et al.* (2005, 2007) and Cochard *et al.* (2006) exploited this relationship to estimate horizontal phase velocities in sliding time windows along the observed time series. Comparison with synthetic traces (rotations and translations) and phase velocities determined in the same way showed good agreement with the observations. These initial results suggested that the determination of Love-wave dispersion curves (and, thus, information on local 1D shear velocity

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structure) may be possible. This is remarkable in the sense that the estimation of the local dispersion curve would thus be based upon a point measurement rather than the observation of the wave field across an array of seismometers. These experimental results are further supported by the recent application of the adjoint method to the combined measurement of rotations and translations (Fichtner and Igel, 2009). This study has shown that the sensitivity of such measurement to structural properties is concentrated around the receiver location, indicating the possibility of developing a novel kind of seismic tomography based upon joint measurements of translations and rotations. In the light of this, it is worth further theoretical exploration of the relationship between rotations and translations and their use to estimate Love-wave dispersion curves.

In this study we simulate fundamental Love-wave rotation and translation motions in global Earth models (both spherically symmetric and 3D). We start by describing rotational motions of seismic waves in a spherically symmetric Earth model using a mode-summation formalism. We then extend this approach to smooth, laterally varying Earth models using surface wave full ray theory (FRT) (Woodhouse, 1974; Woodhouse and Wong, 1986; Tromp and Dahlen, 1992a,b; Ferreira and Woodhouse, 2007). We compare FRT calculations with spectral-element method (SEM) simulations (Komatitsch and Tromp, 2002a,b) and with real data. Furthermore, we use FRT to illustrate the influence of various local structures on Love-wave rotational motions, and we discuss the potential use of FRT in real data applications.

Modeling

Free Oscillations in Spherically Symmetric Earth Models

In a spherically symmetric, nonrotating, perfectly elastic, isotropic (SNREI) Earth model, and for a point source, the ground velocity following an earthquake can be obtained by calculating the excitation of each mode of vibration and then summing the ground velocity associated with each mode at the receiver (e.g., Gilbert, 1970; Gilbert and Dziewonski, 1975). For a given mode of vibration ω_l^m having unit excitation we can write the ground velocity spectrum as

$$\dot{\mathbf{s}}(\mathbf{r},\omega) = \omega U \hat{\mathbf{r}} Y_l^m + \omega V \frac{\nabla_1 Y_l^m}{\sqrt{l(l+1)}} - \omega W \frac{\hat{\mathbf{r}} \times \nabla_1 Y_l^m}{\sqrt{l(l+1)}}, \quad (1)$$

where U, V, and W are the vertical, longitudinal, and transverse scalar modal eigenfunctions evaluated at the Earth's surface, $Y_l^m = Y_l^m(\theta,\phi)$ are the real scalar spherical harmonics, and ∇_1 represents the tangential gradient operator on the unit sphere, $\nabla_1 = \hat{\theta}\partial_\theta + \hat{\mathbf{\Phi}}\mathrm{cosec}\theta\partial_\phi$ (for more details see, e.g., Gilbert and Dziewonski, 1975). It follows that the curl of the ground velocity—the rotational rate motion—is given by (see, e.g., Dahlen and Tromp, 1998)

$$\frac{1}{2}\nabla \times \dot{\mathbf{s}} = \omega \frac{\sqrt{l(l+1)}}{2a} W \hat{\mathbf{r}} Y_l^m + \frac{\omega}{2}\left(\frac{dW}{dr} + \frac{W}{a}\right)\frac{\nabla_1 Y_l^m}{\sqrt{l(l+1)}} + \frac{\omega}{2}\left[\frac{dV}{dr} + \frac{1}{a}(V - \sqrt{l(l+1)}U)\right]\frac{\hat{\mathbf{r}} \times \nabla_1 Y_l^m}{\sqrt{l(l+1)}}, \quad (2)$$

where a is the radius of the Earth. Equations (1) and (2) describe the velocity and rotational rate motion response as a superposition of standing waves or normal modes. Alternatively, it is possible to decompose the response into a sum of traveling waves, which corresponds to the asymptotic limit of large angular order l. For large angular orders $l \gg 1$, the vertical component of the rotation rate vector $\dot{\Omega}_r$ for a given mode of vibration ω_l^m becomes

$$\hat{\mathbf{r}} \cdot \frac{1}{2}\nabla \times \dot{\mathbf{s}} = \dot{\Omega}_r = \omega \frac{(l+1/2)}{2a} W Y_l^m = \omega \frac{k}{2} W Y_l^m, \quad (3)$$

