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Fundamental Deformations in Asymmetric Continuum

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Abstract The complexity of processes in a seismic source zone has inspired us to reconsider a class of basic motions and deformations in an asymmetric continuum, including simple motions (translation and rotation, named spin) and simple deformations. Simple deformations include the axial nuclei, that is, point extension/compression, and the shear nuclei related to string-string point deformations. The point deformation is further redefined as another kind of rotational motion, called twist, representing oscillations of the main shear axes and their amplitudes. Some remarks are added on the recording systems used to measure these motions and deformations.

Introduction

To better understand the complexity of processes in a seismic source zone we will reconsider the fundamental point motions and point deformations: displacements, rotations, shear nuclei, and an axial nucleus. When considering the point motions, it is to be noted that a single couple is not exactly a point source (it is only its derivative that tends to a point), but a double couple already becomes a point deformation: a shear nucleus given by the string-string deformation (see further on). The problem of fundamental point motions gains actuality due to the recent observations of rotation waves derived from the precise instruments able to measure very small spin, up to 10^{-9} rad/sec.

The rotation motions were not included in a consistent way in the classical elasticity, and that is why, since the end of the nineteenth century, many attempts have been undertaken to construct a more adequate and powerful theory of continua. The classical theory has many limitations; the first, noticed by Cartan, was related to the lack of description of more complicated processes. The first attempt to include rotations into the continuum theory is due to Voigt in 1887, while a complete theory with the displacement vector and rotation vector, known as the Cosserat theory of elasticity, was created by the Cosserat brothers in 1896 (see Cosserat and Cosserat, 1909). A very powerful tool is provided by micropolar and micromorphic elastic theories developed by Eringen and his coworkers (see Eringen and Suhubi, 1964; Eringen, 1999, 2001) and Mindlin (1965).

In addition to the micromorphic and micropolar theories, the asymmetric elasticity theory by Nowacki (1986) is also worth mentioning. Some application to seismic waves has been pointed out by Teisseyre (1973, 1974).

Another approach to generalize the classic theory is the Kröner method defining the elastic fields as differences between the total displacement-related fields and the self fields representing a continuum distribution of internal nuclei (Kröner, 1981). Further developments, including asymmetric stresses, strains and rotations, are due to Teisseyre and Boratyński (2003), and finally to Teisseyre (2008).

Considering the fundamental point motions, we believe that each of them shall obey the adequate equations of motion, which shall correspond to the recognized balance laws including that for the moment of momentum; we may also suppose that in an earthquake source region the premonitory and dynamic processes can be interrelated. Further, it is required that a general theoretical approach should include processes leading from a given state of continuum to other states governed by different constitutive laws; for example, we shall include a kind of links between the solid elastic continuum and that subjected to granulation process with its final form similar to a sandlike material. An adequate theoretical approach shall also account for the interactive processes, for example, between shear release and spin rebound. Thus, our aim was also to point out the need to construct a relatively simple and reliable theory, including all the aforementioned elements.

Displacements and Rotations

The problem of rotation motion and its recording was raised very early; when studying the effects of the Lisbon earthquakes (1755) and those of Calabria (1783), the founders of seismology and other well-known scientists, Charles Lyell (1797–1875), Charles Darwin (1809–1882), Robert Mallet (1810–1881), and Alexander von Humboldt (1769–1859), considered a counterpart of the vortical movements or vortex motions induced by earthquakes. We have a precise and still valid explanation from Robert Mallet, at least for the majority of the cases observed, who stated that the observed rotation effects of surface objects are mainly caused by the geometry of the main inertia moments of an object and directions of seismic acceleration. Of importance is also the position of a center of adherence between the ground and distorted objects (see Ferrari, 2006; Kozák, 2006). Numerous attempts have been undertaken to record the vortical motions, that is, the seismic rotation waves; we can mention here the first instrument to record such motions constructed by Filippo Cecchi in 1875, although this electrical seismograph has never managed to record any traces of such wave motions.

According to Gutenberg (1926), the rotation waves cannot propagate as they would be immediately attenuated; of course, there remains rotation of displacements. The Gutenberg statement concerned only the rotation waves independently generated at a source; this statement follows immediately from the assumption that stresses are symmetric and that the constitutive laws describing the material response neglect any action of rotational deformation (the stress moment action is not coherently included in the classical elasticity; one should introduce a length element and a reference rotation point).

Recently, owing to the development of modern, very precise instruments, it has become possible to record extremely small rotation time rates and, of course, the rotation of displacement velocities. This motivated us to reconsider the fundamental motions and deformations.

