Bulletin of the Seismological Society of America, Vol. 99, No. 2B, pp. 1064–1072, May 2009, doi: 10.1785/0120080145

Tutorial on Measuring Rotations Using Multipendulum Systems

by Vladimir Graizer

Abstract This article considers a classical approach of using a combination of pendulums to measure rotations. The idea of using two identical pendulums installed on different sides of the same axis of rotation for separate measurement of rotational and translational motion was apparently first suggested by Golitzin (1912). It was implemented by Kharin and Simonov (1969) in an instrument designed to record strong ground motion (VBPP—a seismograph of large translational motions and rotations). Unfortunately, difficulty in building identical mechanical systems resulted in unreliable measurements of the rotational component. We modified Golitzin's idea by using the same configuration of pendulums (a two-pendulum system) without the requirement that the pendulums be identical (Graizer *et al.*, 1989). Instead of building two identical pendulums, one needs to calibrate the instrument to obtain the natural parameters of each pendulum and apply postprocessing to separate the rotational and translational motions. The two-pendulum system for separate measurements of large amplitude rotations was implemented at the end of the 1980s at the Institute of the Physics of the Earth in Moscow, Russia, using commercially available pendulum instruments. The system was tested using a basic shake table and later successfully applied to measurements in the near field of two large underground nuclear explosions. In this article I updated and generalized the approach to measuring translational and large amplitude rotational motion formulated in previous publications (Graizer, 1989; Graizer *et al.*, 1989). Numerical testing demonstrated that using a combination of pendulums for measuring rotations may be limited for recording relatively large amplitudes of rotations of the order of 10^{-4} and higher for the two-pendulum system of about 100 cm size.

Introduction

During the last half of the twentieth century a number of attempts were made to measure or estimate the rotational component of strong ground motion (e.g., Farrell, 1969; Kharin and Simonov, 1969; Bradner and Reichle, 1973; Niazi, 1986; Oliveira and Bolt, 1989; Graizer *et al.*, 1989; Graizer, 1991; Nigbor, 1994; Takeo, 1998; Huang, 2003; Zahradnik and Plesinger, 2005; Graizer, 2006a; Schreiber *et al.*, 2006; Spudich and Fletcher, 2008), but still there are no consistent measurements of rotations during earthquake shaking. Scientific and technical advances in the recent decade made a number of technologies widely used in inertial navigation, like the combination of gyroscopes and accelerometers, much cheaper and compact and potentially available for use in seismic measurements. Considering different possible directions in rotation measurements is beyond the scope of this study. This article discusses the classic approach to measuring rotations and translational motion by using a multipendulum system. The classic way of measuring rotations by using two identical pendulums was apparently first suggested by Golitzin (1912). It was later implemented by Kharin and Simonov (1969) in an instrument called VBPP (a seismograph of large translational motions and rotations). This instrument used two identical pendulums on the same axis and moving in the same plane (Fig. 1). In the case of purely translational input motion both pendulums are producing exactly the same output signals. In the case of rotation (tilt), the outputs of the pendulums are opposite due to rotational acceleration. The output of the instrument was either a sum of the two signals, or a difference of them. Actually, both versions of the instrument were made. Summation of the two signals was supposed to result in a purely translational signal, and the difference was supposed to result in rotational motion only. In reality, summation of the two signals resulted in reliable translational motions, but the difference of the two signals (of about the same amplitude) produced unreliable rotation measurements. The main problem occurred because of difficulty in constructing two identical mechanical systems (pendulums), and it became clear that the difference in the two signals is mainly determined by nonequality of pendulums. We modified Golitzin's idea and created a two-pendulum system to record large amplitude rotations. The recording system was first

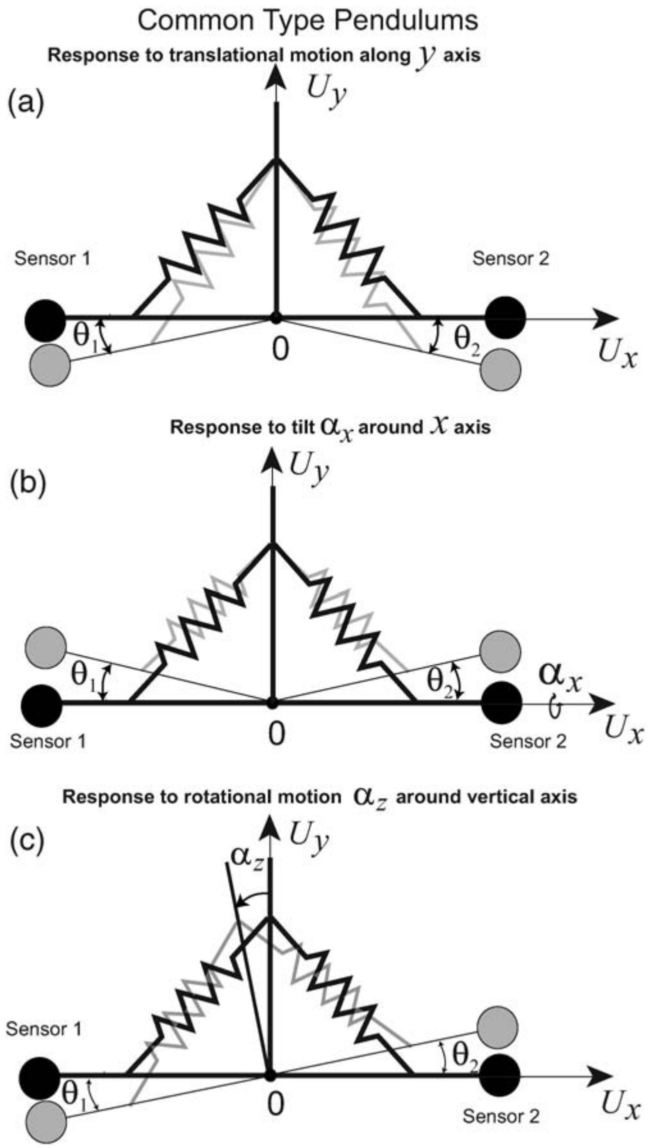

Figure 1. Responses of two identical pendulums on the same axis to (a) horizontal translational motion along the y direction, (b) tilt α_x around the x axis, and (c) rotation (torsion) α_z around the vertical axis.

tested in the laboratory and later used to record strong motion and tilt in the near field of nuclear explosions (Graizer *et al.*, 1989; Graizer, 1991).