where we have used the fact that the asymptotic wavenumber k of a high-degree scalar spherical harmonic is $k = \frac{l+1/2}{a} = \frac{\lambda}{a}$ (see, e.g., Dahlen and Tromp, 1998). On the other hand, the transverse component of ground acceleration, $\ddot{s}_T$, for a given mode of vibration ω_l^m is given by (see equation 1)

$$\ddot{s}_T = \omega^2 W Y_l^m. \quad (4)$$

Thus, dividing equation (3) by equation (4) it follows that the ratio of vertical rotational motion to transverse ground velocity is given by

$$\frac{\dot{\Omega}_r(\omega)}{\ddot{s}_T(\omega)} = \frac{k}{2\omega} = \frac{1}{2c_T(\omega)}, \quad (5)$$

where c_T is the wave's horizontal phase velocity in the transverse direction. This shows that for an SNREI Earth model, at any frequency, the ratio between vertical rotational rate motion and transverse ground acceleration is given by half of the inverse of the transverse wave phase velocity.

Surface Waves in Smooth, Laterally Heterogeneous Earth Models

Let us now focus on seismic surface waves, which correspond to an asymptotic limit of the complete sum over the normal-mode spectrum in equation (1). From such a limit (e.g., Dahlen and Tromp, 1998) and using equation (5) it follows that the vertical component of the rotation rate motion vector of Love waves in an SNREI Earth model in the frequency domain is given by

$$\dot{\Omega}_r(\mathbf{x}_s, \mathbf{x}_r; \omega) = \frac{\omega^2}{2cC}\mathbf{M}{:}E_s^* e^{i\frac{\pi}{4}}\sqrt{\frac{\lambda}{8\pi|\sin\Delta|}}e^{-i\lambda\Delta - \frac{\alpha_l\Delta}{C_l}}W, \quad (6)$$

for a source at position $\mathbf{x}_s$ and a receiver at position $\mathbf{x}_r$. C is the Love-wave angular group velocity, c is the Love-wave phase velocity, $\mathbf{M}$ is the seismic moment tensor in spherical coordinates, and $\mathbf{E}_s$ is the Love-wave strain tensor at the source, which involves the eigenfunction W and its radial derivative evaluated at the source's location. Δ is the angular epicentral distance and α_l is a decay factor related with the decay rate per cycle of the equivalent free oscillations, Q, $\alpha_l = \frac{\omega}{2Q}$.

For a smooth, laterally heterogeneous, transversely isotropic medium, we can write the FRT (or Jeffreys–Wentzel–Kramers–Brillouin) rotation rate motion in a form analogous to equation (6) (e.g., Tromp and Dahlen, 1992b; Ferreira and Woodhouse, 2007). The vertical component of the rotational rate motion response to a moment tensor $\mathbf{M}$ at hypocentral location $\mathbf{x}_s$ then becomes

$$\dot{\Omega}_r(\mathbf{x}_s, \mathbf{x}_r; \omega) = \frac{\omega^2}{2} \underbrace{\frac{1}{\sqrt{C_s}} \mathbf{M}{:}E_s^* e^{i\frac{\pi}{4}}}_{\text{source}} \underbrace{\sqrt{\frac{\lambda}{8\pi S}} e^{\int_{\text{path}} (-i\lambda_l - \frac{\alpha_l}{C_l})\, dl}}_{\text{path}} \underbrace{\frac{W_r}{c_r \sqrt{C_r}}}_{\text{receiver}}. \quad (7)$$

Equation (7) involves a source, a path, and a receiver term, and it differs from the expression for ground acceleration by a factor of $\frac{1}{2} c_r$, where c_r is the Love-wave phase velocity at the receiver. We use a subscript s to denote evaluation at the source and a subscript r to denote evaluation at the receiver.

The Love-wave strain tensor evaluated at the source, $\mathbf{E}_s$, now involves the local eigenfunction, W_s, and its radial derivative evaluated at the source's location (for explicit expressions see, e.g., Ferreira and Woodhouse [2007]). In practice we use the 1D depth profile structure at the source's epicentral location, from the surface of the Earth down to its center as predicted by existing 3D tomographic models. The local eigenfunctions are calculated using a standard normal-mode algorithm (Woodhouse, 1988), and it has been shown that the effect of local structure at the source on surface wave amplitudes is significant (Ferreira and Woodhouse, 2007).

The path amplitude factor, $\sqrt{S}$, accounts for the geometrical spreading of monochromatic surface waves on a sphere, which reflects the focusing and defocusing of surface wave ray tubes due to heterogeneity. The path phase factor, $e^{\int_{\text{path}} (-i\lambda_l - \frac{\alpha_l}{C_l})\, dl}$, represents the phase delay due to propagation along the ray path, taking into account the anelastic attenuation of the waves. The path phase and amplitude are obtained by means of exact kinematic and dynamic raytracing on the sphere (see e.g., Woodhouse and Wong, 1986).