Let us first examine relations between the displacement velocity and spin motions. We can assume that the microfracture processes in the source, including slip and fragmentation events, produce independent displacements, $\mathbf{u}$, and rotations, $\boldsymbol{\omega}$ (for simplicity, we neglect here the axial motions, $\operatorname{div}\mathbf{u} = 0$). However, we shall note that the two independent motions, $\mathbf{u}$ and $\boldsymbol{\omega}$, may contribute to each other in their wave field:

$$\boldsymbol{\omega}' \equiv \operatorname{curl}\mathbf{u} \quad \text{and} \quad \mathbf{u}' \equiv l^2 \operatorname{curl}\boldsymbol{\omega},$$

where we have introduced the intrinsic length l (Cosserat's characteristic length) playing an important role related to the material properties.

Formally, any displacements, $\mathbf{u}$ and $\mathbf{U}$, and rotations, $\boldsymbol{\omega}$ and $\boldsymbol{\Omega}$, of different proveniences can be equivalently described. For example, we may write

$$\boldsymbol{\omega} = \operatorname{curl}\mathbf{U}, \qquad \mathbf{u} = l^2 \operatorname{curl}\boldsymbol{\omega};$$

hence, provided that $\operatorname{div}\mathbf{U} = 0$, we get

$$\mathbf{u} = -\Delta\mathbf{U},$$

and similarly, we write

$$\boldsymbol{\omega} = \operatorname{curl}\mathbf{U}, \qquad \mathbf{U} = l^2 \operatorname{curl}\boldsymbol{\Omega},$$

which leads to

$$\boldsymbol{\omega} = -\Delta\boldsymbol{\Omega}.$$

However, the previous formal relations are an equivalent description only, and these two motions may remain independent.

Moreover, such a possible equivalence revealed by this dual description does not exist at a point source; the generation process may run independently for spin and displacement velocity fields, but we have to point out the importance of physical coaction of these independent motions in the inner granulation and fracture processes.

Thus, given an apparent equivalence of these two descriptions, there appears to be a problem of the scales of these motions generated in earthquake source processes and also at sites on the ground where constructional properties of some objects may give different responses to the displacement motions and rotations. Usually, the observed rotations and their effects are much smaller than those related to displacement motions; however, the reverse proportion may also occur, for example, when considering the tilt motions, the rocking and tilting components observed in some structures hit by strong ground motions, and also for the rotations observed very close to fracture under a compressional load, that is, under the conditions in which the fragmentation processes prevail.

We shall repeat that the rotation and displacement motions in a wave field can be directly interrelated in their formal description; this statement is empirically supported by the almost perfect fit observed between the rotations derived from an array of seismographs, $\operatorname{curl}\mathbf{u}$, and the observed rotations (Igel *et al.*, 2005; Cochard *et al.*, 2006; Igel *et al.*, 2007). Here we shall stress that the classical elasticity perfectly describes the propagation of seismic waves, including the displacement velocity and rotation motion. As stated previously, any spin field can be attributed to rotation of some displacement velocity; hence, the propagation of both these fields, displacement velocity and spin, can be explained in terms of the classical theory as well.

However, from another point of view, these motions differ as to their origins and the effects produced; therefore, we will consider the following cases in which rotations need to be classified separately (Teisseyre *et al.*, 2006):

- Megarotations, related to the ground tilts and tilting of high objects on the ground;
- Macrorotations, related to fragmentation processes at the fracturing under compression load;
- Mesorotations, related to granulation processes and formation of the mylonite zones under the shear load;
- Rotations in the wave motions and directly related to the displacement velocities; here, we may recall the Kröner definition of the total rotation ω^T (see Kröner, 1981) given as rotation of the displacements, $\operatorname{curl}\mathbf{u}$; and
- Microrotations, caused by the internal friction processes, as well as by the microslip motions in dislocations and microcracks.

We have to point out the different counterparts and scales of the rotational processes in microfracturing under confining pressure and under external shears; however, in both cases the role of bond breaking remains similar (Teisseyre, 2007).

Fundamental Point Deformations

We use the tensor notation and the summation convention for the repeating indexes:

$$T_{ss} = \sum_s T_{ss}, \qquad A_k B_k = \sum_k A_k B_k.$$

We may also underline the symmetric and antisymmetric properties of the tensors by using, respectively, the round and square brackets for indexes:

$$S_{(ik)} = S_{(ki)}, \qquad S_{[ik]} = -S_{[ki]}.$$

For any symmetric tensor, $T_{(ik)}$, we can separate its axial and deviatoric parts; the deviatoric part, $T^D_{(ik)}$, having zero value of trace, is defined as

$$T^D_{(ik)} = T_{(ik)} - \frac{1}{3}\delta_{ik} T_{(ss)}, \qquad T^A_{(ik)} = \frac{1}{3}\delta_{ik} T_{(ss)},$$
$$T^D_{(ss)} = \sum_s T^D_{(ss)} = 0,$$

where $T^A_{(ik)}$ is an axial tensor and $T_{(ss)}$ is a trace of tensor.

