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Suggested Notation Conventions for Rotational Seismology

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Abstract We note substantial inconsistency among authors discussing rotational motions observed with inertial seismic sensors (and much more so in the broader topic of rotational phenomena). Working from physics and other precedents, we propose standard terminology and a preferred reference frame for inertial sensors (Fig. 1) that may be consistently used in discussions of both finite and infinitesimal observed rotational and translational motions in seismology and earthquake engineering. The scope of this article is limited to observations because there are significant differences in the analysis of finite and infinitesimal rotations, though such discussions should remain compatible with those presented here where possible. We recommend the general use of the notation conventions presented in this tutorial, and we recommend that any deviations or alternatives be explicitly defined.

Introduction

While the term translation is an unambiguous well-accepted term for nonrotational body motions along a straight line, the situation for inertially observable rotations is less clear, with a history of differing terms too often ill defined. It appears to the authors that some clarification in the matter of inertially observed finite and infinitesimal rotational motions is easily obtained from considerations of first principles and the precedents of physics and traditional seismology and engineering. Our recommendations are illustrated in Figure 1. A glossary of terms can be found in Lee (2009).

Ideally, any such definitions should be descriptive (describing existing practices), as pioneered by the Oxford English Dictionary. However, in this paper we use partially prescriptive definitions in which we attempt to follow established systems where presently discernible. Authors should define their frames and terminology at least as clearly as is done here. In particular, although these standards apply well to finite and infinitesimal rotations observed by inertial sensors anchored to the Earth or a structure (e.g., a building), they will not suffice for the mathematics of finite rotations during analysis, which is beyond the scope of this article.

Fundamentals

Terms like tilt, torsion, rocking, spin, and twist describe various forms of rotation or deformation; however, their definitions are generally not self-evident; moreover, there appear to be inconsistencies in their use. Tilt seems to mean long-period rotations about a horizontal axis to some, while to others, it means only static rotations and to yet others, rotations at any frequency. Torsion seems to mean higher-frequency rotations or strains about the vertical axis of a structure (but with the same variance in the frequency band implied); rocking appears to be the higher frequency rotation of a whole structure about a horizontal axis. Spin and twist are unclear at present—at least in English—but they are widely used in the European literature; when published in English, these terms should either not be used or should be well defined *a priori*. The terms tilt, torsion, and rocking should be used only with full graphically supported definitions inclusive of a frequency band. (Authors are cautioned one one detail of such graphics: the illustration in two dimensions of a three-dimensional gyre [ellipse with arrowhead] is inherently ambiguous unless line breaks are carefully added to indicate which line is in front of another, as in Fig. 1. A similar caveat applies to whether axes point into or out of the page.)

Only one description of observed solid-body rotations (the only quantities that can be measured by inertial sensors) is needed; such definitions are already well established (see, e.g., Morse and Feshbach, 1953).

In Cartesian coordinates, the definitions of rotations directly follow from those of the translational-motion coordinate frame, inclusive of that frame's right- or left-handedness. In keeping with seismological precedent, we elect to use a right-handed translational system with Z pointing upward (i.e., upward case translation and antiazimuth, counterclockwise in map view, case rotation result in positive outputs from the sensor's Z axes; Fig. 1). This choice results in right-hand-rule rotational vectors, also known as pseudovectors or axial vectors, pointing along the same three axes with the same polarities. Thus, our suggested right-hand-rule positive rotational vector axes point in the same directions as the positive translational axes. For example,

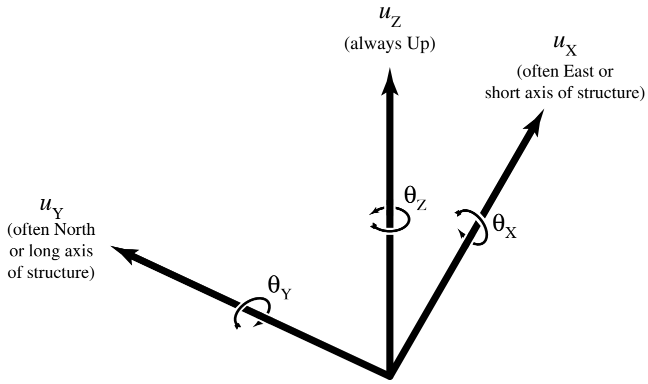

Figure 1. Preferred nomenclature and sign conventions for observed translational and rotational motions in seismology and earthquake engineering. X and Y axes point into the page and are normally in the horizontal plane. Note the positions of rotation gyres with respect to the axes (the line in front breaks the line behind). Axes point in the positive directions of both the translational vectors and rotation vectors. Annotations u and θ are translational displacements and rotation angles, for example, meters and radians.