The receiver term depends on the local displacement eigenfunction at the receiver, W_r, and on the local phase and group velocities. Like for the source term, all the local eigenfunctions and local phase and group velocities are calculated, for a given local model, using a standard normal-mode algorithm (for details on the numerical implementation see Ferreira and Woodhouse [2007]). Calculations are carried out in the frequency domain at 13 frequencies between 5 and 25 mHz, followed by spline interpolation in frequency. The time-domain Love-wave rotational motion synthetics are obtained through an inverse Fourier transformation.

Surface wave FRT assumes that there is no energy transfer from one mode branch to another, nor from Love to Rayleigh waves. While we expect that neglecting mode coupling is a good approximation for fundamental mode surface waves (see, e.g., Tromp, 1994), for higher overtone branches mode coupling is more relevant. In this article we focus on fundamental mode surface waves only, which are the prime data to image the upper mantle of the Earth, so we do not address the issue of mode coupling in detail.

A number of theoretical studies have addressed FRT to calculate seismic displacement seismograms (Woodhouse, 1974; Woodhouse and Wong, 1986; Tromp and Dahlen, 1992b; Ferreira and Woodhouse, 2007), but only two studies have actually implemented it numerically and showed examples of ground displacement synthetic seismogram calculations (Wang and Dahlen, 1994; Ferreira and Woodhouse, 2007). In the next sections we will show illustrative examples of the practical use of FRT to calculate rotational ground motions of fundamental Love waves for a smooth laterally varying Earth model.

Examples

Rotations at Station WET, Wettzell

Here, we consider the vertical component of fundamental Love-wave rotational rate motions at the station Wettzell, WET, at a latitude of 49.15° N and a longitude of 12.88° E, for the 25 September 2003 M 8.1 Tokachi-oki earthquake (Fig. 1). The station WET is equipped with, among other instruments, a ring-laser rotation sensor and a seismic broadband sensor. We calculate rotations using earthquake source parameters from the Global Centroid Moment Tensor catalog (Dziewonski *et al.*, 1981), and we use the 3D mantle model S20RTS (Ritsema *et al.*, 1999) combined with the global crust model CRUST2.0 (Bassin *et al.*, 2000). Within the crust, the local 1D shear-wave velocity model beneath WET predicted by CRUST2.0 and S20RTS differs substantially from the global spherically symmetric model, the preliminary reference Earth model (PREM; Dziewonski and Anderson, 1981) (Fig. 2a). Thus, the fundamental Love-wave dispersion curve beneath WET obtained using the 1D local model is different from the one obtained using PREM, particularly at higher frequencies, which are more sensitive to the crustal structure (Fig. 2b). Figure 3 compares rotational motions calculated using the SEM (Komatitsch and Tromp, 2002a,b) in the 3D Earth model with calculations using FRT in the same model and with mode-summation calculations using PREM. All the traces are band-pass filtered around $T = 40$ sec. We see that there is poor agreement between the PREM synthetic and the SEM synthetic, with large phase and amplitude differences between the two traces. In contrast, FRT calculations match the SEM synthetic well. There are some discrepancies between the two traces in the early part of the wave trains, probably because in this study FRT calculations do not include overtones (only fun-