The fundamental point deformations and motions lead us theoretically to 4 fields including a total of 10 components of motions: displacements (3 vector components), rotations (3 vector components), shear nuclei (3 independent components; this point is explained further on), and axial nucleus (scalar: the strain tensor trace) (see Teisseyre and Górski, 2007).

We consider the following fundamental motions:

- The translation described by the displacement vector;
- The independent rotation: its rate, called spin, relates to the grain and particle rotations; it contributes to the displacement velocity field, $U = u + l^2 \operatorname{curl} \omega$, observed in seismic records as proved by the almost perfect fit between the observed rotations and the rotation motions, $\operatorname{curl} \mathbf{u}$, derived from the array of seismometers (Cochard *et al.*, 2006);
- The axial deformation representing compression/dilatation nuclei, for example, related to thermal anomaly;
- The shear nucleus as related to the deviatoric strain tensor or to the twist motions related to the oscillations of main shear axes represented by twist vector (its invariant definition is considered in a separate article; see Teisseyre [2009]).

These axial oscillation motions relate to that part of motions that can be derived from a scalar potential (see Fig. 1).

We do not consider here superpositions and derivatives of these deformations, like, for example, a string-membrane deformation or a single couple, the latter being given as a derivative of translations that only tends to a point (a double couple appears already as a point-deformation equivalent to a shear nucleus given by the string-string deformation), as schematically drawn in Figure 2.

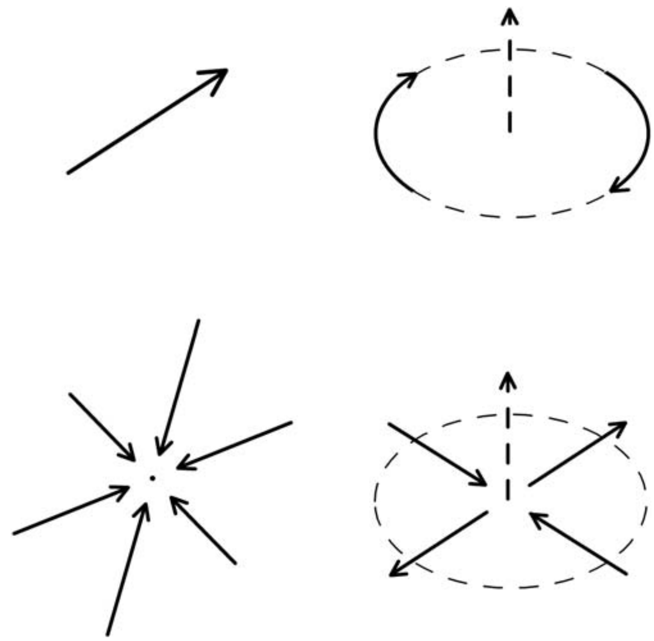

Figure 1. Fundamental point deformations. Upper row: displacement (vector field) and rotation (its vector field is marked by the dashed arrow). Lower row: axial motion (concentric deformation: scalar field) and string-string deformation (its twist vector is marked by the dashed arrow).

Note that the point deformations, including the axial motions, can be described by either the Riemannian curvature or, in more complicated cases, by the torsion tensor in a non-Riemannian space (Kleinert, 1989).

The shear field is given by the deviatoric part of strains; this part may be used to define a new antisymmetric tensor related to the simple deformations that may represent another kind of motion—the twist deformation—belonging to the rotation family. For an ensemble of particles or grains representing points of a continuum, these deviatoric deformations relate to pure shear oscillations.

However, when considering the point motions it is better to relate the shear deformations to the equivalent twist tensor,

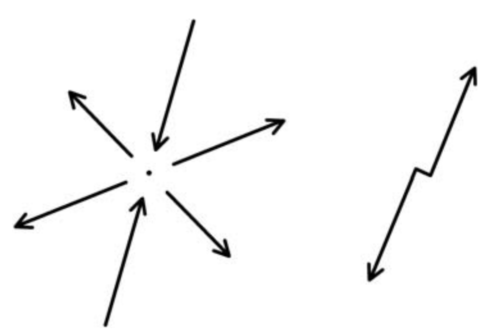

Figure 2. Superposition of the string-string deformations (string-membrane; the left-hand sketch) and the derivative of displacements (single couple, the right-hand sketch).

$\omega_{(ks)}$; such motions contribute to the shear field as observed, for example, in seismology. We believe that this field is directly related to the *S* waves, so for the related displacements we can write $u^s = l\omega$, with l being an effective radius of grains/particles forming the continuum and ω the related rotation.