Theory

Most of the seismological sensors (seismometers and accelerometers) used in conventional seismological instruments are pendulums of the mass-on-rod type (Golitzin, 1912; Savarensky and Kirnos, 1955; Aki and Richards, 1980). A complete equation of small oscillations (i.e., $\sin\theta \cong \theta$) of the horizontal pendulum of the mass-on-rod type can be expressed as (Graizer, 1989, 2005; Graizer and Kalkan, 2008):

$$\varphi_n'' + 2\omega_n D_n \varphi_n' + \omega_n^2 \varphi_n = -u_y'' - g\alpha_x - l_n \alpha_z'' + u_x'' \theta_n, \quad (1)$$

where φ_n is the recorded response of the instrument in the y direction, l_n is the length of pendulum arm, θ_n is the deflection angle of the pendulum relative to the frame of the seismometer from a position of equilibrium, $\varphi_n = l_n \theta_n$ for small angles θ_n, ω_n and D_n are, respectively, the natural circular frequency and the fraction of critical damping of the oscillator, u_y'' is the ground-motion acceleration along the horizontal y direction, u_x'' is the ground-motion acceleration along the horizontal x direction, u_z'' is the ground-motion acceleration along the vertical z direction, α_x is rotation around the x axis, α_y is rotation around the y axis, and α_z is rotation around the z axis. Rotation α_x or α_y around one of the horizontal axes is also called tilt or rocking, and rotation α_z around the vertical axis is also called torsion in engineering practice.

I use the right-hand coordinate system (x, y, z) commonly used in physics and mathematics. In the right-hand system, rotational vector axes should point in the same directions as the translational axes. Figure 2 shows this sign convention for translational and rotational motion, as recommended by the members of International Working Group on Rotational Seismology (Evans and the International Working Group on Rotational Seismology, 2009). Note that a rotation vector along the upward z axis represents counterclockwise rotation in the horizontal plane, as viewed from above.

System of Two Horizontal Pendulums

Let us consider the response of the two-pendulum system shown in Figure 1 and recording ground motion in the horizontal x direction. Both pendulums are oscillating in the horizontal plane. The complete equations for sensors 1 and 2 shown in Figure 1 are

$$\begin{aligned} \varphi_1'' + 2\omega_1 D_1 \varphi_1' + \omega_1^2 \varphi_1 &= -u_y'' - g\alpha_x - l_1 \alpha_z'' + u_x'' \theta_1, \\ \varphi_2'' + 2\omega_2 D_2 \varphi_2' + \omega_2^2 \varphi_2 &= -u_y'' - g\alpha_x + l_2 \alpha_z'' + u_x'' \theta_2. \end{aligned} \quad (2)$$

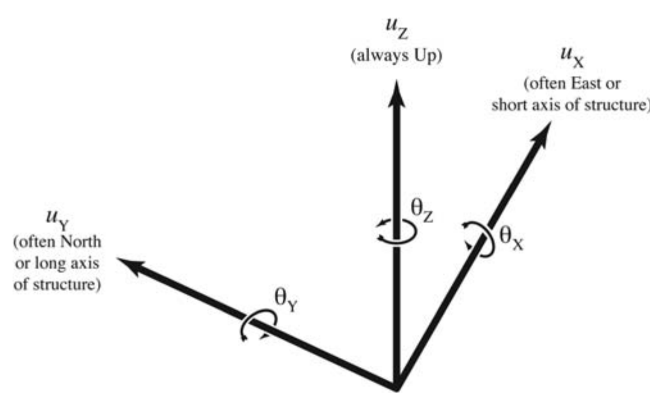

Figure 2. Sign convention for translational and rotational motion (from Evans and the International Working Group on Rotational Seismology, 2009).

The left sides of both equations in (2) represent responses of the sensors.

The right sides of both equations in (2) are complex inputs that include

- Acceleration along the y axis,
- Tilt α_x of the base around the x axis,
- Inertial force due to rotation α_z'' around the vertical axis, and
- Cross-axis sensitivity.

As was previously shown by Graizer (1989, 2006b), Todorovska (1998), Trifunac and Todorovska (2001), and Graizer and Kalkan (2008), cross-axis sensitivity (the fourth term on the right side of both equations in (2) is relatively low for recent instruments and can be neglected (most of the recent instruments are using force-balance feedback system minimizing actual motion of a pendulum). Effective equations of the two-pendulum system can be written

$$\begin{aligned} \varphi_1'' + 2\omega_1 D_1 \varphi_1' + \omega_1^2 \varphi_1 &= -u_y'' - g\alpha_x - l_1 \alpha_z'', \\ \varphi_2'' + 2\omega_2 D_2 \varphi_2' + \omega_2^2 \varphi_2 &= -u_y'' - g\alpha_x + l_2 \alpha_z''. \end{aligned} \tag{3}$$

The system of two equations in (3) contains three unknown terms (u_y, α_x, and α_z). Mathematically, this system can be resolved for rotations around the vertical axis by subtracting equations (3):

$$\begin{aligned} &(\varphi_2'' - \varphi_1'') + (2\omega_2 D_2 \varphi_2' - 2\omega_1 D_1 \varphi_1') + (\omega_2^2 \varphi_2 - \omega_1^2 \varphi_1) \\ &\quad = (l_1 + l_2)\alpha_z''. \end{aligned} \tag{4}$$

Theoretically, for small angles of rotations, the difference is not sensitive to tilt and is only sensitive to angular acceleration.