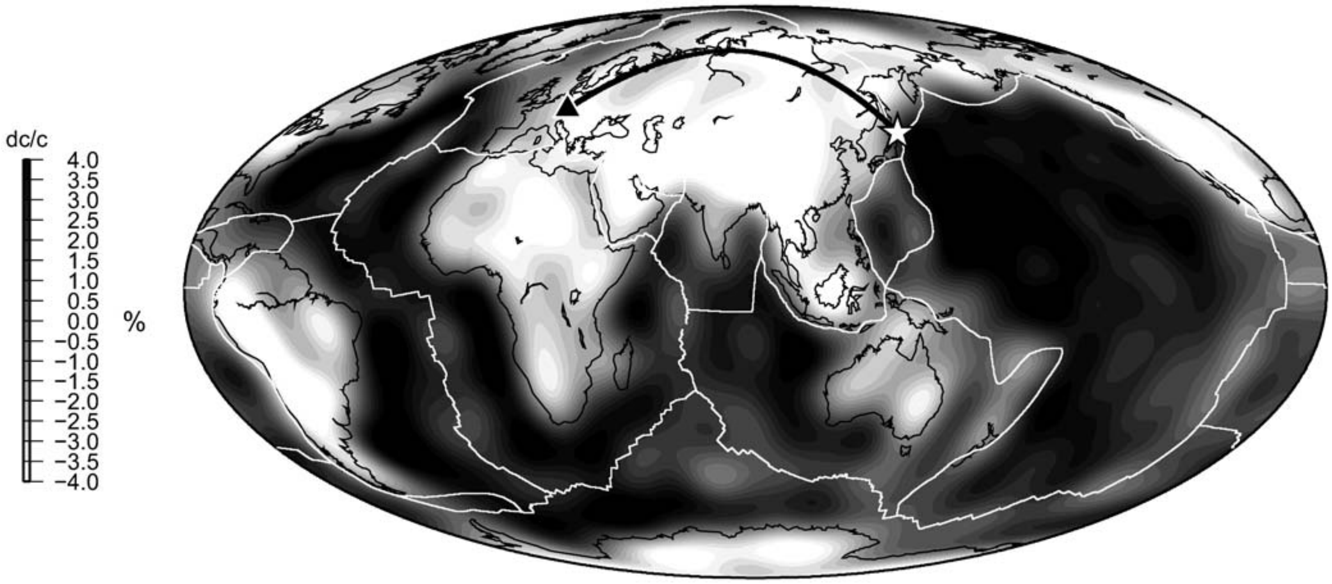

Figure 1. Great-circle path (solid curve) between the epicenter of the 25 September 2003 M 8.1 Tokachi-oki earthquake, Japan (star), and seismic station WET (latitude = 49.15°, longitude = 12.88°) near Wettzell, Germany (triangle), at an epicentral distance of 80°. These are superimposed on a $T = 40$ sec fundamental Love-wave phase velocity map calculated from the crust model CRUST2.0 combined with the mantle model S20RTS.

damental modes are calculated). We then compare FRT calculations with real data recorded by the ring-laser instrument at WET (Fig. 4). There is reasonable agreement between the two waveforms, with some discrepancy in the amplitudes. Because the FRT calculations agree well with SEM predictions, these discrepancies must be due to uncertainties in the earthquake source model and/or to incorrect Earth structure rather than to theoretical limitations.