in Figure 1, a positive rotation of the instrument case about the Z axis causes particles of that solid body to arc from X toward Y within a horizontal plane.

These rotation vectors should generally be annotated as θ, ϕ, φ, ψ, or a similar character with a subscript indicating the axis (1, 2, and 3, or X, Y, and Z). Such motions are termed rotation angles, preferably in units of radians. For example, θ_X is the right-handed rotation angle about the X axis. (We note in passing that the International System of Units abbreviation for radians is rad and for seconds is s, not sec, so that units in the style mrad/s are preferred to milliradians/sec.)

The capitalized and scripted forms of these letters should generally be reserved for transformed equivalents (Fourier, etc.), while rotation rates (rotational velocities), rotational accelerations, etc., should be indicated as their time derivatives (e.g., $\ddot{\theta}_X$ for a rotational acceleration about the X axis). Boldface commonly means a vector of rotations while italics generally imply single components and scalars. Thus, for example, boldface $\boldsymbol{\theta}$ might represent the column vector

$$\begin{bmatrix} \theta_{\mathbf{X}} \\ \theta_{\mathbf{Y}} \\ \theta_{\mathbf{Z}} \end{bmatrix},$$

while $\Im$ or $\mathbf{\Theta}$ might represent transforms of θ or $\boldsymbol{\theta}$. Some journals will enforce or authors will prefer other usage, of course, but such usage should be described in the articles themselves.

Right-hand-rule rotational rotation vectors are widely used in physics. Any such rotation vector is positive in the direction of the right hand's thumb when the particle motions are in the direction that the wrapped right-hand fingers point. For example, a rotation vector along the positive Y axis represents particle motion in a vertical plane from Z toward X.

We urge the use of the right-handed system $[X, Y, Z]$ as the coordinate frame for both translations and rotations (Fig. 1). These axes are commonly oriented (east, north, or up) relative to the Earth (transverse, longitudinal, or up), relative to the axes of a structure (the long axis of the structure being longitudinal), or rotated about Z resulting in two noncardinal orthogonal horizontal axes (e.g., Y is oriented N25°W and X at N65°E).

Normally, this coordinate system is the slowly rotating, gravitationally (upward) accelerating frame of the Earth's surface or a built structure; any other reference frame should be indicated by the authors with a figure. While this is not an inertial frame, usage in seismology and engineering commonly disregards this fact. Similarly, the name inertial sensor is very widely used and adopted here—indeed, such sensors are inherently sensitive only to accelerations, though many output velocity; the term is proper notwithstanding that gravitational acceleration and Earth rotation are either commonly compensated or too small to measure. Some variation in terminology is likely to be preferred by some, particularly those dealing with theory and analysis; thus, terms should be properly defined.

Conclusions

We urge authors to use the terms and notation of Figure 1 and the terms included in the glossary (Lee, 2009). We also recommend that they explain or eliminate use of the terms tilt, torsion, rocking, spin, twist, yaw, pitch, and roll. If those or any other notation or coordinate frames are used, they must be fully described by the authors in the text and with a figure in the style of Figure 1. Many authors properly distinguish between finite and infinitesimal rotations, but these distinctions are beyond the scope of this article; the notation defined here is sufficient for describing whole-body motions of any magnitude as measured by inertial sensors, seismometers, and accelerometers, with axes fixed to the Earth or to a structure.

Data and Resources

No data were used in this tutorial.

Acknowledgments

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References

Lee, W. H. K (2009). A glossary for rotational seismology, *Bull. Seismol. Soc. Am.* **99,** no. 2B, 1082–1090.

Morse, P. M., and H. Feshbach (1953). *Methods of Theoretical Physics, Part I*, McGraw-Hill, New York.

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