Assuming that sensors 1 and 2 are equal,

$$\omega_1 = \omega_2 = \omega, \qquad D_1 = D_2 = D, \qquad l_1 = l_2 = l,$$
$$\varphi_D = \varphi_2 - \varphi_1, \qquad \varphi_S = \varphi_2 + \varphi_1,$$

results in

$$\begin{aligned} \varphi_D'' + 2\omega D \varphi_D' + \omega^2 \varphi_D &= 2l\alpha_z'', \\ \varphi_S'' + 2\omega D \varphi_S' + \omega^2 \varphi_S &= -2(u_y'' + g\alpha_x''). \end{aligned} \tag{5}$$

When purely translational motion along the y axis is applied to a system of identical pendulums, both sensors are moving in the same negative direction and their outputs are identical (Fig. 1a). When tilt α_x around the x axis is applied to the system, both pendulums are moving in the same negative direction (Fig. 1b). When purely rotational motion around the vertical axis α_z is applied to the same system, the sensors are moving in opposite directions (Fig. 1c): sensor 1 in the positive direction and sensor 2 in the negative direction. The Russian instrument VBPP (Kharin and Simonov, 1969) was based on this principle of measuring the difference in electrical outputs of the two identical pendulums.

Based on equations (3)–(5) the following observations can be made:

- Sensitivity to rotations α_z'' around the vertical axis is higher for a pendulum with a long pendulum arm.
- If the rotational signal is low and the sensors are not identical, the system based on equation (5) is measuring errors instead of rotations.
- Summation of the signals from the two sensors results in acceleration plus tilt (if tilt exists). Tilt sensitivity of both horizontal sensors is the same.

System of Two Vertical Pendulums

As was shown in a number of publications (e.g., Graizer, 2005, 2006a), sensitivity of a vertical sensor to tilt is relatively low and can be neglected for small tilt angles. A combination of the two vertical sensors can be described by the following system of equations:

$$\begin{aligned} \varphi_1'' + 2\omega_1 D_1 \varphi_1' + \omega_1^2 \varphi_1 &= -u_z'' - l_1 \alpha_x'', \\ \varphi_2'' + 2\omega_2 D_2 \varphi_2' + \omega_2^2 \varphi_2 &= -u_z'' + l_2 \alpha_x''. \end{aligned} \tag{6}$$

The system of equations in (6) contains two unknown terms, and consequently, can be resolved against both ground-motion parameters (vertical acceleration and angular acceleration of tilt).

Assuming that the sensors are equal results in

$$\begin{aligned} \varphi_D'' + 2\omega D \varphi_D' + \omega^2 \varphi_D &= 2l\alpha_x'', \\ \varphi_S'' + 2\omega D \varphi_S' + \omega^2 \varphi_S &= -2u_z''. \end{aligned} \tag{7}$$

In contrast to the horizontal sensors, the combination of the two identical vertical sensors allows resolution of both vertical and angular acceleration. Unfortunately, due to the difficulty of building two identical mechanical pendulums, reliable measurements of rotations using VBPP were not obtained (Kharin and Simonov, 1969). A similar two-pendulum system was recently described by Wiszniowski (2006).

Based on previous negative experience we decided to use another approach to measuring rotations (Graizer *et al.*, 1989; Graizer, 1991). We used advantages provided by the new technologies because it is easier to measure natural parameters of each sensor (ω_n, D_n, and l_n) with relatively high precision than to build identical pendulums. Our two-pendulum system was based on a post measurement processing. We used the same sensor arrangement as proposed by Golitzin (1912) and Kharin and Simonov (1969), but instead of trying to build two identical mechanical systems, we measured the parameters of each sensor and applied postcorrection to get the rotation and translation.

Integrating both sides of the equations in (6) twice and assuming

$$F_1(t) = \varphi_1 + 2\omega_1 D_1 \int_0^t \varphi_1 \, d\tau + \omega_1^2 \int_0^t d\tau \int_0^t \varphi_1 \, d\tau,$$
$$F_2(t) = \varphi_2 + 2\omega_2 D_2 \int_0^t \varphi_2 \, d\tau + \omega_2^2 \int_0^t d\tau \int_0^t \varphi_2 \, d\tau, \quad (8)$$

results in

$$F_1(t) = -u_z(t) - l_1\alpha_x(t), \qquad F_2(t) = -u_z(t) + l_2\alpha_x(t), \quad (9)$$

and

$$u_z(t) = -\frac{l_2F_1(t) + l_1F_2(t)}{l_1 + l_2}, \qquad \alpha_x(t) = \frac{F_2(t) - F_1(t)}{l_1 + l_2}. \quad (10)$$

Because the sum of the linear and angular displacement functions on the right side of the equations in (9) should demonstrate the same type of behavior as the displacement function alone, the same integration scheme as used in strong motion to get displacements can be used for data processing and baseline correction (e.g., Trifunac, 1971; Graizer, 1979) and, consequently, calculation of functions $F_1(t)$ and $F_2(t)$. As can be seen from equations (8)–(10), there is no need to build identical pendulums, but there is a need to measure parameters of each of the two instruments (pendulums). Calibration of current instruments can be accomplished with the error of about 1% or less and will provide those required parameters.

System of Six Pendulums

The system of six pendulums shown in Figure 3 combining three vertical and three horizontal sensors allows resolution of all six components of ground motion. The corresponding system of equations can be written

$$\begin{aligned}
\varphi_1'' + 2\omega_1 D_1 \varphi_1' + \omega_1^2 \varphi_1 &= -u_z'' - l_1\alpha_y'',\\
\varphi_2'' + 2\omega_2 D_2 \varphi_2' + \omega_2^2 \varphi_2 &= -u_z'' + l_2\alpha_y'',\\
\varphi_3'' + 2\omega_3 D_3 \varphi_3' + \omega_3^2 \varphi_3 &= -u_z'' + l_3\alpha_x'',\\
\varphi_4'' + 2\omega_4 D_4 \varphi_4' + \omega_4^2 \varphi_4 &= -u_x'' + g\alpha_y - l_4\alpha_z'',\\
\varphi_5'' + 2\omega_5 D_5 \varphi_5' + \omega_5^2 \varphi_5 &= -u_x'' + g\alpha_y + l_5\alpha_z'',\\
\varphi_6'' + 2\omega_6 D_6 \varphi_6' + \omega_6^2 \varphi_6 &= -u_y'' - g\alpha_x - l_6\alpha_x''.
\end{aligned} \quad (11)$$

The system of equations in (11) can be resolved step by step. The first two equations allow for resolution of the vertical ground motion u_z and tilt α_y around the y axis. The third equation will allow resolution relative to another component of tilt α_x around the x axis. The difference of the fifth and fourth equations will result in the calculation of rotation around the vertical axis α_z. The sum of the fourth and fifth equations will result in the horizontal motion along the x axis plus tilt $g\alpha_y$. Because tilt α_y is already computed from the first and second equations, one can get the horizontal component u_x. The sixth equation corrected for tilts and rotations around the vertical axis will give the ground motion along the u_y axis. A similar approach to separate measurements of all six components of ground motion was recently presented by Todorovska and Trifunac (2007).