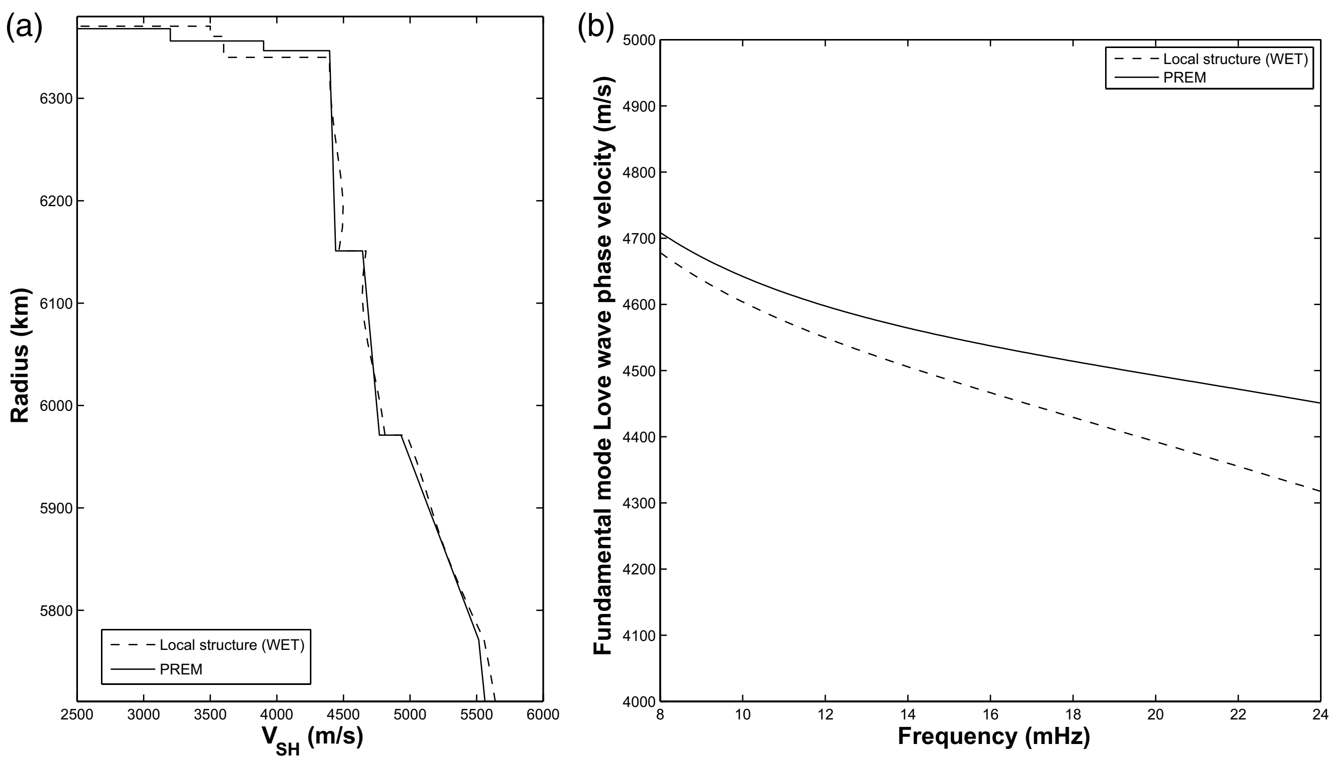

Figure 2. (a) Comparison between PREM shear-wave velocity model (solid line) and the local model beneath the seismic station WET predicted by the crustal model CRUST2.0 combined with the mantle model S20RTS (dashed line). (b) Theoretical local dispersion of fundamental Love waves as predicted by PREM (solid line) and by the local model beneath WET (dashed line).

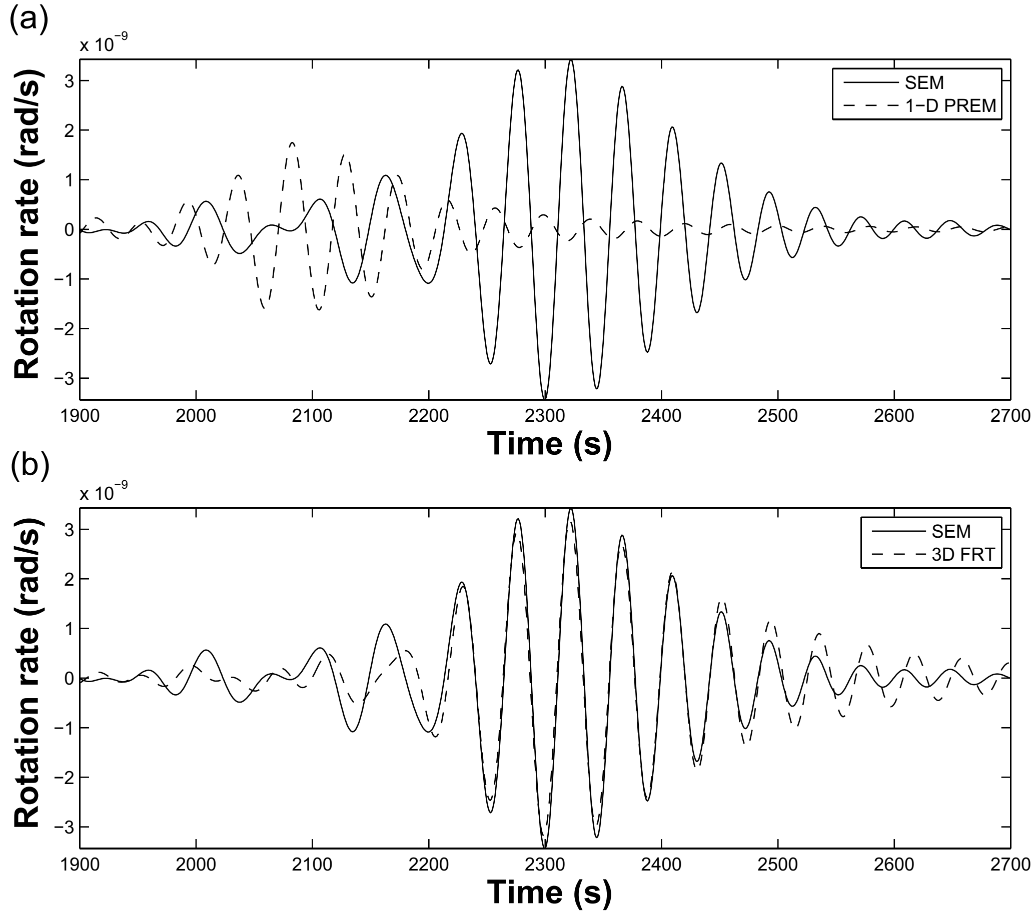

Figure 3. (a) Comparison between SEM rotation rate synthetics around a vertical axis using the crustal model CRUST2.0 combined with the mantle model S20RTS (solid curve) and mode-summation PREM rotation rate synthetics (dashed curve) following the 25 September 2003 M 8.1 Tokachi-oki earthquake. (b) Comparison between SEM rotation synthetics using the crustal model CRUST2.0 and the mantle model S20RTS (solid curve) and FRT rotation rate synthetics using the same Earth model (dashed curve). All the traces have been band-pass filtered around $T = 40$ sec.