Having defined the spin and twist, we can now define the complex rotation field; twist describes the angular oscillations of shear axes and related amplitude variations. In the limit related to the pointlike deformations we arrive at the string-string type motions (see Fig. 1) representing in a different way the twist motion as described by a vector $\omega_{(k)}$ perpendicular to the string-string plane and having an appropriate magnitude of a string-string deformation (a twist vector can be constructed from three components of the shear tensor in its off-diagonal form). The invariant representation of the twist vector is discussed in a separate article (Teisseyre, 2009).

Except for the axial deformation, all the previous deformations may be described by the displacement field, but this is not true for the point-related spin and string-string deformation originated as independent motions in the focus.

We repeat that we have not considered a number of the first and higher order moments of these basic motions and deformations; we shall also note that such source models tend to a point source as a limit but cannot be treated as exact point nuclei.

Conclusions and Experimental Evidence

We have discriminated the two vector fields, translation and rotation, the strain tensor trace (scalar axial field), and the deviatoric strain (shears). The shear motion may be described by relations for the twist vector (see Teisseyre, 2009); therefore, these deformations should obey a total of 10 independent balance equations. At their origin, these motions may be interrelated because they are all governed by a momentary source mechanism process. In a wave field, these motions appear as the *P*- and *S*-wave constituents, but some of them may also be detected by rotation sensors and strain meters owing to modern sensitive instrumentation techniques. Thus, the axial motions can be detected by the strain meters; for small amplitudes we need to use the laser devices.

As concerns the rotation/spin motions, we may note that perhaps the first rotation seismogram has been attained in an indirect way: the azimuth array of horizontal seismographs, installed in one of the coal mines in Upper Silesia, Poland, to record the very near-by tremors, permitted the deduction of the rotational component of motions (see Teisseyre, 1974). However, the first directly recorded rotation motions are those obtained at the two fundamental geodetic stations in Germany (Wetzell) and in Australia at the end of the last century (e.g., Igel *et al.*, 2005; Cochard *et al.*, 2006; Schreiber *et al.*, 2006; Igel *et al.*, 2007) equipped with ring-laser interferometers based on the Sagnac principle. These systems, having sensitivity up to 10^{-9} rad/sec, were able to record the rotation motions related even to many distant earthquakes.

The fiber-optic interferometers, also based on the Sagnac principle, were used by Takeo (2006) for seismic observations in a near field.

T. Moriya (e.g., Moriya and Teisseyre, 2006; Teisseyre, 2007) has constructed the first rotation seismograph system consisting of a pair of antiparallel seismographs; such a system with a common suspension axis of the two antiparallel pendulums was improved in later constructions (Wiszniowski, 2006; Wiszniowski *et al.*, 2008).

Using the rotation wave records of different events in the very near field (Teisseyre *et al.*, 2003) it was possible to find that for some events, for example, the shallow volcanic and the explosion type ones, we obtain records differing from the common features by the extremely small rotation components.

We shall keep in mind that using the rotation seismographs we can measure both spin and twist motions; the spin motions are represented by average values from two perpendicularly situated systems of rotation seismometers and the twist motions by the difference of these records. Having analyzed numerous records we observed that both motions show oscillations of the same order of magnitudes (Moryia and Teisseyre, 2006; Teisseyre and Suchcicki, 2006). Yet we must stress that the twist field measured in this way gives only the angular variations of the off-diagonal axes of shears, while their amplitude is not well defined and depends on orientation of the rotation seismometer system in relation to the main external shear axes; however, an additional rotational seismometer system situated on the same plane may solve this problem. Only when measuring the shear deformations with a system of strain meters, we can achieve more reliable and independent data on the shear-twist variations. Moreover, the strain meter system can also measure the axial deformations. It is to be noted that when recording simultaneously the strains and spin we can derive some new information on the earthquake source processes (see Teisseyre, 2009).

Finally, the question arises of how the invariant twist field can be compared with the observed shear variations. An exact procedure requires that the six components of the shear strain, E_{ik}, determined in an observation site system be transformed, at each time moment, into the off-diagonal system:

$$\{E_{11}, E_{22}, E_{33}, E_{23}, E_{31}, E_{12}\} \rightarrow \{E_{23}, E_{31}, E_{12}\},$$

where we should keep in mind that the trace of the shear tensor vanishes for the shear strain ($E_{11} + E_{22} + E_{33} = 0$).

We want to underline once again the fact that the problem of rotation motions has found its theoretical basis in the Cosserat theory and later a strong support in the micropolar and micromorphic theories. Following those approaches, Shimbo (1975) introduced the constitutive relation joining the rotations with the antisymmetric stresses; this law permits us to establish a new approach to a theory describing asymmetric continuum.

Data and Resources

No data were used in this article.

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