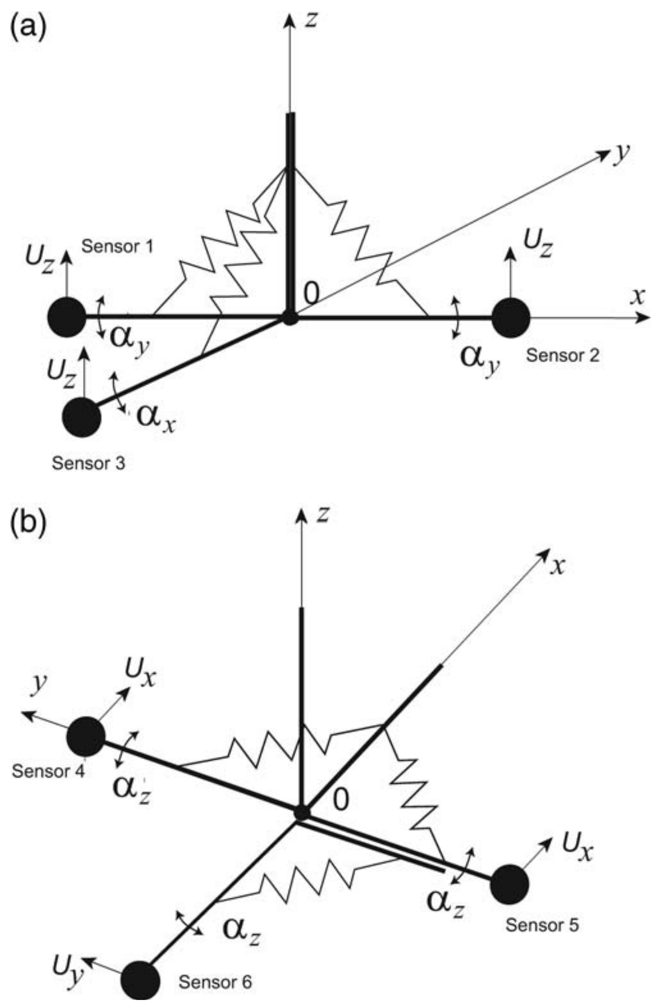

Figure 3. Schematic configuration of the six-pendulum system for measuring translational motion and rotation.

Theoretically, the previously described six-pendulum system allows separation of the rotational and translational motions. Practical realization of this system is challenging and requires large dimensions of the measuring system for increasing the resolution.

Increasing Sensitivity to Rotations

As was shown previously the biggest problem in measuring rotations using a two-pendulum system is the relatively low level of rotation signal. To increase the sensitivity of the system to rotations α_z'' around the vertical axis (torsion), we can put sensor 1 at a distance S_1 and sensor 2 at a distance S_2 from the center of rotation 0 (Fig. 4). Assuming the center of rotation at the same place 0, sensitivity to rotations will increase and

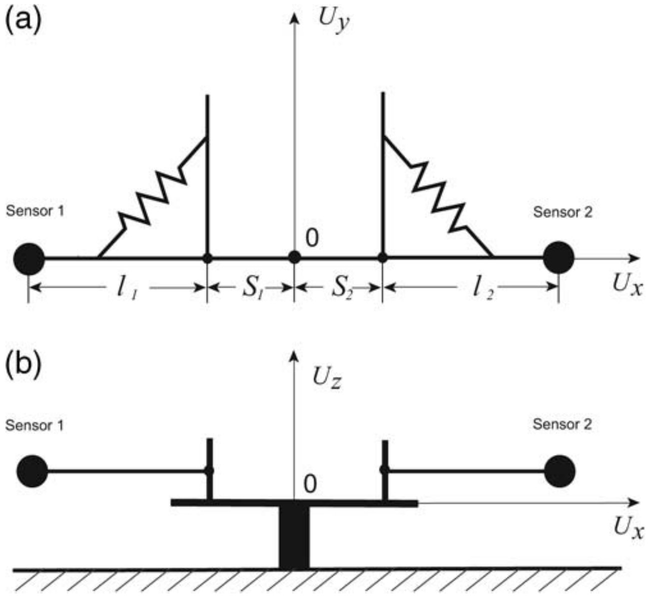

Figure 4. Increasing sensitivity to rotations around the vertical axis: (a) plane view from the top along the vertical z axis and (b) prospective view along the horizontal y axis.

$$(\varphi_2''-\varphi_1'') + (2\omega_2 D_2 \varphi_2' - 2\omega_1 D_1 \varphi_1') + (\omega_2^2 \varphi_2 - \omega_1^2 \varphi_1) = (l_1 + S_1 + l_2 + S_2)\alpha_z''. \quad (12)$$

Accordingly, mounting two pendulums at a certain distance from the center of rotation will increase sensitivity to rotation by a factor of k:

$$k = \frac{S_1 + S_2 + l_1 + l_2}{l_1 + l_2}.$$

This approach allows increasing sensitivity of pendulum systems to rotations by increasing the dimension of the system.

Laboratory Testing and Field Measurements

In 1985–1991 a number of attempts were made at the Institute of the Physics of the Earth in Moscow, Russia, to create two-pendulum systems for separate measurements of translational motion and rotation (Graizer *et al.*, 1989; Graizer, 1991). We built two-pendulum systems based on the three types of commercially available sensors at this time in the former Soviet Union. Two of the systems were based on a combination of the two accelerometers (ASZ—a regular accelerometer and APT—a piezo-accelerometer), and the third one was based on a combination of the two seismometers (SM-3—a 2 sec period seismometer). The systems were tested at the Institute of the Physics of the Earth using a specially designed relatively basic shake table shown in Figure 5 (Graizer *et al.*, 1989), allowing direct registration of displacement and tilt. Comparison of the angles computed using those two-pendulum systems with the directly registered ones has shown that for the well-calibrated instruments, like SM-3, tilts were calculated with the error not exceeding 20%. It was up to 30% when piezo-accelerometers were used. We targeted measurements of relatively large amplitude rotations on the order of 10^{-4} rad and higher.