Influence of Local Structure beneath the Receiver

Because FRT allows one to express waveforms in terms of a source, a path, and a receiver term, it is an ideal tool to study the influence of local structure on observed waveforms. Here, we give examples of the effect of local structure beneath receivers on rotational ground motions by comparing two types of fundamental Love-wave calculations:

1. FRT synthetics in the 3D Earth model;
2. Receiver PREM synthetics, which are calculated using FRT upon the 3D Earth model applied only to the source and the path terms in equation (7). The receiver term entering the FRT formulation (see equation 7) is calculated using PREM.

Thus, for a given earthquake and recording station, discrepancies between these two types of theoretical calculations are only due to deviations of the local structure beneath the station from PREM.

We carry out calculations for the 25 September 2003 M 8.1 Tokachi-oki earthquake at a number of stations located in distinct tectonic environments with different crustal and mantle structures. Figure 5 shows comparisons of vertical component rotational rate motion FRT synthetics with Receiver PREM synthetics at four stations from the Global Seismic Network. Stations WET and LSA are located in distinct environments within the interior of continents, with LSA being in the Tibetan plateau. Station GUMO is located in the Marianas ocean island, and station LCO is located in the continental margin of Chile. For most stations we see that deviations in the local structure beneath the receiver from PREM lead to considerable differences in amplitude between the two types of rotational motion synthetics. This emphasizes the strong sensitivity of rotational motions to local structure at the receiver even for those predicted by smooth, long-wavelength Earth models such as CRUST2.0 combined with S20RTS. Comparing the differential synthetics for stations LSA and LCO with those in Figure 4, we see that the differences due to local structure at the receiver are comparable to differences between realistic 3D calculations and real data. Hence, the effect of local structure at the receiver on Love-wave rotational motion records must be strong enough to be observable, suggesting that rotational ground records