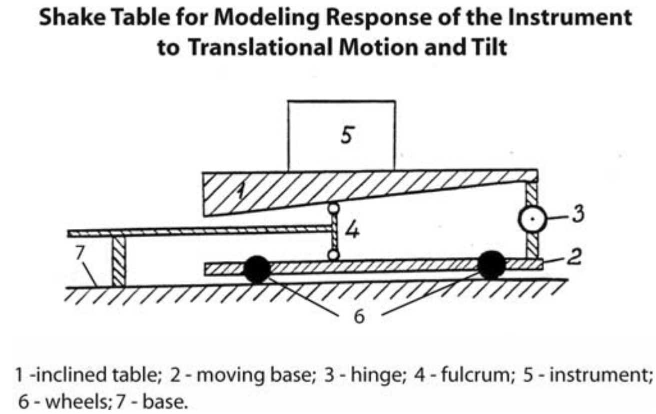

Figure 5. Shake table for testing response of the instrument to translational motion and tilt (modified from Graizer *et al.*, 1989).

A registration system similar to that described previously consisting of two pairs of accelerometers was built to record large ground motions. One pair of accelerometers was sensitive to vertical ground motion and another one to horizontal ground motion. It was based on commercially made Soviet accelerometers ASZ (similar in parameters to the American SMA-1). The system was first tested at the Institute of the Physics of the Earth using the shake table shown in Figure 5 (Graizer *et al.*, 1989). It was later applied to measuring ground motion in the near field of the two underground nuclear explosions. Explosions of different power (m_b of 4.5 and 4.4, respectively) were recorded at the same station at the hypocentral distance of less than 1 km (at reduced distances of 14.3 and 18.6 $\mathrm{m/kg^{1/3}}$).

The results of separate determination of displacements and tilts are shown in Figure 6. The maximum amplitude of displacement reached 14 mm at the vertical component, and tilt reached 3.7×10^{-3} rad (0.21°). The maximum tilt during the second less powerful explosion reached 9.2×10^{-4} rad (0.053°). The similarity in the shapes of tilts and horizontal displacements for both explosions gives additional confidence in the results. The ratio of the amplitudes of tilt motions between the two explosions is ~4, and the amplitudes of displacements differ by 2.5 times. It demonstrates more rapid decay of tilt with distance to the source and is consistent with the theory of an explosion. Residual tilts reached 1.5×10^{-3} and 2.4×10^{-4} rad for the first and second explosions, correspondingly. Those results are not contradicting published data. For example, amplitudes of relatively slow tilts measured in lakes at distances of a few kilometers from the epicenter of the CANNIKIN underground explosion were on the order of 10^{-5} to 10^{-4} rad (Dickey *et al.*, 1972).

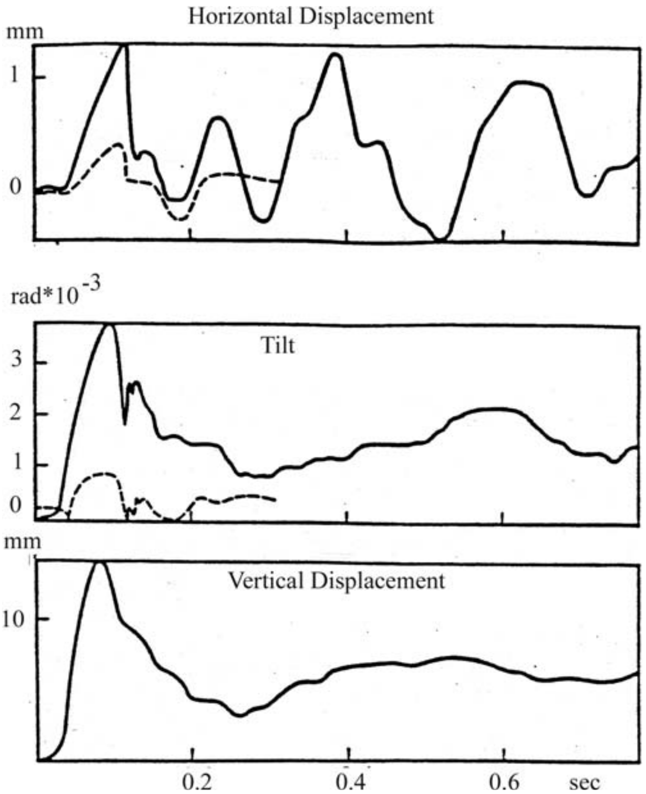

Figure 6. Motions in the near field of the two underground explosions: horizontal displacement, tilt, and vertical displacement (modified from Graizer *et al.*, 1989).

Numerical Tests on Recovering Rotation from a Two-Pendulum System

In addition to shake-table tests and field experiments, I performed numerical testing to study the limitations of using two pendulums for measuring rotation. Numerical testing is an important tool allowing the study of the separate influence of different types of errors on the proposed method. As a test signal I used the data from the 1994 M_w 6.7 Northridge earthquake recorded at the Pacoima Dam station. Ground-motion acceleration at this station reached 1.8g at one of the horizontal components with very significant tilting of more than 3°. It was concluded that such a large tilt was actually a local site effect induced by strong ground shaking and not a source-generated phenomenon. In Graizer (2006a) I described the way to extract the rotational component from this record.

As a first step I modeled the response of a hypothetical two-pendulum instrument similar to that shown in Figure 4. Both pendulums are oriented to record the vertical component of ground motion with a total length of the pendulum arm $(S + l) = 100$ cm. Figure 7 demonstrates the input translational (Fig. 7a) and rotational (Fig. 7b) motions, response of pendulums 1 and 2 (Fig. 7c), and the calculation of rotation (Fig. 7d). As can be seen from the Figure 7c, the difference between the outputs of the two pendulums (recording acceleration plus angular acceleration multiplied by $[S + l]$) is not large. As expected, the rotational signal calculated from the difference of the two ideal signals is the same as the original rotation.

Let us assume that the pendulums are not exactly same (or are not calibrated precisely). This will result in the translational acceleration signal leaking into the calculated rotational signal. The first example demonstrates the case of 1% error (the difference between the two pendulums), and the second example is 10%. As can be seen from Figure 7d, the error of 1% does not produce significant error in the final rotational signal. On the other side, the initial error of 10% produces about 25% error in the determination of the rotational signal (Table 1). Clearly, in the case of such high amplitudes of rotations relative to translational motion, recovery of the rotational signal can be achieved. The previously described numerical test is probably not very representative because it corresponds to one of the most dramatic cases of extremely high amplitude rotation.

Let us consider the more realistic case of 4 times lower maximum vertical acceleration of about 0.33g (Fig. 8a) and 10 times lower tilting with a maximum on the order of 10^{-3} rad (Fig. 8b). In this case the difference between the records of the two pendulums is smaller (Fig. 8c), and as a result of inaccurate calibration (or leaking of the acceleration signal into the rotational component) the final results are calculated with much higher errors than in the previous example (Fig. 8d and Table 1).

The performed numeric testing demonstrates the limitations of using pendulums for measuring rotations. It requires installation of pendulums at large distances from each other and a high level of calibration. Our estimates demonstrated that using a combination of two pendulums for measuring rotations may be limited for recording relatively large amplitudes of rotations on the order of 10^{-4} and higher for a system of about 100 cm in size. Summarizing the results of numerical testing, one can conclude that using a combination of pendulums to record rotations is a very complicated task requiring high-precision instrumentation and may be limited for recording relatively large amplitudes of rotations.

Discussion and Conclusions

Measuring rotations of the ground and structures during earthquake shaking is not part of common strong-motion measurement practice, and there are only a few measurements (mostly estimates) of rotations during strong ground shaking. We considered the modified classical approach to rotation measurements by using pairs of pendulums. The registration system based on these principles was built and tested at the Institute of the Physics of the Earth (Moscow, Russia) using a specially designed basic shake table and later successfully used to record translational motion and tilts in the vicinity of two large underground explosions, with maximum tilts reaching 3.7×10^{-3} and 9.2×10^{-4} rad, respectively.

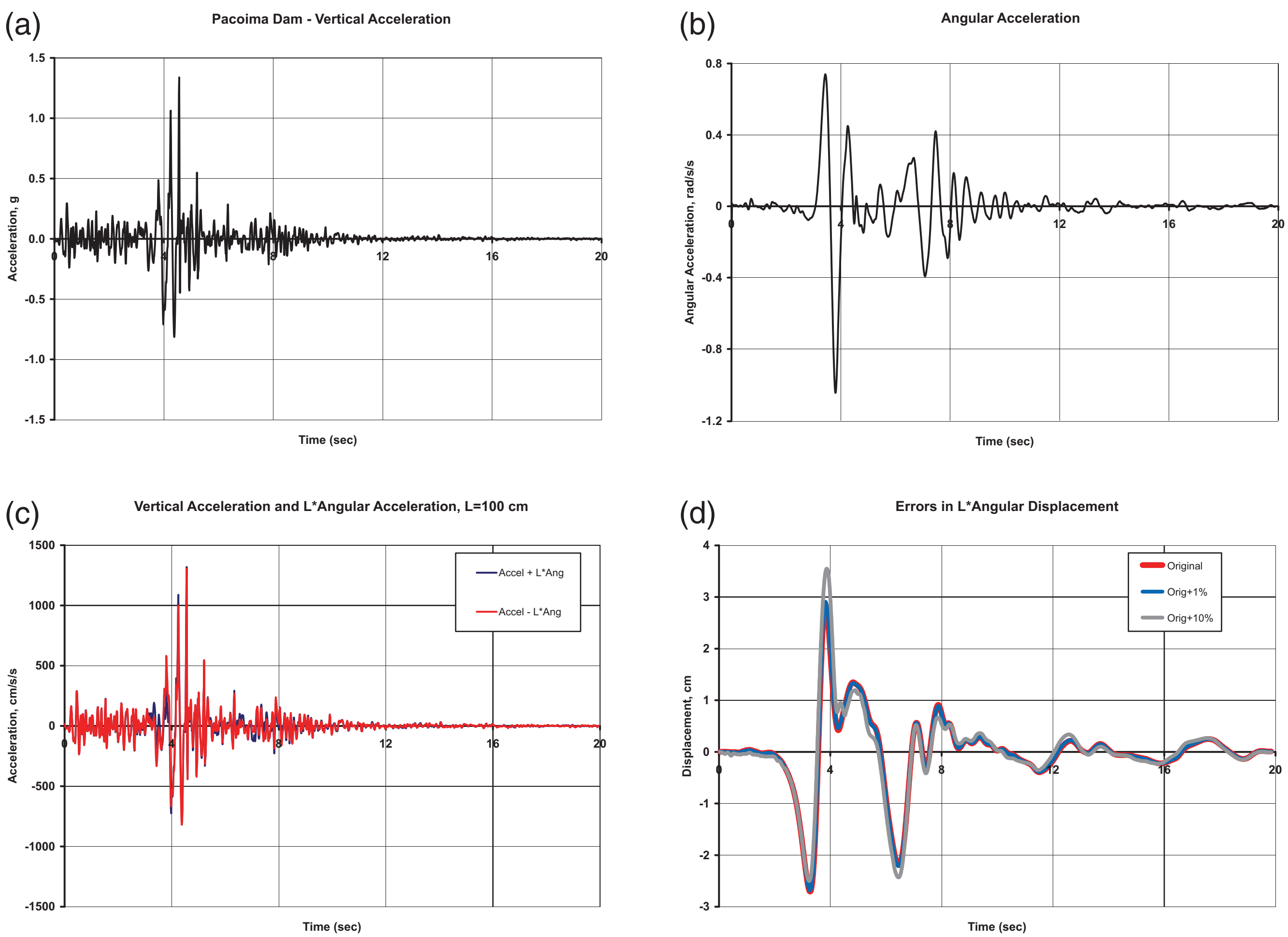

Figure 7. Numerical testing of the recovery of rotational motions from the response of the two-pendulum system: (a) vertical acceleration recorded at the Pacoima Dam Upper Left Abutment station during the Northridge earthquake, (b) angular acceleration recovered from the record (Graizer, 2006a), (c) responses of the two pendulums to the combination of translational and rotational motion, and (d) recovery of angular displacement from ideal signals and signals contaminated by noise.