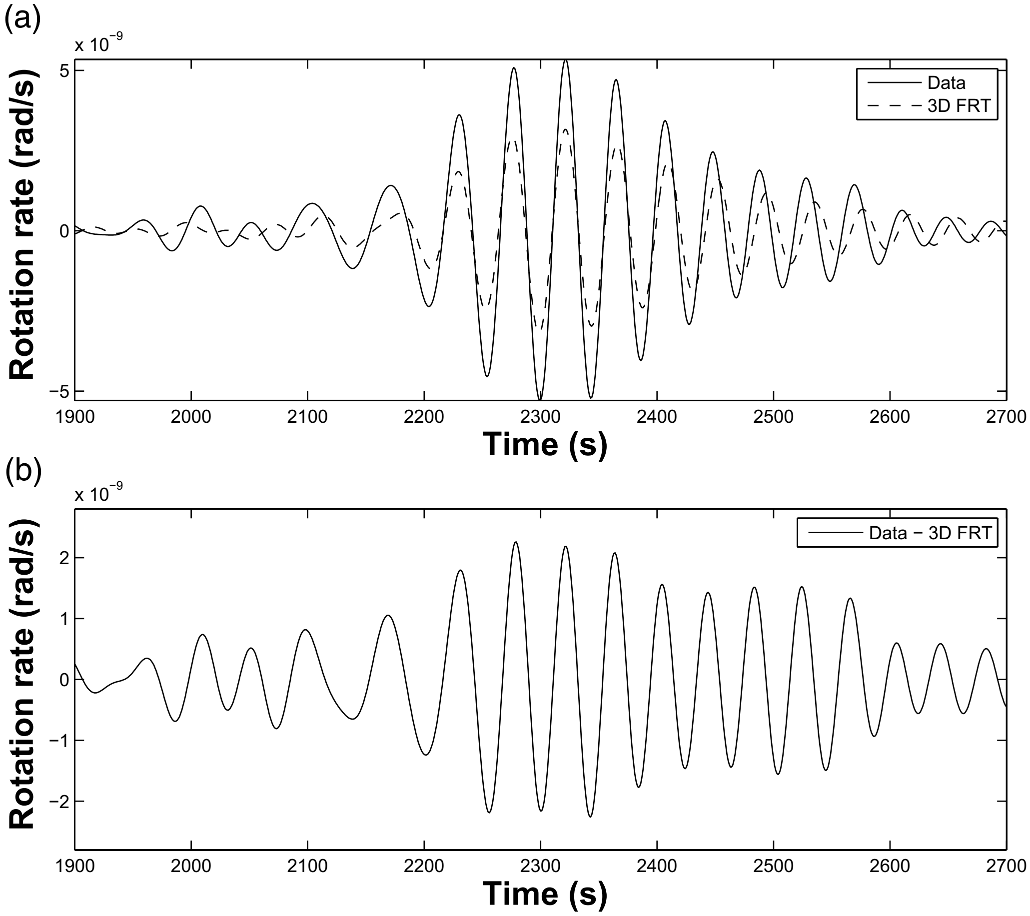

Figure 4. (a) Comparison between observed rotation rate motions around a vertical axis at station WET (solid curve) and FRT rotation rate synthetics calculated using the crust model CRUST2.0 combined with the mantle model S20RTS (dashed curve) for the 25 September 2003 M 8.1 Tokachi-oki earthquake. (b) Difference between the two traces in the top diagram. All the traces have been band-pass filtered around $T = 40$ sec.

can be used in practical applications to estimate the local structure beneath seismic receivers, particularly when combined with records of transverse ground acceleration at the same site.

Discussion

While applications to real data are beyond the scope of this article, it is worthwhile to discuss some important practical aspects on how this work may be extended to real records of Love-wave rotational rate and ground velocity (and particularly to their amplitude spectra analysis). Seismic surface wave amplitudes are rarely used in seismological applications because they are difficult to model and to measure. For example, sharp lateral variations in Earth structure along the source–receiver propagation path may lead to complications such as ray focusing and defocusing, ray bending, multipathing, coupling of Rayleigh waves with Love waves, and interference with overtones and scattered waves. Thus, in order to isolate the fundamental mode of Love waves in rotation rate and acceleration records, careful data selection and data analysis is needed to minimize these effects. For example, it is important to use earthquakes that are shallow to minimize overtone contamination and mode coupling effects and that are large enough to generate low-noise surface Love waves. FRT is a valuable tool to select and analyze the data. In contrast to complete numerical wave field calculations such as SEM, it allows us to simulate the propagation of fundamental mode Love waves alone using realistic 3D Earth models. Thus, waveform comparisons between such calculations and real data allow us to assess the level of overtone contamination and interferences between the fundamental mode and other waves. Moreover, ray focusing and defocusing and ray bending effects are taken into account in FRT calculations and a multipathing detection is also included, enabling us to discard source–receiver pairs for which multipathing is predicted to occur (Ferreira, 2005; Ferreira and Woodhouse, 2007). FRT calculations do not require substantial computational facilities and are computationally inexpensive, so it is feasible to use such calculations to select and analyze large data sets.

In contrast to global tomography, the problem of estimating local 1D models involves a relatively small number