Numerical testing was performed to assess the limitations of using a combination of pendulums for measuring rotations. Testing shows that it requires installation of pendulums at a large distance from each other (and correspondingly increasing the size of the instrument) and a high level of calibration. Based on modeling errors in calibration, it was estimated that the lowest level of rotational signal that can be reliably recovered from the two-pendulum system of about 100 cm size is about 10^{-4} rad, corresponding to extremely high amplitudes of rotation. Summarizing the results of numerical testing, one can conclude that using a combination of pendulums to record rotations is a very complicated task requiring high-precision instrumentation and may be limited for recording relatively large amplitudes of rotations.

Table 1
Effect of Errors in Pendulum Calibration on Errors in Recovery of Rotations

Calibration Error	Recovery Error for $D_{max}/\alpha_{max} = 384$ cm	Recovery Error for $D_{max}/\alpha_{max} = 960$ cm
1%	2.4%	5.8%
5%	12.3%	32%
10%	25%	75%

In recent years the situation has begun to change. Researchers are realizing the necessity of recording translational and rotational earthquake motions simultaneously. Recent technological advances provide new opportunities for rotation measurements because some developments previously available only for defense industries have become cheaper and more widely available. The six-component strong-motion measuring systems that include three translational and three rotational sensors should bring new insights into earthquake engineering studies. A two- or a six-pendulum system similar to that described previously can possibly be used for measuring rotations during very strong ground shaking, including measurements on structures.

Data and Resources

All of the data used in this study were collected by me or in collaboration with my previous colleagues from the Insti-

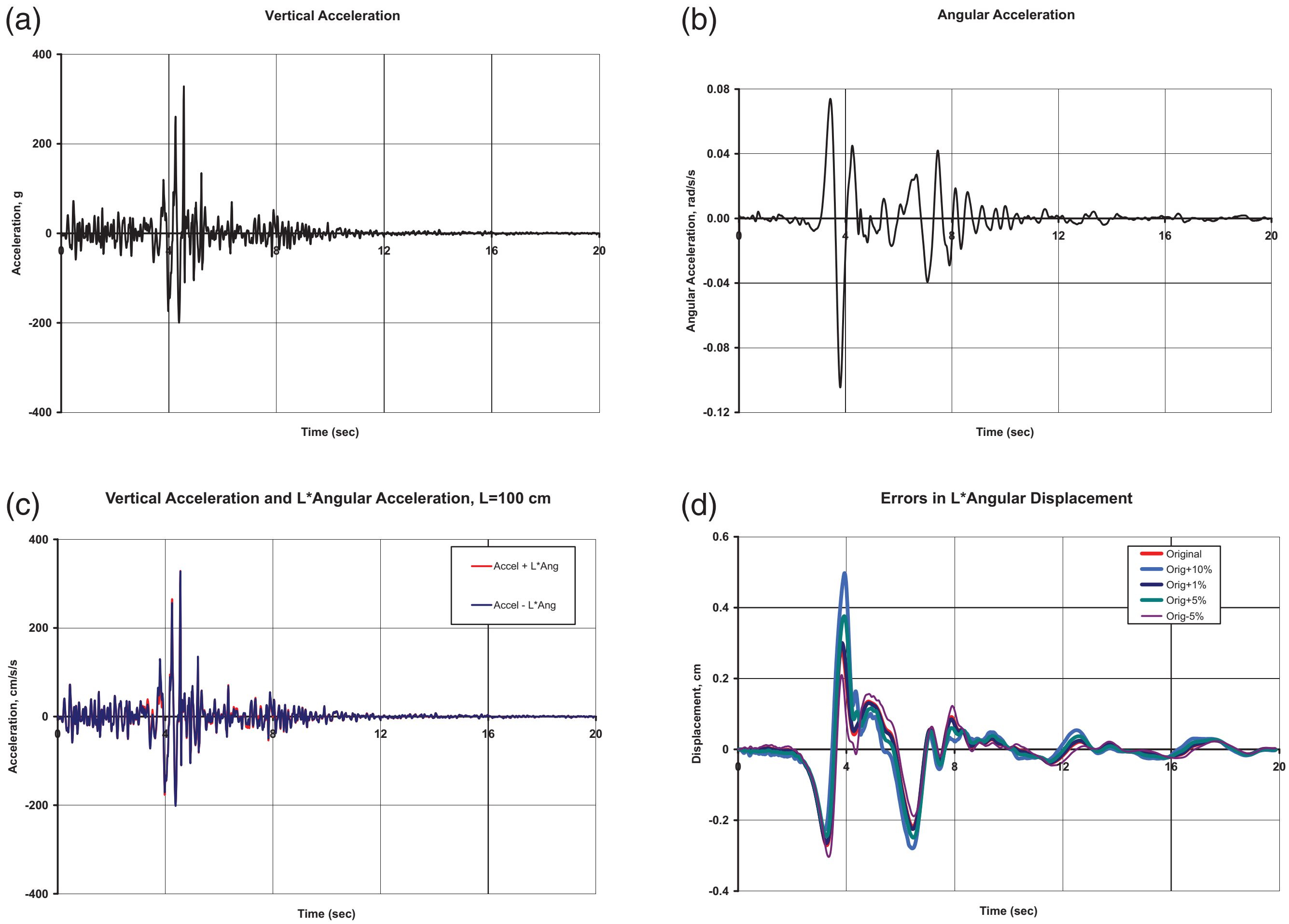

Figure 8. Numerical testing of the recovery of rotational motions from the response of the two-pendulum system: (a) vertical acceleration, (b) angular acceleration, (c) responses of the two pendulums to the combination of translational and rotational motion, and (d) recovery of angular displacement from ideal signals and signals contaminated by noise.

tute of the Physics of the Earth in Moscow, Russia (appropriate references included).

Acknowledgments

I am grateful to Oleg Kuznetsov and Djamil Sultanov from the Institute of the Physics of the Earth in the former Soviet Union for their participation in studies of rotational effects. I appreciate very much the critical comments of the two anonymous reviewers that helped significantly improve the quality of the manuscript. Any opinions, findings, and conclusions expressed in this material are those of the author solely and do not necessarily reflect the views of the U.S. Nuclear Regulatory Commission.

References

Aki, K., and P. G. Richards (1980). *Quantitative Seismology*, Vol. **I** and **II**, Freeman, San Francisco.

Bradner, H., and M. Reichle (1973). Some methods for determining acceleration and tilt by use of pendulums and accelerometers, *Bull. Seismol. Soc. Am.* **63,** 1–7.

Dickey, D. D., F. A. McKeown, and R. C. Bucknam (1972). Preliminary results of ground deformation measurements near the CANNIKIN explosion, *Bull. Seismol. Soc. Am.* **62,** 1505–1518.

Evans, J. R., and International Working Group on Rotational Seismology (2009). Suggested notation conventions for rotational seismology, *Bull. Seismol. Soc. Am.* **99,** no. 2B, 1073–1075.

Farrell, W. E. (1969). A gyroscopic seismometer: measurements during the Borrego earthquake, *Bull. Seismol. Soc. Am.* **59,** 1239–1245.

Golitzin, B. B. (1912). *Lectures on Seismometry*, Russian Academy of Science, St. Petersburg (in Russian).

Graizer, V. M. (1979). Determination of the true displacement of the ground from strong-motion recordings, *Izv. USSR Acad. Sci., Phys. Solid Earth* **15,** no. 12, 875–885.

Graizer, V. M. (1989). Bearing on the problem of inertial seismometry, *Izv. USSR Acad. Sci., Phys. Solid Earth* **25,** no. 1, 26–29.

Graizer, V. M. (1991). Inertial seismometry methods, *Izv. USSR Acad. Sci., Phys. Solid Earth* **27,** no. 1, 51–61.

Graizer, V. M. (2005). Effect of tilt on strong motion data processing, *Soil Dyn. Earthq. Eng.* **25,** 197–204.

Graizer, V. (2006a). Tilts in strong ground motion, *Bull. Seismol. Soc. Am.* **96,** 2090–2106.

Graizer, V. M. (2006b). Equation of pendulum motion including rotations and its implications to the strong-ground motion, in *Earthquake Source Asymmetry, Structural Media, and Rotation Effects*, R. Teisseyre, M. Takeo, and E. Majewski (Editors), Springer-Verlag, Berlin, 471–485.

Graizer, V., and E. Kalkan (2008). Response of pendulums to complex input ground motion, *Soil Dyn. Earthq. Eng.* **28,** 621–631.

Graizer, V. M., O. P. Kuznetsov, N. I. Nedoshivin, and D. D. Sultanov (1989). Bearing on ground tilt measurements near the explosion source, *Rep. USSR Acad. Sci.* **305,** no. 2, 314–318 (in Russian).

Huang, B.-S. (2003). Ground rotational motions of the 1999 Chi-Chi, Taiwan earthquake as inferred from dense array observations, *Geophys. Res. Lett.* **30,** no. 6, 1307–1310.

Kharin, D. A., and L. I. Simonov (1969). VBPP seismometer for separate registration of translational motion and rotations, in *Seism. Instrum.* **5,** 51–66 (in Russian).

Niazi, M. (1986). Inferred displacements, velocities and rotations of a long rigid foundation located at El Centro differential array site during the 1979 Imperial Valley, California earthquake, *Earthq. Eng. Struct. Dyn.* **14,** 531–542.

Nigbor, R. L. (1994). Six-degree-of-freedom ground-motion measurements, *Bull. Seismol. Soc. Am.* **84,** 1665–1669.

Oliveira, C. S., and B. A. Bolt (1989). Rotational components of surface strong ground motion, *Earthq. Eng. Struct. Dyn.* **18,** 517–526.

Savarensky, E. F., and D. P. Kirnos (1955). *An introduction to seismology and seismometry*, State Press of Techinical–Theoretical Literature (GITTL), Moscow, USSR, 543 pp (in Russian).

Schreiber, K. U., G. E. Stedman, H. Igel, and A. Flaws (2006). Ring laser gyroscopes as rotation sensors for seismic wave studies, in *Earthquake Source Asymmetry, Structural Media and Rotation Effects* R. Teisseyre, M. Takeo, and E. Majewski (Editors), Springer-Verlag, Berlin, 377–390.

Spudich, P., and J. B. Fletcher (2008). Observation and prediction of dynamic ground strains, tilts and torsions caused by the M_w 6.0 2004 Parkfield, California, earthquake and aftershocks derived from UPSAR array observations, *Bull. Seismol. Soc. Am.* **98,** 1898–1914.

Takeo, M. (1998). Ground rotational motions recorded in near-source region of earthquakes, *Geophys. Res. Lett.* **25,** no. 6, 789–792.

Todorovska, M. I. (1998). Cross-axis sensitivity of accelerographs with pendulum like transducers—mathematical model and the inverse problem, *Earthq. Eng. Struct. Dyn.* **27,** 1031–1051.

Todorovska, M. I., and M. Trifunac (2007). Instrument correction for 6 DOF seismic sensors (Abstract), in *First International Workshop on Rotational Seismology and Engineering Applications*, 18–19 September 2007, U.S. Geological Survey, Menlo Park, California.

Trifunac, M. D. (1971). Zero baseline correction of strong-motion accelerograms, *Bull. Seismol. Soc. Am.* **61,** 1201–1211.

Trifunac, M. D., and M. I. Todorovska (2001). A note on the usable dynamic range of accelerographs recording translation, *Soil Dyn. Earthq. Eng.* **21,** 275–286.

Wiszniowski, J. (2006). Rotation and twist motion recording-couple pendulum and rigid seismometers system, in *Earthquake Source Asymmetry, Structural Media, and Rotation Effects*, R. Teisseyre, M. Takeo, and E. Majewski (Editors), Springer-Verlag, Berlin, 451–468.

Zahradnik, J., and A. Plesinger (2005). Long-period pulses in broadband records of near earthquakes, *Bull. Seismol. Soc. Am.* **95,** 1928–1939.

U.S. Nuclear Regulatory Commission
Mail Stop T-7E18, Washington, DC, 20555-0001
Vladimir.Graizer@nrc.gov
graizer@comcast.net

Manuscript received 24 June 2008