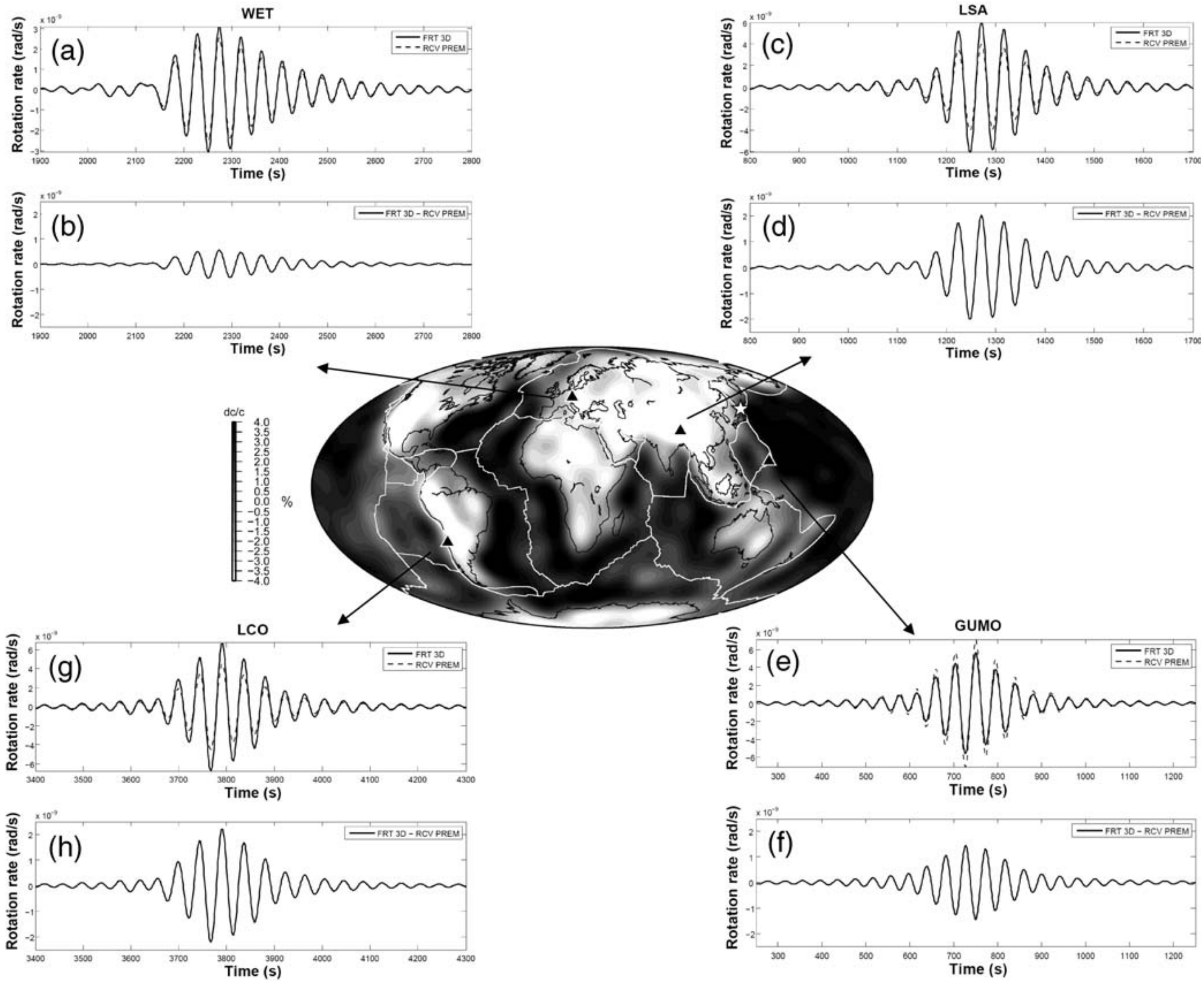

Figure 5. Influence of local structure beneath the receiver for stations within different tectonic environments (triangles): WET (Germany), LSA (Tibet), GUMO (Marianas Islands), and LCO (Chile). Seismograms are calculated for the 25 September 2003 M 8.1 Tokachi-oki earthquake (star). (a), (c), (e), and (g) compare rotation rate FRT synthetics using the 3D Earth model (solid curve) with Receiver PREM synthetics, that is, calculations using the 3D Earth model to calculate the source and path terms in the FRT equation (7) and using PREM to calculate the receiver term (dashed curve); see section titled Influence of Local Structure Beneath the Receiver for details. The traces in (b), (d), (f), and (h) show the differences between the two seismograms in (a), (c), (e), and (g). These differences are only due to deviations of the local shear-wave velocity structure beneath the receiver from PREM.

of model parameters. Thus, Monte Carlo methods could be used to find the optimal local structure that minimize the misfit between theoretical and measured dispersion curves. Although only a standard normal-mode algorithm would be needed for this purpose, FRT remains useful as it could be used to additionally assess the improvement in the fit between synthetics and recorded rotational waveforms during the model search. In contrast to numerical methods such as SEM, it is straightforward to implement different Earth models and notably to change local 1D structures used in FRT calculations.

FRT has limitations and a restricted domain of applicability, being valid in smooth media, when the wavelength of the propagating wave is smaller than the scale length of heterogeneity in the Earth model along the propagation path. Thus, it remains to determine the exact domain of validity of the linear relationship between fundamental mode surface wave rotation rate and transverse acceleration. Future comparisons between FRT and SEM calculations should allow us to establish exactly how small the wavelength must be compared with the scale length of heterogeneity for the relationship to hold.

Conclusions

In this study we apply surface wave FRT to the problem of simulating Love waves (rotation and translations) for global Earth models (spherically symmetric and 3D). We demonstrate that for smooth, laterally heterogeneous media the fundamental Love-wave dispersion relation naturally appears as being proportional to the spectral ratio between transverse acceleration and rotation rate. The accuracy of FRT is confirmed by comparison with complete calculations using the SEM. The sensitivity of rotational motions to the local structure beneath the receiver is demonstrated with synthetic calculations for a global 3D velocity model. This work suggests that FRT is a useful tool to help estimate the velocity structures of local 1D shear waves beneath seismic stations.

Data and Resources

The records of the Tokachi-Oki event used in this study were provided by the Fundamentalstation Wettzell and the Geophysics Section, Ludwig-Maximilians-University Munich and also previously discussed in Igel *et al.* (2005, 2007).

Acknowledgments

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